Abstract— An image registration becomes more and more important in many biomedical imaging applications. Image registration is classically performed by optimizing a similarity criterion over a given spatial transformation space. Accurate definition of similarity measure is a key component in image registration. This study compares three measures of similarity: Sum of Squared Differences (SSD), Sum of Absolute Difference (SAD) and Maximum of Absolute Difference (MAD). They are used in same environment to adequate comparison. An unexpected result is that one of the simplest measures of similarity presents very good results in the process.

Keywords - Image Processing, Medical Image Registration; Metrics, Similarity Measures; Sum of Squared Differences.

I. INTRODUCTION

Medical images are increasingly being used within health care for diagnosis, planning treatment, guiding treatment and monitoring disease progression. Within medical research they are used to investigate disease processes and understand normal development and aging. In many of these studies, multiple images are acquired from patients at different times, and often with different acquisition modalities. Computerized approaches offer potential benefits, particularly by accurately aligning the information from different images, and providing tools for visualizing the combined images. Such process is named alignment or registration [4]. Tough, registration is the process of finding the spatial transform that maps points form one image to the corresponding point in another image.

Medical image registration has many clinical or practical and academic applications. For example, magnetic resonance (MR) images have good soft tissue discrimination for lesion identification, while CT images provides bone localization useful for surgical [13].

In the literature, many criteria have been used as basic for aligning two images. Generally these criteria can be landmark, segmentation or intensity-based registration [6] [7]. Landmark-based registration uses salient features selected by user. Segmentation-based methods attempt either rigidly or deformable transformations to align binary structure obtained by segmentation. The procedure involved in these methods minimizes the distance between physical points. On the other side, intensity-based methods operate directly on the image intensity. Such methods imply minimizing a cost function that measures the similarity between the image intensity of corresponding points between images.

Accurate definition of similarity measure is a key component in image registration [8]. Most commonly used intensity-based similarity measures, including Sum of Squared Differences (SSD), Correlation Coefficient (CC), Correlation Ratio (CR) and Mutual Information (MI), rely on the assumption of independence and stationarity of the intensities from pixel to pixel [1].

This paper reports results of comparison of three similarity measures: Sum of Squared Differences (SSD); Sum of Absolute Difference (SAD) and Maximum of Absolute Difference (MAD). The first one, Sum of Squared Differences (SSD), is the simplest similarity measures which are widely used for sets of Magnetic Resonance (MR) registration and available in Insight Toolkit (ITK).

ITK is a C++ object-oriented open-source system for image processing, segmentation and registration [10]. The Sum of Absolute Difference (SAD) and Maximum of Absolute Difference (MAD) are not available in ITK, we implemented this. The accuracy of the three measures is compared here using MR brain images. Registration quality is estimated by three evaluators: mean square error, peak signal to noise ratio and correlation coefficient.

II. SIMILARITY MEASUREMENTS

Registration can involve the calculation of an image transformation T achieved by optimization of some measure computed directly from intensity values of the
images [8]. An important distinction when using similarity measures is the modalities involved in the registration. Mutual information and normalized mutual information [12] are the most popular image similarity measures for registration of multi modal images. Cross-correlation, sum of squared differences and ratio image uniformity are commonly used for registration of images in the same modality.

A. Sum of squared differences

When the images to be registered are from the same type, the image intensity at corresponding points between the two images should be similar. One of the simplest similarity measures is the sum of squared intensity differences (SSD) between images which is minimized during registration [12]. Mathematically, this is defined by (1):

\[
SSD = \frac{1}{N} \sum_{x,y \in \Omega_{A,B}} |A(x,y) - B^T(x,y)|^2
\]

Where A is the fixed image intensity function, B represents the transformed image B, that is the B image under the current transformation on consideration, T.

The registration process involves recovering the spatial transformation T which maps x_B to x_A over the entire domain of interest, i.e., that maps from \( \Omega_A \) (domain) to \( \Omega_B \) with the overlapping portion of the domains. We refer \( \Omega_{A,B} \) to this overlap domain [8].

