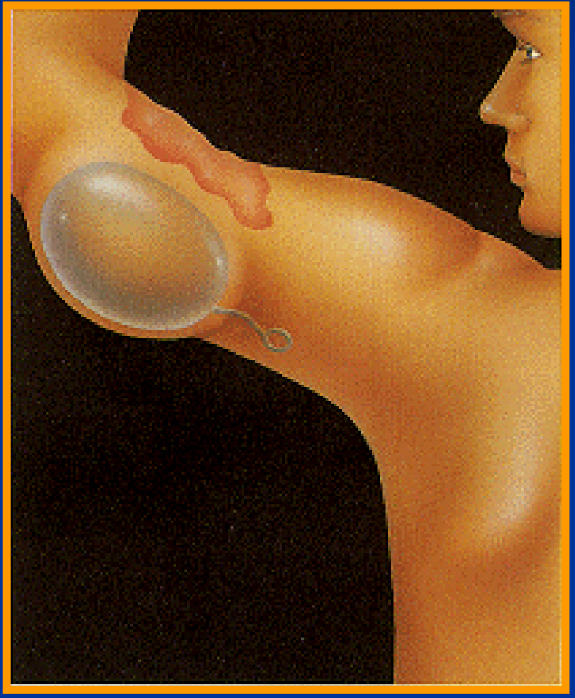
Prof. Djenane C. Pamplona
Daniel El-Jaick de Souza Mota

NUMERICAL AND EXPERIMENTAL ANALYSIS OF INFLATED MEMBRANE OVER ELASTIC FOUNDATION

PACAM 2010
Foz do Iguaçu, January 4-8

Motivation



The expansion of the skin is a physiological process, defined as the capacity that the skin has to increase its superficial area due to an imposed deformation.

The research is focused in Biomechanical Engineering and investigates the behaviour of a inflated skin expander over elastic foundation.

It was motivated by the puzzlingly results obtained during skin expansion done in certain regions of patients. During the expansions it could be observed that depending on the region where the expander is implanted its behavior can be very different.

Scope

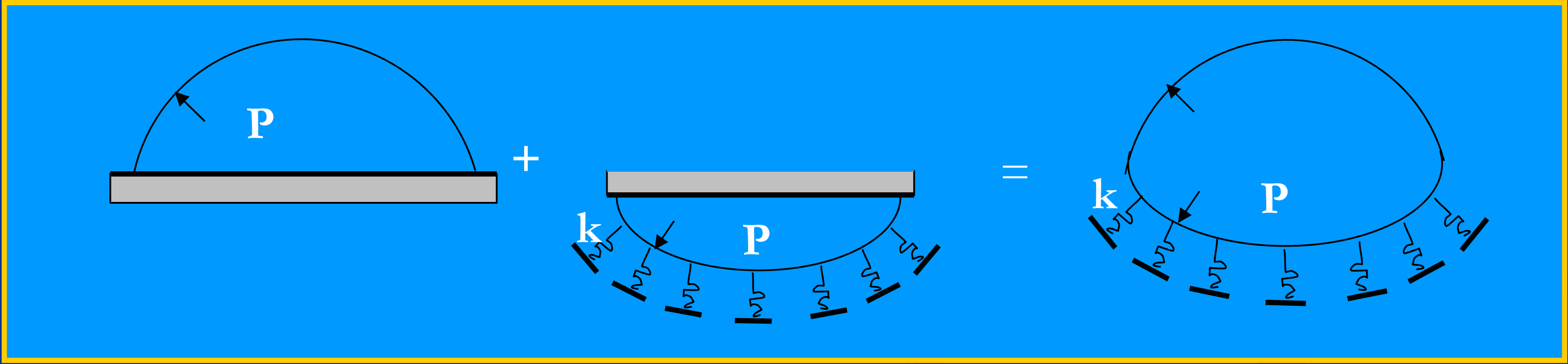
The purpose of this work is to investigate numerically and experimentally the behavior of a circular flat membrane, of radius a, expanded by a circular skin expander that can reach an internal volume of 200 ml. The coordinates r and z were made dimensionless when divided by the external radius a.

