# A Metric for Evaluating the 3D Reconstruction of the Length of Artificially Generated Electric Arcs 

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#### Abstract

Studies with artificially generated electrical arcs includes the variation of the length of the arc as an important information. In this paper, it is done a 3D reconstruction (by 3D-snake) followed by the measurement of the length of the arc in evolution based on stereo pairs of images. The accuracy of such measurement depends strongly on the halting of the iterative relaxation process that adjust the 3D-snake and the key aspect to make such decision is detecting when each projection of the 3D-snake matches special features in the pair of images. For this sake, this paper proposes an strategy based on the knowledge of projection matrices of the cameras and tables named here as Maps of Distances (MDs) which are obtained by image processing.


Keywords- Electric Arc; Computer Vision; Image Processing.

## I. Introduction

The major part of faults in Brazilian electrical system are single-phase and transitory what makes possible to apply an electrotechnical maneuver called Single-Phase Auto-Reclosure (SPAR) switching to normalize the system. Provided that such maneuver consists in opening just the faulty phase during a short time, minimizing the disturbance in the electrical system, the National Electrical Energy Regulatory Agency (ANEEL) has recommended that new transmission lines must be capable to carry out such switching, since then a mathematical model of the electric arcs has been pursued and the elongation of the length of the electrical arc is considered an important parameter for this model [1], [2], [3]. The research in this field has been carried out and/or supported mainly by COPPE/UFRJ, FURNAS (Brazilian utility) ANEEL 2001/2002, UNICAMP (University of Campinas), CEPEL (Brazilian High Power Laboratory) and, more recently, UDESC (Santa Catarina State University).

This work presents an unseen application of a Computer Vision approach founded on a 3D-snake which models the medial axis of an electrical arc in spatial evolution. The 3D-snake model is based on the same physical-inspired principles of the precursory twodimensional case [4] i.e. the dynamic of this 3D active contour relies on a functional of energy whose optimization looks for a minimum of total energy to the system and, at the same time, determines the spatial adjustment of the 3D contour. The optimization process is regulated by external forces that guide the adjustment of the 3D active contour and internal forces that guarantee its continuity (smoothness). The stereo images are the primary sources of external forces and each pair of them is processed generating the respective pair of repositories, named Vector Maps (VMs), used for carrying the external image forces.

The minimum energy of the system is found when the projections of the 3D-snake are consistent with the current stereo pair of images, at this time the 3D adjustment of the active contour is completed for this pair and a measurement of the 3D-snake is done determining the demanded length of the medial axis of the electrical arc captured by the current pair of images.

Since the optimization is implemented on an iterative procedure, it is necessary a criteria to identify when this repetitive process must be halted. Such criteria must detect the best (or sufficient good) 3D reconstruction of the electrical arc. For this sake, this paper proposes an strategy based on the knowledge of the projective matrices and tables so called Maps of Distances (MDs).

The remainder of this paper is organized as follows. In the next Section (II) the functional of energy is described. In Section (III) it is made a description of the iterative optimization procedure and the criteria used for halting it. In Section (IV), the experiments developed for testing the approach is discussed and conclusions are discussed in Section (V).

## II. THE FUNCTIONAL OF ENERGY

In this paper the 3D-snake is spatially represented by a B-spline with a set of control points V and basis functions matrix N . The 3D-snake adjustment is described by the functional of energy shown in (1), where $\mathrm{E}(\mathrm{Q})$ defines the total energy of the system, Q corresponds to the points generated by the B-spline, $\mathrm{E}_{\mathrm{int}}$ represents the internal energy which preserves smoothness and $\mathrm{E}_{\text {ext }}$ corresponds to the external energy. The external forces attract the 3D-snake pushing it in a direction that makes its projections match the features of interest captured in a stereo pair of images [5].

The balance of energies must be optimized looking for a minimum of total energy to the system and this procedure is done by the relaxation shown in (2) and (3) under constraints of internal and external forces [6].

The characteristics of the first pair of images of the arc facilitate to recover a set of 3D points and least squares are applied to them for determination of the first set of control points [5]. The second set of control points is obtained by (2) and (3) over the first set and continuing in same way a new set $\mathrm{V}_{\mathrm{t}}$ is calculated based on the old set $\mathrm{V}_{\mathrm{t}-1}$., from one set to the other the 3Dsnake is moved.

$$
\begin{align*}
& \mathrm{E}(\mathrm{Q})=\int_{0}^{1} \mathrm{E}_{\text {int }}(\mathrm{Q})+\mathrm{E}_{\mathrm{ext}}(\mathrm{Q}) \mathrm{ds}  \tag{1}\\
& \mathrm{~V}_{\mathrm{t}}=(\mathrm{H}+\gamma \mathrm{I})^{-1}\left(\mathrm{~F}_{\mathrm{ext}-3 \mathrm{D}}^{\mathrm{cp}}\right)  \tag{2}\\
& \mathrm{F}_{\mathrm{ext}-3 \mathrm{D}}^{\mathrm{cp}}=\gamma \mathrm{V}_{\mathrm{t}-1}-g\left(Q_{t-1}\right) \tag{3}
\end{align*}
$$

## A. Composing the external force in the space of control points

In (3), $F_{\text {ext-3D }}^{c p}$ corresponds to the vector of 3D external forces in the space of control points and capable to act upon them for guiding the 3D-snake to adjust its projections in accordance to the current pair of images. To obtain such a vector, firstly is necessary to compose the ordinary vector of 3D external forces $\left(F_{\text {ext-3D }}\right)$ from the pair of current VMs (a Vector Map is a bidimensional matrix with the horizontal and vertical components of gradient vectors) and transform these forces to space of control points giving $F_{e x t-3 D}^{c p}$ in demand. This process is described next.

