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Foundations of the Interval-Valued Image Model to Model and Process Uncertainty in Image Capture

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***Abstract*— The conditions under which images are captured are not always ideal or stable. This can lead to uncertainty regarding the measured pixel values, which in some cases is also related to the uncertainty regarding the spatial position of an object in an image. Such uncertainty can also be caused by technical limitations, in particular by the fact that in practice we always deal with a limited and finite number of pixels and pixel values, which leads to numerical and spatial approximations. In order to deal with this uncertainty we need appropriate image models, which also allow image processing without losing the information regarding the uncertainty. The ability to propagate the uncertainty information during image processing can be important in applications, since information on the level of uncertainty will influence an expert's attitude. In this paper we present the interval-valued image model for grayscale images. This model is based on interval-valued fuzzy set theory, and provides tools to build corresponding morphological operators. We will discuss the foundations of both the image model and the morphological theory, and point out the challenges for future research.**

Keywords: image processing, interval-valued, image acquisition, fuzzy logic, morphology.

I. The Problem: Uncertainty in Image Capture

Consider the situation in which you have different takes of the same scene, as illustrated in Figure 1: you have the cameraman image with a cloudy sky, a sunny sky, and a slight distortion, and consequently you are faced with numerical and/or spatial uncertainty regarding the measured pixel values. How do you deal with this uncertainty? One option could be to choose one of the takes, but how do you know which one is the best representation of the real situation? Another option could be to average the values of the three takes, but then you are working with a forced approximation of the real situation and your information on which pixel values are rather certain or uncertain (and to what degree) is lost in the process. Furthermore, the resulting image can become quite blurry.

This example illustrates that image capture can be accompanied with a large amount of uncertainty due to capture circumstances such as illumination and camera position. Furthermore it is clear that approximations w.r.t. the number of pixels and pixel values, which are forced upon us due to technical limitations, also contribute to this uncertainty. In order to take this uncertainty into account – in such a manner that it is incorporated in the image model and can be processed together with the image – new image models are required. In this paper we discuss the interval-valued image model which we recently introduced, and the morphological theory that is associated with it. This image model is based on interval-valued fuzzy set theory, which also provides the tools to build interval-valued morphological operators (Section III). Having established the new image model and its basic properties, the field is open for further theoretical and practical research (Section IV). First, we start with an introductory discussion of the considerations that lead to the interval-valued image model.

II. A Solution: the Interval-Valued Image Model

The value of a pixel in a grayscale image indicates the amount of black or white present at that specific location in the image. However, one always assumes that these values are *certain*, although in practice the measured values might be uncertain and merely indicate a *likely* value of the image at a specific position. The uncertainty regarding the value is an immediate fact if one takes into account that any device will round captured values up or down to the finite set of allowed values. The uncertainty grows if several takes of an image reveal different values for some pixels. This might be the case under identical recording circumstances, and will surely arise when these circumstances change (e.g., due to weather conditions). Not only the environmental circumstances can play a role. Indeed, pixels that belong to the edge of an object might slightly shift position in different takes (e.g., when the camera slightly shifts position). This could result in large differences in the measured value of a specific pixel, and consequently in a large uncertainty regarding the real value of that pixel, i.e., for that specific spatial position in the image.

For all these reasons, it can be useful not to work with grayscale *values* but with grayscale *intervals*, where the interval represents the set to which the actual grayscale value

Fig. 1. Different captures of the cameraman image: top = take with cloudy sky, middle = take with sunny sky, bottom = take with distortion. This example illustrates that the capture circumstances can cause uncertainty regarding the real pixel values. Also, all recorded values and positions are approximations of the real situation due to technical limitations.

belongs. Such an interval will be small for a pixel that belongs to a larger object in the image and that was captured under more or less identical circumstances, but can be large for a pixel that was captured under different circumstances or that belongs to the edge of a larger object in the image.

We illustrate this construction with an example. Starting from the three different takes (cloudy/sunny/distorted) shown in Figure 1, the lower and upper bounds of the grayscale interval for every pixel are obtained by selecting the lowest and highest value from the images for that pixel. The difference between the lower bound and upper bound images, which is shown in Figure 2, has a very nice interpretation: the higher the difference for a certain pixel (i.e., the higher the width of the corresponding interval, and the brighter the pixel in the corresponding image), the higher the uncertainty regarding that pixel. This example illustrates the natural way in which the interval-valued approach makes sense in image processing.