The expansion is realized over an elastic continuum flat surface, in this way it is possible to compare numerically and experimentally the results.

The material of the membrane is considered as homogenous, isotropic, and is modeled as being Neo-Hookean, $W=C_1\,(I_1-3)$, where I_1 is the first invariant.

Numerical Formulation

The problem was divided in two parts and added afterwards



1. ***The first deals with the expansion of the membrane over rigid foundation.***
2. ***The second deals with the inflation of the membrane over rigid foundation but inside of an elastic continuum.***
3. ***The two configurations, for the same pressure P, are summed .***

$$\frac{1}{\lambda_3}(\lambda_3\sigma_{11})' + \frac{r'}{r}(\sigma_{11}-\sigma_{22}) = 0$$

The equilibrium equations that describe an inflated membrane with internal pressure P are:

$$\frac{1}{\lambda_3}(\lambda_3\,\sigma_{11})' + \frac{r'}{r}(\sigma_{11}-\sigma_{22}) = 0$$

$$\frac{(r''z' - r'z'')}{(\lambda_1)^3}\sigma_{11} - \frac{z'}{r\lambda_1}\sigma_{22} + \frac{P}{\lambda_3 H} = 0$$

Where H is the reference thickness, λ_i are the principal stretches, W the constitutive equation, W_i the derivatives of W in relation to λ_i.

The principal stresses are:

$$\sigma_{ii} = \lambda_i W_i$$

Those equations can be rewritten (Haughton (1996)) by:

$$r'' = \frac{\left(r'F + z'G\right)}{\lambda_1^2}$$

$$z'' = \frac{\left(z'F - r'G\right)}{\lambda_1^2}$$

Where F and G are:

$$G = \frac{\lambda_1^2}{W_1}\left(-\frac{\lambda_1\lambda_2\left(P - \frac{kaz\,r'}{\lambda_1}\right)}{H} + \frac{z'}{r\lambda_1}\lambda_2 W_2 \right)$$

$$F = \frac{\lambda_1}{W_{11}}\left[\lambda_2{}'\left(\frac{W_1}{\lambda_2} - W_{12}\right) - \frac{r'}{r\lambda_1}\left(\lambda_1 W_1 - \lambda_2 W_2\right) \right]$$

With the boundary conditions:

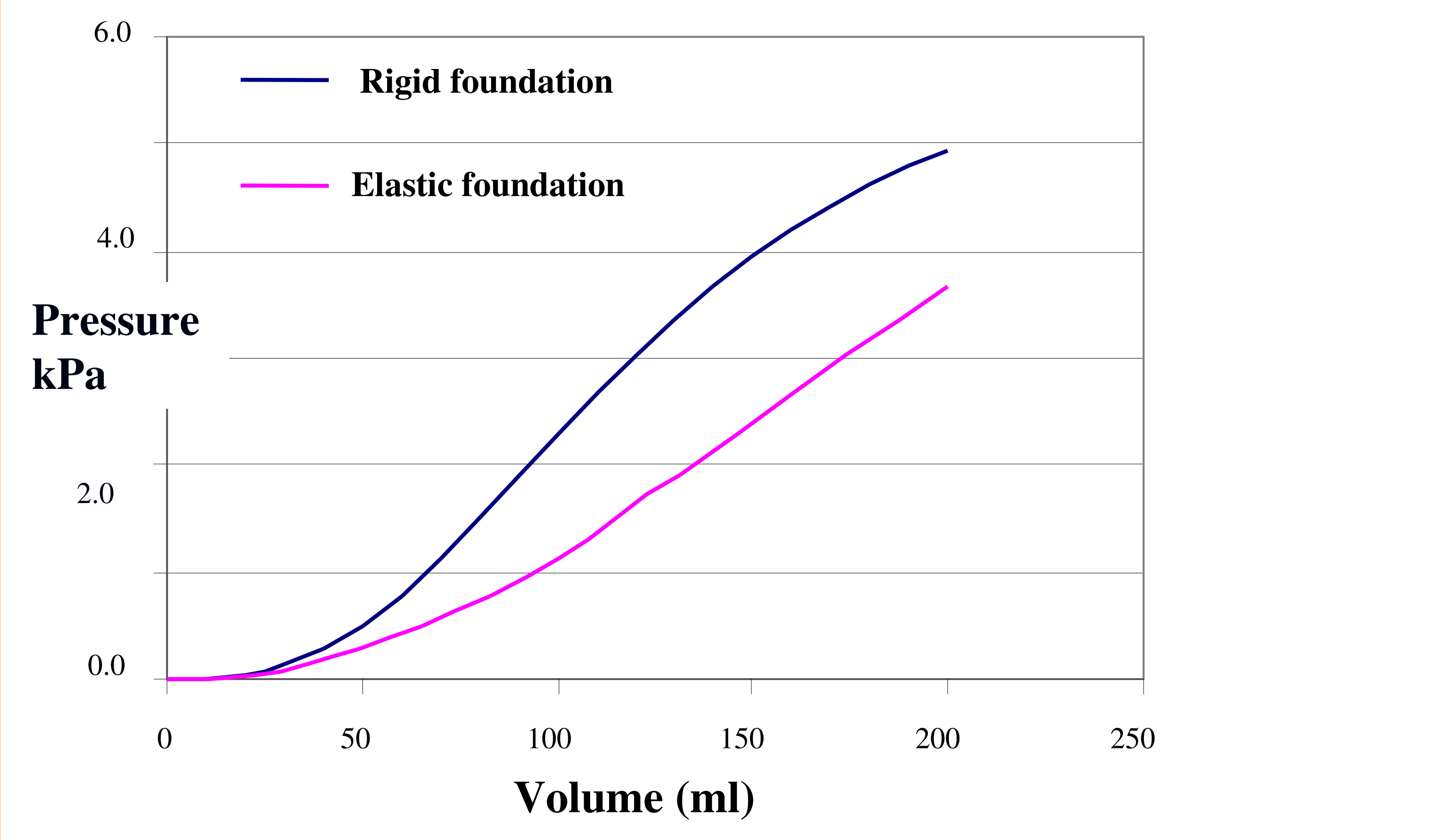
$$r(0) = 0,\ z(0) = 0,\ z'(0) = 0 \ \text{ and } \ r(a) = a.$$

Numerical Results

The numerical solution is done through the Runge-Kutta Method coupled with the Newton Raphson Method.