Taking advantage of the bright of the discharge (Fig. 1 ), it is applied a threshold for segmenting it from the original images followed by morphological operations (thinning and pruning). These procedures emphasize the feature of interest in each frame, i.e., the medial-axes which characterizes the longitudinal length of the electrical discharge. Next, the distance transform is used in order to generate a matrix whose cells represent a pixel in the respective frame and store the distance from this pixel to the medial-axes obtained. Then, this MD is operated by the gradient operator originating the bidimensional matrix named in this paper as Vector Map (VM).


Figure 1. Image of a recent generated electrical arc and its histogram. The brightest pixels corresponds to the discharge.

Considering the Fig. 2, the point E has the points $\mathrm{E}_{1}$ and $E_{2}$ as its projections upon the pair of current VMs. Assuming that $\mathrm{F}_{\text {ext-2D }}(\mathrm{E} 1)$ is the image force associated to $\mathrm{E}_{1}$ by the $\mathrm{VM}_{1}$ and the same for $\mathrm{F}_{\text {ext-2D }}(\mathrm{E} 2)$ about the $\mathrm{VM}_{2}$, the points $\mathrm{F}_{1}$ and $\mathrm{F}_{2}$ are homologous points and the retroprojection [7] of them gives the point F .

The vector $E F$ corresponds to the wanted $\mathrm{F}_{\text {ext-3D }}$ which is transformed to the space of control points by (4) [6] and the vector $\operatorname{Vg}(Q)$ corresponds to the 3D external force that acts upon the control points moving the 3D-snake in order to adapt its projections over the current pair of images. The vector $F_{e x t-3 D}^{c p}$ represents the set of all these 3D external forces in the space of control points.
$\mathrm{g}(\mathrm{Q}) \cong \mathrm{N}^{\mathrm{T}} \mathrm{F}_{\mathrm{ext}-3 \mathrm{D}}(\mathrm{Q})$

The matrix H represents the stiffness of the 3D-snake model. The parameters $\alpha$ and $\beta$ which control the flexibility are embedded in H , so H acts on the control points determining the flexibility of the 3D-snake. The formulas for determination of matrix H can be obtained in [6] or [5].

## III. Tracking the Electrical Arc

Being a model for the actual media-axes of the discharge, the 3D-snake should track it in stages associated to each pair of images, the lengths can be estimated by measuring the B-spline contained in the 3D-snake model at the end of each stage. The points of the B-spline are generated by DeBoor's algorithm [8] and the sum of the Euclidean distance between two adjacent points gives the measure of the length.


Figure 2. The image forces Fext-2D represented in VMs are used for recovering the external force $\mathrm{F}_{\text {ext-3D }}$ which is converted by $\phi^{-1}$ [6] to the space of control points.


Figure 3. In A, each stereo pair of images corresponds to a stage of adjustement $\left(e_{i}\right)$ of the 3D-snake. In B, each stage is describe as an iterative process for minimizing the energy of the 3D-snake and measurement of the length of the arc.

In Fig. 3-A, each stage of adjustment $e_{i}$ is associated to one correspondent stereo pair of images that captured a snapshot of the arc in evolution. Besides, each stage also corresponds to a 3D-snake spatial evolution looking for the 3D configuration of the arc captured in the same stereo pair of images.
By its turn, each stage implements (4) e (5) in a iterative procedure (Fig. 3-B) looking for a minimum of total energy to the system when the projections of the 3Dsnake match the medial-axes extracted from the current pair of images.

The measurement on demand is done just after the ending of the iterative process when the optimal (or acceptable) 3D reconstruction of the 3D axes of the electrical arc has been gotten. Therefore, it is crucial to know the right time to halt the iterations, for this sake is proposed the use of the pairs of MDs (already obtained during the construction of the VMs) for calculating how much each projection of the 3D-snake is adjusted to the current pair of images and use such parameter for stopping the iterative process.

Each MD works like a function that gives the correspondent distance between a point (line, column) at image and the respective medial-axes. So that, in order to get the parameter to halt the iterations, the points of the 3D-snake are projected upon the current pair of MDs and considering $\mathrm{D}_{1}$ the set of distances obtained from $\mathrm{MD}_{1}$ and $\mathrm{D}_{2}$ the set obtained from $\mathrm{MD}_{2}$, these two sets are concatenated giving a big vector with all distances, the norm of such vector is calculated providing the measure of the adjustment wanted. Without loss of generality, the Fig. 4 depicts this strategy for one point $E$ which is projected over the pair of images and it was obtained $d_{1}$ and $\mathrm{d}_{2}$ based on its coordinates in images.