The optimal value of this measure of similarity is zero. Poor matches between images A and B result in large values. Such measure of similarity relies on the assumption that intensity representing the same homologous point must be the same in both images.

B. Sum of absolute differences

The sum of absolute differences, defined in (2), works by taking the absolute value of the difference between each pixel in the original image A and the corresponding pixels in the transformed image under for comparison B^T.

The SSD is very sensitive to a small number of pixels presenting very large intensity differences between images A and B. This could arise, for example, when contrast fluids are injected into the patient between the acquisition of images A to B, or if the images are acquired during an intervention and instruments are in different positions relative to the subject in the two acquisition. To reduce the impact generated by this sensitivity the sum of absolute difference (SAD) can be used [8]. This is defined by (2):

\[
SAD = \frac{1}{N} \sum_{x,y \in \Omega_{A,B}} |A(x,y) - B^T(x,y) |
\]

Smaller the values of SAD represent more similar images. This similarity measure requires that the two images are from the same modality [8].

C. Maximum of Absolute Difference

Equation (3) shows this measure. It is known as the Maximum of Absolute Difference (MAD) and uses the maximum absolute values of the differences between pixels in the original image A and the corresponding pixels in the image being translated B^T for comparison.

\[
MAD = \frac{1}{N} \max |A(x,y) - B^T(x,y) |
\]

III. METHODOLOGY

To perform the measurements we use the group of images shown in Fig. 1. Fig. 1(a) represents the original images A and the others the B^T images. Fig. 1(b) is the result of applied to the source image translations of 13mm in the x-axis and -10mm in the y-axis. In Fig. 1(c) these displacements are: 40mm and 50mm in the x-axis and the y-axis. While in Fig. 1(d) to 1(f), rotation is the rigid body movement applied: it was 15, 30 and 45 degrees clockwise, respectively.

To quantify the results of the registration process, image-subtraction is accomplished (see middle columns of Fig. 2 to Fig. 4), since the images are of the same modality registration. Moreover, to evaluate the quality of the registration, the mean square error (MSE) (4) the correlation coefficient (CC) (5) and the peak signal to noise ration (PSNR) (6) are computed [5]. These values are presented in Table I, II, III, IV and Table V.

\[
MSE = \frac{1}{NM} \sum_{i} \sum_{j} [I(i,j) - \hat{I}(i,j)]^2
\]

\[
CC = \frac{\sum_{i} \sum_{j} (I(i,j) - \bar{I})(\hat{I}(i,j) - \bar{\hat{I}})}{\sqrt{\sum_{i} \sum_{j} (I(i,j) - \bar{I})^2} \sqrt{\sum_{i} \sum_{j} (\hat{I}(i,j) - \bar{\hat{I}})^2}}
\]
The ITK toolkit is used for testing. In ITK, registration is performed within a framework of components that can be easily interchanged. This flexibility offers the possibility to combine and create a great variety of registration methods [10]. The registration method requires the following set of components: two input images (moving image and fixed image), a transformation, a metric (measure similarity), an interpolation and an optimizer. Here, we used translation and rotation, linear interpolation, an optimizer Regular Step Gradient Descent and three evaluators SSD, SAD and MAD. All components are present in the framework, except the SAD and MAD metrics. The SAD and MAD metric were implemented and added to the framework.

IV. RESULTS

Fig. 2 shows the result for the process registration for the images of Fig. 1. In Fig. 2, images (a), (d), (g), (j) and (n) show the result of resampling the moved image in order to map it onto the fixed image space using SSD. Fig. 2 images (b), (e), (h), (l) and (o) show the difference between the fixed image and the original moving image. That is, the difference before the registration process. Fig. 2 images (c), (f), (i), (m) and (p) show the difference between the fixed image and the transformed moving image. That is the difference after the registration is performed. Fig. 3 shows the same results, but using SAD. In Fig 4, MAD is used.