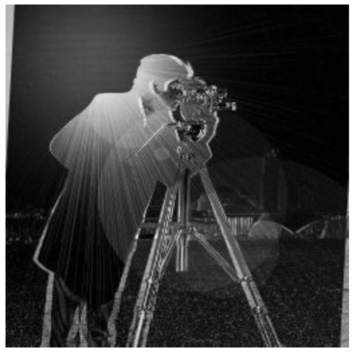

Fig. 2. Representation of the interval width, resulting from the interval-valued representation of the cloudy/sunny/distorted take of the cameraman image.

III. Foundations of the Interval-Valued Image and Morphological Model

A. Interval-Valued Fuzzy Set Theory

The interval-valued image model that is described in this paper finds its origin in fuzzy set theory [15]. The concept of fuzzy sets was introduced to deal with imprecise information: instead of thinking in terms of "yes/no" or "black/white". Fuzzy set theory allows a gradual transition between both extreme states. Formally, a fuzzy set A in a universe $\mathcal{U}$ is characterized by a $\mathcal{U} \rightarrow [0,1]$ mapping, that associates with every element x of $\mathcal{U}$ a membership degree $A(x)$. This can be generalized to the notion of an $\mathcal{L}$-fuzzy set, where $\mathcal{L} = (L, \leq_L)$ represents a complete lattice [4]; a complete lattice is a partially ordered set in which every family of elements has a supremum and infimum. An $\mathcal{L}$-fuzzy set A in $\mathcal{U}$ is characterized by an $\mathcal{U} \rightarrow L$ mapping; note that the class $\mathcal{L}^{\mathcal{U}}$ of $\mathcal{L}$-fuzzy sets in $\mathcal{U}$ forms a complete lattice if $\mathcal{L}$ is a complete lattice. When $L = [0,1]$, $\mathcal{L}$-fuzzy set theory reduces to classical fuzzy set theory.

An interval-valued fuzzy set (IVFS) corresponds to a mapping A from $\mathcal{U}$ into the class of closed intervals $[\mu_1, \mu_2] \subseteq [0,1]$. Thus, $A(x) = [\mu_1(x), \mu_2(x)]$ for every $x \in \mathcal{U}$. Evidently, if $\mu_1(x) = \mu_2(x)$ for all $x \in \mathcal{U}$ then the interval-valued fuzzy set reduces to a classical fuzzy set. The class of IVFS can be regarded as $\mathcal{L}^I$-fuzzy sets, with the complete lattice $\mathcal{L}^I = (L^I, \leq_{L^I})$ defined by:

$$L^I = \{[x_1, x_2] | [x_1, x_2] \subseteq [0,1]\},$$
$$[x_1, x_2] \leq_{L^I} [y_1, y_2] \Leftrightarrow x_1 \leq y_1 \ and \ x_2 \leq y_2.$$

Infimum and supremum of a set $C = \{[a_s, z_s] \subseteq [0,1] | s \in I_S \subseteq \mathbb{N}\}$ of closed intervals are given by $\bigwedge C = [\inf_s a_s, \inf_s z_s]$ and $\bigvee C = [\sup_s a_s, \sup_s z_s]$; smallest element $0_{L^I} = [0,0]$, largest element $1_{L^I} = [1,1]$. For a complete lattice $\mathcal{L} = (L, \leq_L)$, the inclusion of $\mathcal{L}$-fuzzy sets S_1 and S_2 in $\mathcal{U}$ is defined by: $S_1 \subseteq S_2 \Leftrightarrow S_1(u) \leq_L S_2(u)$, for all u in $\mathcal{U}$. The intersection and union of two $\mathcal{L}$-fuzzy sets S_1 and S_2 in $\mathcal{U}$ are defined by: $(S_1 \cap S_2)(u) = S_1(u) \wedge S_2(u)$ and $(S_1 \cup S_2)(u) = S_1(u) \vee S_2(u)$, for all u in $\mathcal{U}$.

It is quite clear that in the context of the interval-valued image model as described in Section II, grayscale images are actually characterized by interval-valued fuzzy sets. Consequently, techniques and tools from interval-valued fuzzy set theory can be used to construct a corresponding morphological model to process interval-valued images [8], [11], [13], [14].

As crisp set theory has Boolean logic as underlying logical framework, the extension to fuzzy set theory comes along with the extension of Boolean logic to fuzzy logic. The basic Boolean logical operators (negation, conjunction, implication) were extended to fuzzy logical operators, which respectively model the complement, the intersection, and the inclusion of fuzzy sets. These definitions can be easily generalized to $\mathcal{L}$-fuzzy logical operators which play an important role in $\mathcal{L}$-fuzzy mathematical morphology [3].