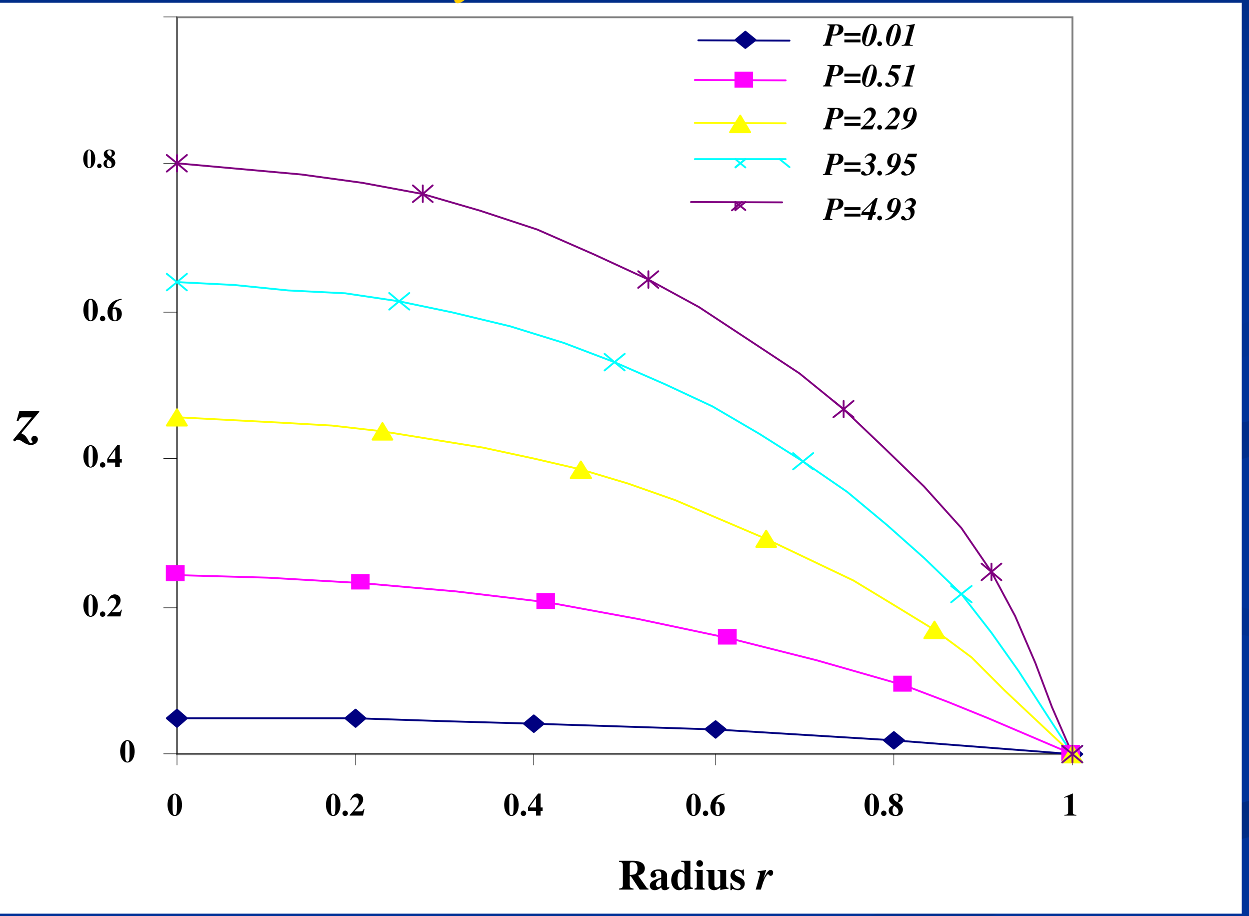
Both membrane and elastic continuum had their elastic properties characterized experimentally.

k is considered 2.0 kPa/cm and the elastic constant C_1= 359.63 kPa.