\[
PSNR = 10 \log \frac{255^2}{MSE} \tag{6}
\]
TABLE I. RESULTS TRANSFORMATION 13 MM X-AXIS AND -10 MM Y-AXIS

<table>
<thead>
<tr>
<th>Measure similarity</th>
<th>MSE</th>
<th>CC</th>
<th>PSNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSD</td>
<td>3296.8703</td>
<td>0.8526</td>
<td>12.9498</td>
</tr>
<tr>
<td>SAD</td>
<td>3347.5790</td>
<td>0.8492</td>
<td>12.8835</td>
</tr>
<tr>
<td>MAD</td>
<td>3292.2447</td>
<td>0.8531</td>
<td>12.9559</td>
</tr>
</tbody>
</table>

TABLE II. RESULTS TRANSFORMATION 40 MM X-AXIS AND -50 MM Y-AXIS

<table>
<thead>
<tr>
<th>Measure similarity</th>
<th>MSE</th>
<th>CC</th>
<th>PSNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSD</td>
<td>6682.6491</td>
<td>0.65163</td>
<td>9.88132</td>
</tr>
<tr>
<td>SAD</td>
<td>6631.0287</td>
<td>0.65424</td>
<td>9.91499</td>
</tr>
<tr>
<td>MAD</td>
<td>6682.0605</td>
<td>0.65162</td>
<td>9.88170</td>
</tr>
</tbody>
</table>

TABLE III. RESULTS ROTATION OF 15 DEGREES

<table>
<thead>
<tr>
<th>Measure similarity</th>
<th>MSE</th>
<th>CC</th>
<th>PSNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSD</td>
<td>4903.42050</td>
<td>0.74731</td>
<td>11.22581</td>
</tr>
<tr>
<td>SAD</td>
<td>4903.26013</td>
<td>0.74731</td>
<td>11.22595</td>
</tr>
<tr>
<td>MAD</td>
<td>4902.21396</td>
<td>0.74734</td>
<td>11.22688</td>
</tr>
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</table>

TABLE IV. RESULTS ROTATION OF 30 DEGREES

<table>
<thead>
<tr>
<th>Measure similarity</th>
<th>MSE</th>
<th>CC</th>
<th>PSNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSD</td>
<td>6336.62461</td>
<td>0.66692</td>
<td>10.11222</td>
</tr>
<tr>
<td>SAD</td>
<td>6336.38361</td>
<td>0.66694</td>
<td>10.11239</td>
</tr>
<tr>
<td>MAD</td>
<td>6335.65682</td>
<td>0.66697</td>
<td>10.11289</td>
</tr>
</tbody>
</table>

TABLE V. RESULTS ROTATION OF 45 DEGREES

<table>
<thead>
<tr>
<th>Measure similarity</th>
<th>MSE</th>
<th>CC</th>
<th>PSNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSD</td>
<td>6774.61082</td>
<td>0.644456</td>
<td>9.82196</td>
</tr>
<tr>
<td>SAD</td>
<td>6771.20823</td>
<td>0.644624</td>
<td>9.82414</td>
</tr>
<tr>
<td>MAD</td>
<td>6774.67421</td>
<td>0.644448</td>
<td>9.82192</td>
</tr>
</tbody>
</table>

Obtained results indicate that all of them (i.e. the SSD, the SAD and the MAD) achieved satisfactory results. Since a perfect registration would have produced a null difference image. In work after registration was produced a difference image near zero.

Note, also, that the process of registration images obtained better results on the value of the transformation (translation or rotation) applied in the original image was small.

Moreover, we can observe that the SSD, the SAD and the MAD produce similar results in terms of the MRE, the CC, and PSNR in process registration for each image of Fig. 1. This proves effectiveness of the three measure similarity for same modality registration.

V. CONCLUSION

In this paper, we report the comparison results of the three approaches: SSD, SAD and MAD. We evaluate the accuracy of the measures using MR brain images and compare the performance by similarity metrics. We have used MSE, CC, PSNR. Results shows that SSD measure is the simplest and efficient. The MAD shows better results in terms of MSE, CC, PSNR.

Although there are few studies related similarity measures SAD and MAD, our results show that for same modality images registration, they produce similar results to the SSD.

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