## IV. VALIDATING THE PROPOSAL

Some initial experiments were done with analytical functions evolving in 3D space, such curves imitate the
crescent waviness and elongation presented by the electrical arcs generated at CEPEL. An image acquisition system were simulated in order to capture the evolution of these curves and the 3D-snake were applied using the strategy described for halting the iterative relaxation process. In these cases the true length of the curves were knew and compared to the measures obtained by 3D-snake, the discrepancy found were at most $3 \%$, which means that the measures obtained by 3D-snake method deviates at most $3 \%$ of the actual ones. The results were considered acceptable and the approach was applied for electrical generated in the facilities of CEPEL during which a pair of cameras captured a sequence of image pairs of the electrical arcs.

The experiments with true electrical arcs presented in this paper used a pair simple handycams (Sony ${ }^{\text {T.M. }}$ model HDR-SR10) with low sample rate (approximately 30 fps ) and resolution of $720 \times 570$ pixels (after conversion from manufacturer video format to a popular usable format, such as AVI). The images are manually synchronized after extraction of frames sequence. The synchronization is based on identifying the first pair of frames where the discharge appears, and that can result in a frame match synchronization error which varies from 0 to 60 ms . The experiments were carried out in open environment using actual high voltage transmission towers.

projection of medial axes of the arc
projective transformation

Figure 4. Projecting one point of the 3D-snake upon the MDs gives a pair of distances d1 and d2.

## A. Automatic initialization

The electrical arc is generated along a fuse wire and it resembles a straight line linking two points (Fig. 1-A). At this initial moment the medial-axes of the discharge can be understood as a 3D vector whose extremities are the points $A$ and $B$. Fortunately, this vector $A B$ can be recovered by triangulation of two pairs of homologous points at the pair of images taken. This is not a hard problem considering that the projections of the arc look like straight lines too. So that, it is a simple task to detect the extremities of such projections and triangulate them by retroprojection [7] in order to get A and B and the equation of the vector AB . By such equation is generated a set of points that are approximated by a third order B-
spline ( $\mathrm{Q}_{0}$ in Fig. 3-A) and the first spatial representation of the 3D-snake and electrical arc at its initial instants.

## B. Camera Calibration

The approach discussed demands knowledge of the projection matrices. In the case of the analytical curves these matrices were determined because the cameras were simulated. On the other hand, for actual arcs, some points located at the tower (the experiments are implemented in true transmission lines at CEPEL facilities) had its 3D coordinates pre-determined and used for calibrating process in order to get the projection matrix. An image of these points is captured by each camera and used for identification of the correspondence between each 3D point and its respective projection, the projection equation system obtained is solved [9] and gives the wanted projection matrix.

## C. Results

There is no way to get the actual length of an electrical arc, however, according to [1],[2] and [3] the voltage between arc terminals per unit length is almost constant in a long arc such as the arcs analyzed here. Basically, it means that the effective first pseudo harmonic order arc voltage $\mathrm{V}_{\text {rms }}$ can be considered roughly proportional to the arc length, therefore, apart a scale factor, the curve representing the lengths of the arc via 3D reconstruction (3D-snake) has a profile similar to the curve $\mathrm{V}_{\mathrm{rms}}$ and this feature can be used for analysis.

Initial experiments have shown a similar profile between measurements based on $\mathrm{V}_{\text {rms }}$ and 3D-snake, as can be seen at Fig. 5, where both curves are plotted in the same graphic (axis (OY) presents length in meters and $\mathrm{V}_{\mathrm{rms}}$ in kilovolts). The lengths of the discharges measured by 3D-snake rise from four meters (the length of the insulator string where the arcs started) to four times this initial length which is a consistent behavior, since the wind and the convection heat force the plasma to elongate.

## V. CONCLUSIONS

Halting the relaxation is a crucial aspect with severe implications on the accuracy of the length measurements in 3D reconstruction of electrical arcs. The present paper describes an innovative computer vision application based on 3D-snake that uses Maps of Distances (MDs) for detecting the best time to halt the iterative relaxation process. Different experiments were carried out and not only the results of the experiments with analytical curves but also those with true artificially generated electrical arcs show the potential of the strategy proposed. The measurements obtained in the case of the analytical curves were considered acceptable in comparison with the true lengths already known. For the case of electrical arc, even though the true lengths were not known, the results are coherent with those ones obtained by application of the electrotechnical strategy, so that results show that the halting criteria based on MDs is reliable.

It is important to emphasize that, in the experiment with true electrical arcs, the image acquisition system used was far from ideal, besides, these are the very first
results of the experiments in the field of measurement of electrical arcs based on 3D-snake, so that improvements will be implemented in order to get better results.

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Figure 5. For the experiment number1906, plotted at the same graph: V1rms [kV] and length [m] measured by 3D-snake. The highlighted region at voltage curve represents the disturbance in electrical data at the beginning of the voltage measurement.