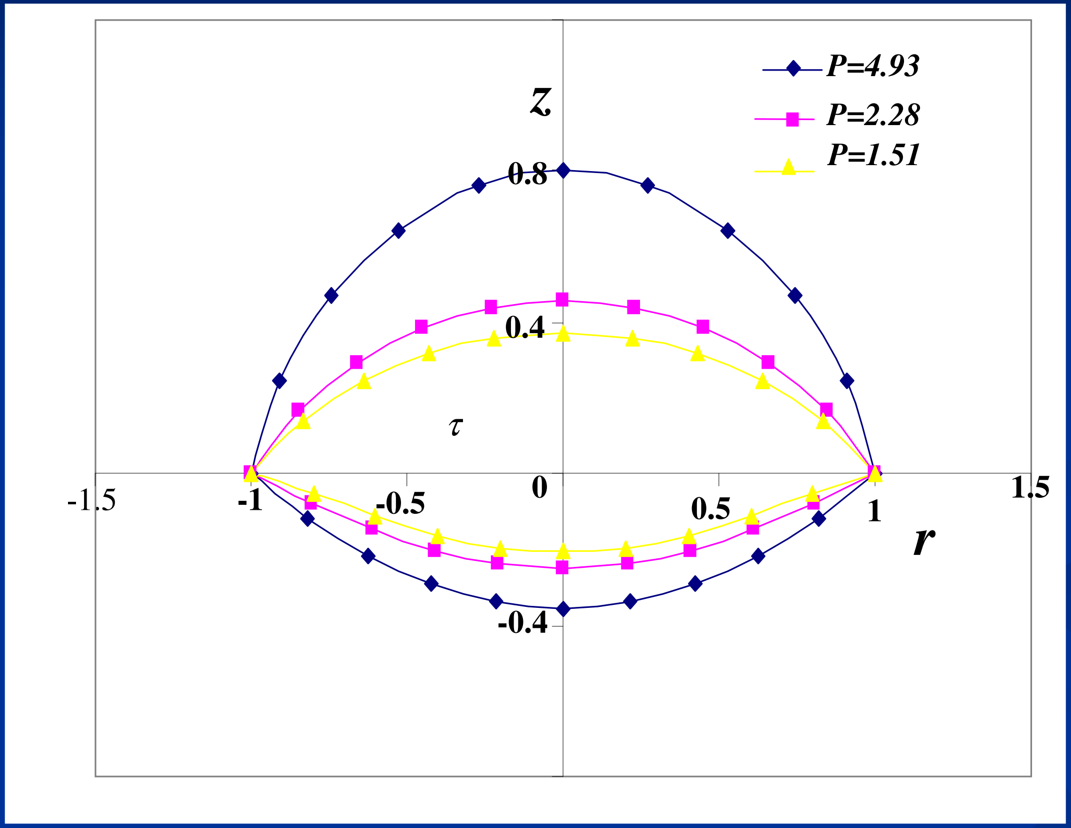


Shapes of the expanded membrane, over rigid foundation for different pressures, P, in kPa.

The coordinates r and z were made dimensionless when divided by the external radius a.

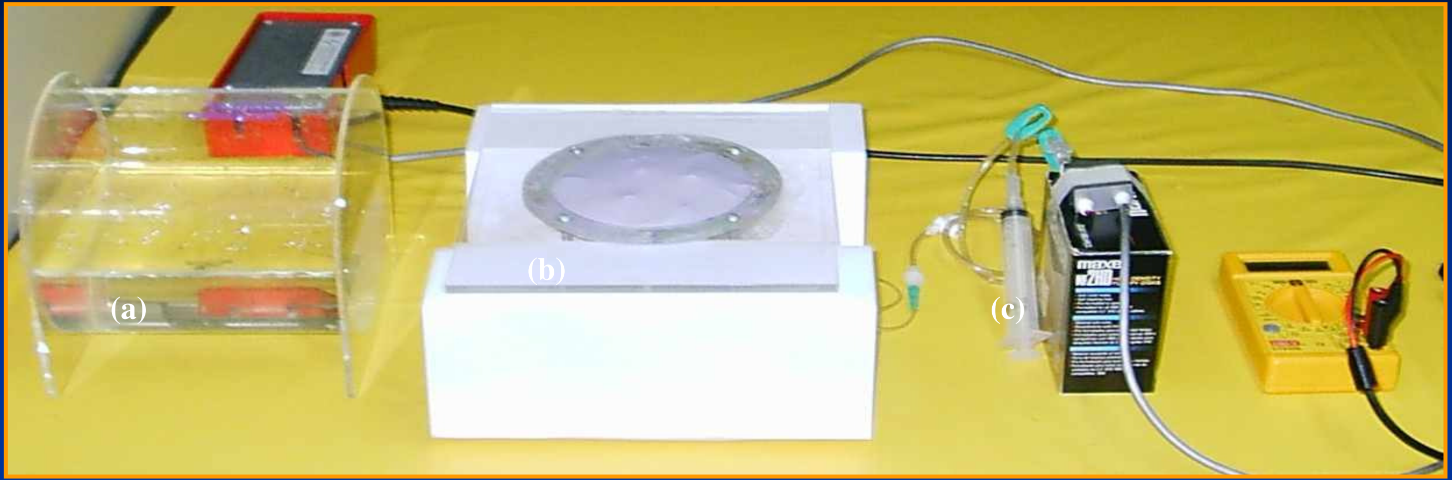


Shapes of the expanded membrane, over elastic foundation for different pressures, P, in kPa.



Since the geometry is dimensionless, for a pressure of 4.93 kPa, in a rubber sheet of radius 5 cm the height reached is 0.8 times 5 cm, i.e. 4 cm.

Experimental Formulation



To measure the internal pressure, an apparatus was especially developed for this purpose.

The membrane rubber sheet used is from Hygenic, model Fiesta Medium purple, with initial thickness of 0.208 mm, modeled as neo-Hookean with elastic constant *C1*= 359.63 kPa.

