Hilbertian Energy: a method for external energy calculation on radial active contours

Auzuir Ripardo de Alexandria
Departamento da Indústria / G5
Instituto Federal de Educação, Ciência e Tecnologia do Ceará (IFCE)
Fortaleza, Brasil
auzuir@ifce.edu.br

Paulo César Cortez, John Hebert da Silva Félix, Tarique da Silveira Cavalcante, Pedro Pedrosa Rebouças Filho, and José de Arimatéia Carvalho Silva Júnior
Dep. de Engenharia de Teleinformática (DETI/LESC)
Universidade Federal do Ceará (UFC)
Fortaleza, Brasil
{cortez, john, tarique, pedrosa, junior}@lesc.ufc.br

Abstract—Active contours method has been successfully applied to image segmentation even under low signal/noise ratio situations. These methods consist of total energy minimization, calculated by the sum of internal energy, which is a function of active contour geometry, and external energy which is function of pixel intensity of the image to be segmented. Traditionally, external energy is calculated by input image gradient. Among the methods of active contours, radial methods have lower computational complexity and have real time applications. In this work we propose the use of Hilbert Transform calculation for external energy in radial active contours. This external energy is named Hilbertian energy. Tests were performed synthetic images and show that Hilbertian energy is able to avoid the use of derivative and balloon force, performing segmentation properly.

Keywords- radial active contours; Hilbert transform; Hilbertian energy;external energy; image segmentation; Computer Vision.

I. INTRODUCTION

Computer Vision systems are able to perform automatic analysis in digital images. These systems have applications in many areas such as Robotics [1], Biomedical Engineering (medical images) [2] and Industry [3]. In a typical Computer Vision system, image segmentation's block is the most important one, because its success is primary to obtain quality results [4].

Among the techniques for image segmentation, the active contour method, well-known as snakes, is very used for border segmentation and real time applications. It consists in a curve (snake) deformation, adjusting it to the border of the present object in a digital image to be segmented. Contour deformation is performed through the lowest total energy search. This energy depends on snake geometry (internal energy) and image characteristics (external energy) [5]. Snake external energy is traditionally calculated as a gradient calculation of the input image.

Radial active contours were developed in order to decrease computational complexity of active contour methods and consequently real-time applications [6]. Energy calculation and its minimization are performed in one dimension (1D), what makes them faster [7].

The Hilbert transform [8] is well-known in Mathematics and Digital Signal Processing areas, having peculiar characteristics for signal border detection [9], [10].

This work proposes a method for external energy calculation in radial snakes named Hilbertian energy, based on Hilbert transform.

In section 2 it is performed a bibliographic review about active contour methods and snake energy equations. A review of the main works about radial snakes is presented in section 3. In section 4, it is performed an explanation about Hilbert transform and its peculiar behavior in border detection. In section 5, Hilbertian energy and the proposal of its use as external energy in radial snakes are presented. The method test methodology and its obtained results are presented in section 6. Discussions about these results are performed in section 7. At last, in section 8, this work is concluded, indicating future works to be produced.

II. TRADITIONAL ACTIVE CONTOUR METHOD

The traditional active contour method is based on variational methods. Its objective is to minimize a function that represents snake energy. The curve then evolves, so its energy decreases with each new iteration [5, 11-12]. Snakes model is the 2D parameterization of a geometric curve

$$\begin{cases} [0,1] & \rightarrow & \mathbb{R}^2 \\ s & \rightarrow & c(s) = (x(s), y(s)). \end{cases} \tag{1}$$

The model is called deformable because it is described by an energy function *E(s)* which varies in

$$\begin{cases} \mathbb{R}^2 & \xrightarrow{E(s)} & \mathbb{R} \\ c & \rightarrow & \int_{[0,1]} [e_1|c'(s)|^2 + e_2|c''(s)|^2 \\ & & \quad + E_{ext}(c(s))]ds, \end{cases} \tag{2}$$

where apostrophes represent derivative operations and E_{ext} is the energy associated to external forces [5].