Definition 1 (logical operators): Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice with smallest element 0_L and largest element 1_L.

- A decreasing $L \to L$ mapping $\mathcal{N}$ is called a negator on $\mathcal{L}$ if $\mathcal{N}(0_L) = 1_L$ and $\mathcal{N}(1_L) = 0_L$; it is called involutive if $(\forall x \in L)$ $(\mathcal{N}(\mathcal{N}(x)) = x)$.
- An $L^2 \to L$ mapping $\mathcal{C}$ is called a conjunctor on $\mathcal{L}$ if $\mathcal{C}(0_L, 0_L) = \mathcal{C}(1_L, 0_L) = \mathcal{C}(0_L, 1_L) = 0_L$, $\mathcal{C}(1_L, 1_L) = 1_L$, and if it has increasing partial mappings. It is called a t-norm (usually denoted as $\mathcal{T}$) if it also is commutative, associative and satisfies $(\forall x \in L)(\mathcal{C}(1_L, x) = x)$.
- An $L^2 \to L$ mapping $\mathcal{I}$ is called an implicator on $\mathcal{L}$ if $\mathcal{I}(0_L, 0_L) = \mathcal{I}(0_L, 1_L) = \mathcal{I}(1_L, 1_L) = 1_L$, $\mathcal{I}(1_L, 0_L) = 0_L$, and if it has decreasing first and increasing second partial mappings.

For the complete lattice $([0,1], \leq)$ the standard negator N_s is defined by $N_s(x) = 1 - x$ $(x \in [0,1])$. Popular t-norms on $[0,1]$ are $T_M(x,y) = \min(x,y)$ and $T_W(x,y) = \max(0, x+y-1)$ (Lukasiewicz), popular implicators on $[0,1]$ are $I_{KD}(x,y) = \max(1-x, y)$ (Kleene-Dienes) and $I_W(x,y) = \min(1, 1-x+y)$ (Lukasiewicz), always with $x, y \in [0,1]$.

For our purposes we are interested in logical operators on the complete lattice $\mathcal{L}^I = (L^I, \leq_{L^I})$. There are different ways of representing operations on interval-valued fuzzy sets by corresponding operations on classical fuzzy sets. We refer to [3] for a nice overview, and limit ourselves here to some specific examples. Let $x = [x_1, x_2]$ and $y = [y_1, y_2]$, then the standard negator N_s leads to the following interval-valued negator:

$$\mathcal{N}_s(x) = [N_s(x_2), N_s(x_1)] = [1 - x_2, 1 - x_1].$$

The minimum T_M and the Kleene-Dienes implicator I_{KD} lead to the following interval-valued t-norm and implicator:

$$\begin{aligned} \mathcal{T}_M(x,y) &= [\min(x_1, y_1), \min(x_2, y_2)], \\ \mathcal{I}_{KD}(x,y) &= [\max(1 - x_2, y_1), \max(1 - x_1, y_2)]. \end{aligned}$$

Here, the construction of the interval-valued operators from their single-valued counterparts is straightforward. Less obvious constructions are possible and useful as well: the Lukasiewicz t-norm T_W and the Lukasiewicz implicator I_W lead to the following interval-valued t-norm and implicator:

$$\begin{aligned} \mathcal{T}_W(x,y) &= [\max(0, x_1 + y_1 - 1), \\ &\qquad \max(0, x_1 + y_2 - 1, x_2 + y_1 - 1)], \\ \mathcal{I}_W(x,y) &= [\min(1, 1 - x_1 + y_1, 1 - x_2 + y_2), \\ &\qquad \min(1, 1 - x_1 + y_2)]. \end{aligned}$$

The t-norm is called "pessimistic" (due to its modified upper bound), the implicator is called "optimistic" (due to its modified lower bound) [3]. These examples illustrate that there are quite a lot of possibilities to construct interval-valued logical operators.

B. Interval-Valued Morphology

Mathematical morphology was originally introduced for binary images and binary structuring elements, i.e., for objects that can only have values 0 (representing black) and 1 (representing white). In the binary case, images and structuring elements are represented as crisp subsets of a universe $\mathcal{U}$. The dilation and erosion are defined as follows.

Definition 2 (binary dilation and erosion): [12] Let A, B be crisp subsets of $\mathcal{U}$. The binary dilation $D(A,B)$ and the binary erosion $E(A,B)$ are defined by:

$$\begin{aligned} D(A,B) &= \{v \in \mathcal{U} | T_v(B) \cap A \neq \emptyset\}, \\ E(A,B) &= \{v \in \mathcal{U} | T_v(B) \subseteq A\}, \end{aligned}$$

with $T_v(B) = \{u \in \mathcal{U} | u - v \in B\}$ the translation of B by the point v.