III. RADIAL SNAKES REVIEW

Buda et al. (1983) wrote one of the pioneering works about border detection through radial search [6]. However, Active Rays are one of the best known and most relevant works about radial snakes [7,13]. This technique is applied to object border tracking in real-time. Its idea is to define an origin point inside a contour and find characteristic points by searching along rays that diverge from a central origin m. For this purpose, traditional active contours equations were adapted. The contour *c(s)* can be defined as

$$\begin{cases} [0,1] & \rightarrow & \mathbb{R}^2 \\ s & \rightarrow & c(s) = c_m(\phi(s), \lambda(s)), \end{cases} \quad (3)$$

where *c(s)* is an active contour and $c_m(s)$ is a contour defined since origin *m*, in polar coordinates (ϕ, λ) [13]. The contour internal energy is calculated by equation

$$E_i(c_m(\phi)) = \alpha(\phi)\Big|\frac{d}{d\phi}\lambda(\phi)\Big|^2 + \beta(\phi)\Big|\frac{d^2}{d\phi^2}\lambda(\phi)\Big|^2, \quad (4)$$

where $\alpha(\phi)$ and $\beta(\phi)$ are real constants for a certain angle ϕ. The first term of equation 4 is continuity energy definition and the second one, the curvature energy for *Active Rays*. We perceive, through last equation that these energies can be calculated along a beam. Then, all calculations can be done using only one dimension (1D).

Another relevant work is the optimal radial active contours one [14]. It uses dynamic programming for contour energy optimization and object tracking is its main application. Total energy definition *(E)* of the contour is given by

$$E(r_m(\phi)) = \int_0^{2\pi} (E_i(r_m(\phi)) + E_e(r_m(\phi)))d\phi, \quad (5)$$

where $r_m(\phi)$ is the distance from origin *m* to a contour, considering an angle ϕ; E_i is the internal energy and E_e is the external one. The external energy E_e of the active contour is a function of image gradient that will be segmented

$$E_e(r_m(\phi)) = \alpha_e \cdot g(-\Big|\frac{d}{d\lambda}\rho_m(\phi,\lambda)\Big|^2)$$
$$= \alpha_e \cdot g(-(\rho_m(\phi,\lambda+1) - \rho_m(\phi,\lambda))^2), \quad (6)$$

where λ e ϕ are polar coordinates of an active contour control point (the actual node); *g* is a non-linear monotonically crescent function and ρ_m is the active ray.

The continuity energy E_i^{cont} for the *i-th* node of an active contour is calculated by the expression

$$E_i^{cont}(r_m(\phi)) = \alpha_i \cdot \Big|r_m(\phi_i) - r_m(\phi_{i-1})\Big|^2, \quad (7)$$

where α_i is a real constant.

Curvature energy E_i^{curv} is given by equation

$$E_i^{curv}(r_m(\phi)) = \beta_i \cdot \Big|((r_m(\phi_i) - r_m(\phi_{i-1})) - ((r_m(\phi_{i-1}) - r_m(\phi_{i-2})))\Big|^2, \quad (8)$$

where β_i is a real constant.

IV. THE HILBERT TRANSFORM

The Hilbert transform of a real function is defined by

$$\hat{f}(t) = \frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{f(\tau)}{t-\tau}d\tau, \quad (9)$$

where *P* denotes principal value of Cauchy, since there is a singularity in the integral for *t = 0* [15-16]. Another form to Hilbert transform definition is through

$$\hat{f}(t) = \frac{1}{\pi t} * f(t). \quad (10)$$

Fourier transform of *1/πt* function is given by *F(1/πt) = -j·sgn(ω)*, where signal function *sgn(ω)* is given by

$$sgn(\omega) = \begin{cases} +1, & \text{se } \omega > 0, \\ 0, & \text{se } \omega = 0, \text{ and} \\ -1, & \text{se } \omega < 0. \end{cases} \quad (11)$$

Hence Hilbert transform is generally implemented through the application of Fourier transform over the result of *-j·sgn(ω)* multiplicated by Fourier transform of *f(t)*, i. e., $\hat{f}(t) = F^{-1}(-j \cdot sgn(\omega) \cdot F(\omega))$.

One important feature of Hilbert transform is that it can be used as a good signal border detector, even at noise presence.