These definitions can be extended to non-binary objects [1], [2], [7], in particular to interval-valued objects [8], [14], by fuzzifying the underlying logical framework (cfr. Section III-A). Note that it makes sense to include interval-valued structuring elements in this extension. Also regarding the values of those pixels some uncertainty might exist, even though it is chosen by the user. Indeed, if one wants the structuring element to reflect the importance or weight that is associated with a pixel at a certain position w.r.t. the center of the structuring element, one might not be completely sure how to estimate that weight. The use of an interval with likely values might be a solution in that case.

Definition 3 (interval-valued dilation and erosion): Let $\mathcal{L}^I = (L^I, \leq_{L^I})$ be the complete lattice corresponding to interval-valued fuzzy set theory, let $\mathcal{T}$ be a t-norm on $\mathcal{L}^I$ and let $\mathcal{I}$ be an implicator on $\mathcal{L}^I$. Let A be an image and B be a structuring element, both represented as interval-valued fuzzy sets in $\mathcal{U}$. The interval-valued fuzzy dilation $D^I_{\mathcal{T}}(A,B)$ and the interval-valued fuzzy erosion $E^I_{\mathcal{I}}(A,B)$ of A by B are defined by:

$$D^I_{\mathcal{T}}(A,B)(v) = \bigvee_{u\in\mathcal{U}} \mathcal{T}(B(u-v),A(u)),$$
$$E^I_{\mathcal{I}}(A,B)(v) = \bigwedge_{u\in\mathcal{U}} \mathcal{I}(B(u-v),A(u)),$$

for all v in $\mathcal{U}$.

With every choice of the interval-valued t-norm and implicator, a different interval-valued morphological model is associated. Of course, in practice one is usually interested in those models that satisfy some specific properties [8]. We refer to [9] and [10] for a study of the morphological models based on the Lukasiewicz-operator ($\mathcal{T}_W$ and $\mathcal{I}_W$) and on the minimum-operator ($\mathcal{T}_M$ and $\mathcal{I}_{KD}$), respectively. Both models satisfy properties w.r.t. expansivity and restrictivity, monotonicity, interaction with intersection, interaction with union, and duality.

In [5], [6] we have also investigated decomposition and construction properties of interval-valued morphological operators. For example, it turns out that the interval-valued dilation based on $\mathcal{T}_M$ can be decomposed into binary dilations and, reversely, that this is the only interval-valued dilation that can also be constructed from its binary counterpart.

Having a dilated and eroded image, one can take the difference to result in a gradient-image, just as in the case for regular grayscale images. Note that the difference between two intervals $[x_1,x_2]$ and $[y_1,y_2]$ is defined as the interval $[x_1-y_2, \max(x_1-y_1, x_2-y_2)]$. We refer to our work [11] for some visual examples and discussion. In any case, and this is an important conclusion, one can observe that the uncertainty that was present in the original representation of the cameraman image is propagated through the (interval-valued) morphological operators and the edge detection application. This means that the information regarding the uncertainty is not lost, but is fully taken into account and can be used and exploited in further processing of the images.

IV. Some Challenges for Future Research

The interval-valued image model and the corresponding morphological theory – both based on concepts from extended fuzzy set theory – succeed in modeling the uncertainty regarding grayscale pixel values, and enables us to propagate this uncertainty through morphological operators. Several specific morphological models have been investigated and provide the tools for applications (e.g., edge detection). In such applications, experts will be able to judge the uncertainty of the results, and can modify their attitude towards the information accordingly.

It is clear that this image model is also faced with many challenges and requires broader and deeper research on different topics. For example, what is the relation of the interval-valued image model with defuzzification? In other words, if we defuzzify the interval-valued representation (leading to single values for the pixels) and then apply regular morphological operations, to what extent will this result be different from working with the interval-valued representation and interval-valued morphological operations followed by defuzzification in the end? The answer to this question (which might depend on conditions imposed on the involved operators) not only improves our theoretical insight in the model, but is also important in the context of computational efficiency.

Regarding the application of edge detection [11], we have the advantage that the interval-valued approach gives us insight in the (un-)certainty of the gradient-image, but additionally we would like to know the source of that uncertainty. For example, in case of the cameraman image the upper bound gradient-image will contain real edges, but also false edges caused by the spatial distortion. Both kind of edges will be accompanied with a degree of uncertainty, and the challenge is to make an automated intelligent decision about the nature (real or false) of these edges.

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