V. HILBERT TRANSFORM AS EXTERNAL ENERGY IN RADIAL SNAKES

The Hilbert transform has characteristics that make it very efficient when used as external energy in radial active contours. For this purpose, we need to apply the 1D Hilbert transform along active rays of the represented image using polar coordinates. We advice that Hilbert transform should be normalized between 0 and 1, taking its absolute value to be used as external energy. Therefore, we propose that external energy for radial active contours would be given by

$$E_{ext}(r,\theta) = |\hat{f}(r)|, \quad (12)$$

that can be normalized by the expression

$$E_{ext}^n(r,\theta) = 1 - \frac{|\hat{f}(r)|}{max(|\hat{f}(r_i)|)}, \quad (13)$$

for $r_i \in [0; r_{max}]$, where r_{max} is the greatest radius achieved by the beam in which the external energy is being calculated.

In this case we consider that the radial beam divergence point is inside the border of the image to be segmented, Hilbert transform tends to achieve negative values near from the border and positive ones beyond the descent border. Then, we can also use the following expression for external energy calculation, despising transform values outside object border, i.e.,

$$E_{ext}^n(r,\theta) = \begin{cases} 1 - \frac{|\hat{f}(r)|}{max(|\hat{f}(r_i)|)}, & \hat{f}(r) < 0, \\ 0, & \hat{f}(r) \geq 0. \end{cases} \quad (14)$$

The absolute value of normalized Hilbert transform of pixel intensity along a radial beam is called Hilbertian field or Hilbertian energy. This energy is used as external energy for radial snakes in this work.

In Fig. 1 we illustrate a typical case of Hilbertian energy behavior. The valley seen in Fig. 1(b) corresponds to the border of the object shown in Fig. 1(a). According to this referenced figure, the field intensity increases on the left of the mentioned valley. Then it is used as guide to minimum energy search of the snake, corresponding to the desired segmentation. Hence, we use equations 13 or 14 as external energy definition. A node (control point) of a radial snake, under the action of this energy, tends to move towards the image border.

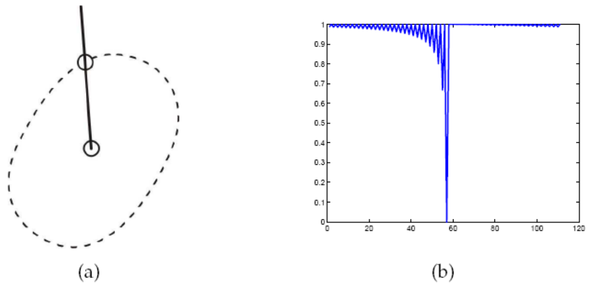

Figure 1. Hilbertian energy along a beam passing through a cavity - typical situation, a) beam passing through the object contour and b) Hilbertian energy along the beam.

VI. Tests And Obtained Results

Towards the efficiency verification of the Hilbertian energy's use as external energy in radial active contour methods, radial snakes method is implemented. Equations 7 and 8 are used to internal energies calculation, with $\alpha_i = \beta_i = 1$. The normalized Hilbertian energy, according to equation 14, is used as external energy. Greedy algorithm has been chosen to minimize the total energy of the radial snake, applying search space along the radium. This space is compounded by the neighborhood of 2 points: actual point, two points above and two bellow, being 5 ones in total.

Test images are presented in Fig. 2. These images are generated automatically, according to [17], as shown

$$r = a + b \cdot cos(m \cdot \theta + c), \quad (15)$$

where *a, b, c* and *m* are constants and *(r, θ)* are polar coordinates of each generated curve point. In this work, the constant values $a = 50$, $b = 10$, $c = 0$ and $m \in \{0, 2, 4, 8\}$ are used.

In Fig. 3, we can see the original point of the beams (Active Rays), localized in the center of each image test.

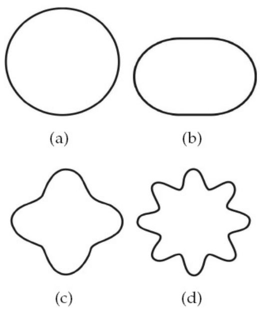

Figure 2. Test images generated for a) m = 0, b) m = 2, c) m = 4 e d) m = 8.

Dots represent Active Rays passing through border. 80 beams with radium of 110 pixels are used to the method application as shown in the mentioned figure. The initial snake is obtained by manual selection directly on the image, using pointer device (mouse).

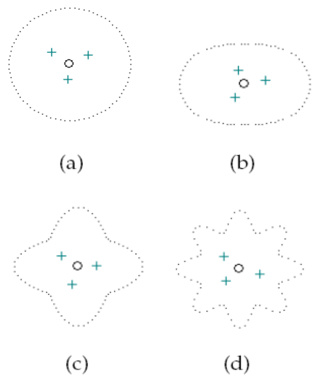

Figure 3. Beams, initial contour and divergence point for each test image.

In Fig. 4 are presented 23 iterations performed toward the convergence of the method and the final result for each test image. We can observe that the active contour follow the object border to be segmented properly.

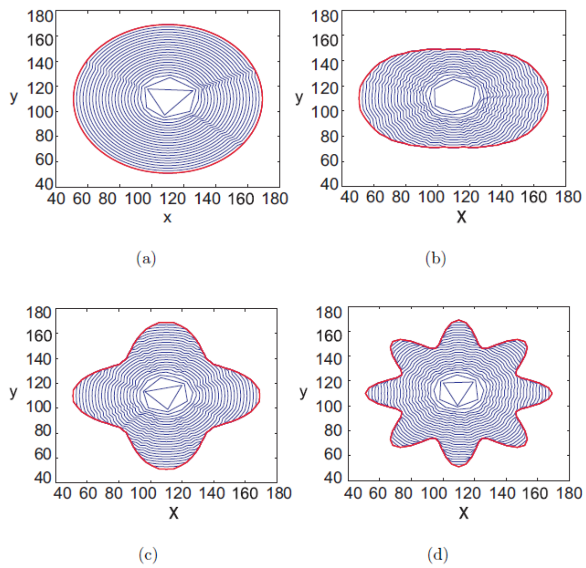

Figure 4. Convergence dynamics of radial snakes with Hilbertian energy (23 iterations).

Hilbertian energy for each test image is shown in Fig. 5. Considering, in this figure, that the clearer are the tones the greater is the intensity of Hilbertian energy, we can observe that when the point is going more distant from the border of the object to be segmented, the energy decay softly and its influence remains even faraway from the object border.

VII. Discussion

The use of Hilbert transform as external energy in radial active contour methods is very efficient regarding to the application of more classic definitions. One of the main advantages of this approach is that it can substitute the use of derivative as external energy. Derivative used in the works of [14] and [7] is defined only very close to borders, making its convergence more difficult when initial snake is far from these borders.

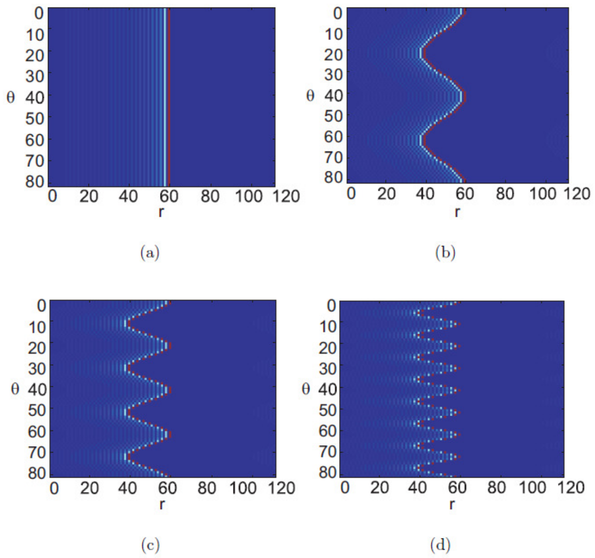

Figure 5. Beams, initial contour and divergence point for each test image.

Hilbert transform application also becomes advantageous when compared to [18], since it avoids the use of gaussian mask for spreading derivative influence along the beams. Besides, Hilbertian energy influence is present in a greater distance regarding to the proposed method in that mentioned reference, with less computational effort.

Hilbertian energy also permits a faraway snake initialization, compared to GVF method for radial snakes [19], [20]. Another advantage for this approach is that it is suitable for real-time application, since Hilbert transform 1D has a low computational cost.

The use of a search space greater or equal to 5 during energy minimization lets active contour dynamics immune to high and low local peaks of Hilbert transform. These local peaks can be observed in Fig. 1.

VIII. CONCLUSION

Hilbertian energy calculated through Hilbert transform application along a radial beam is successfully used as external energy in radial snakes. This method is evaluated using syntectic images. The obtained results are very efficient for those images. As future works, we can suggest a study about noise robustness of the method and an application in medical images as lung computerized tomography, cardiac magnetic resonance and echocardiography images.

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