The expansion was done with a round skin expansor from Silimed, its circular basis has 9.6 cm of diameter and its volume can reach 200 ml.

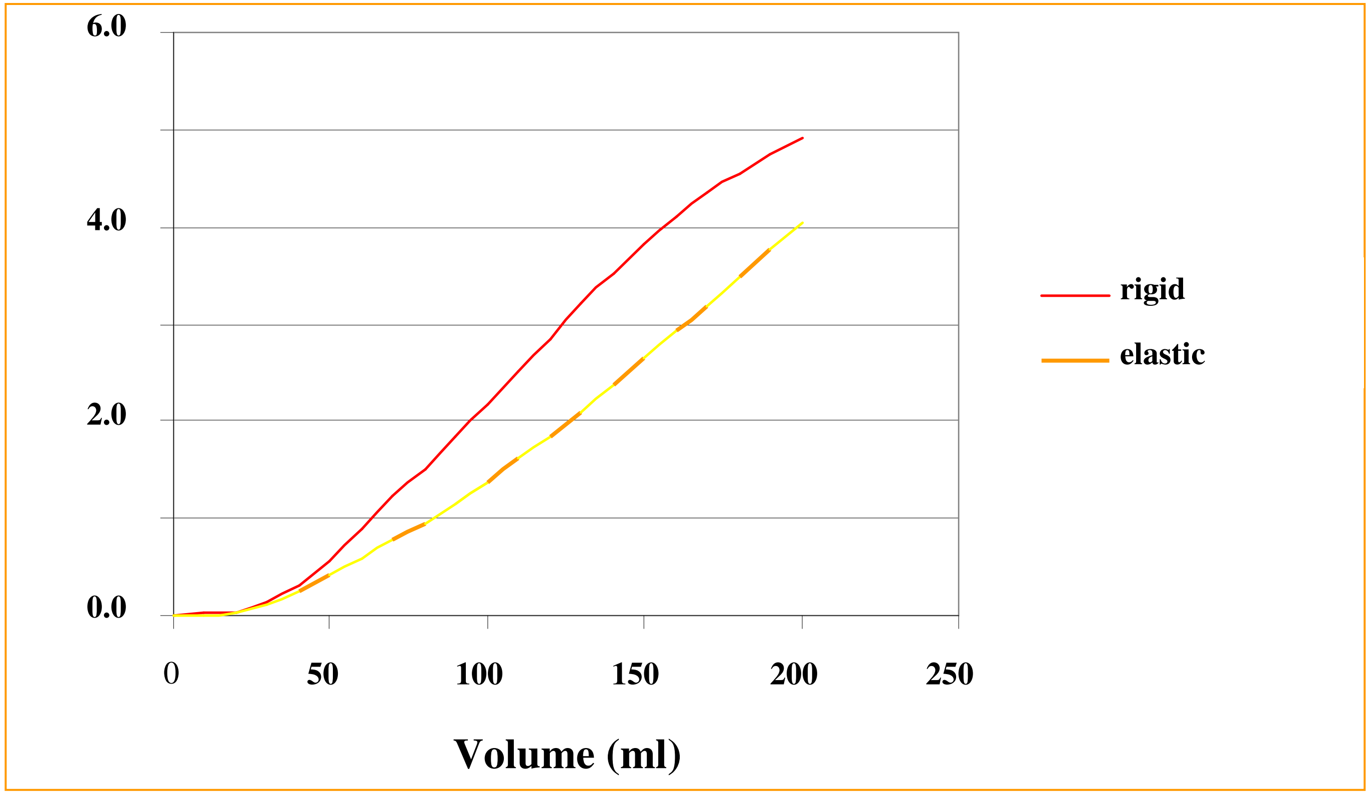
Experimental Formulation

To model the elastic foundation, and simulate the fatty tissue, a rubber balloon was filled with a visco elastic material.

The set was tested by the ASTM Standard Test Method D 1194 and the elastic constant *k* for Winkler´s equation obtained was 2.0 kPa/cm.

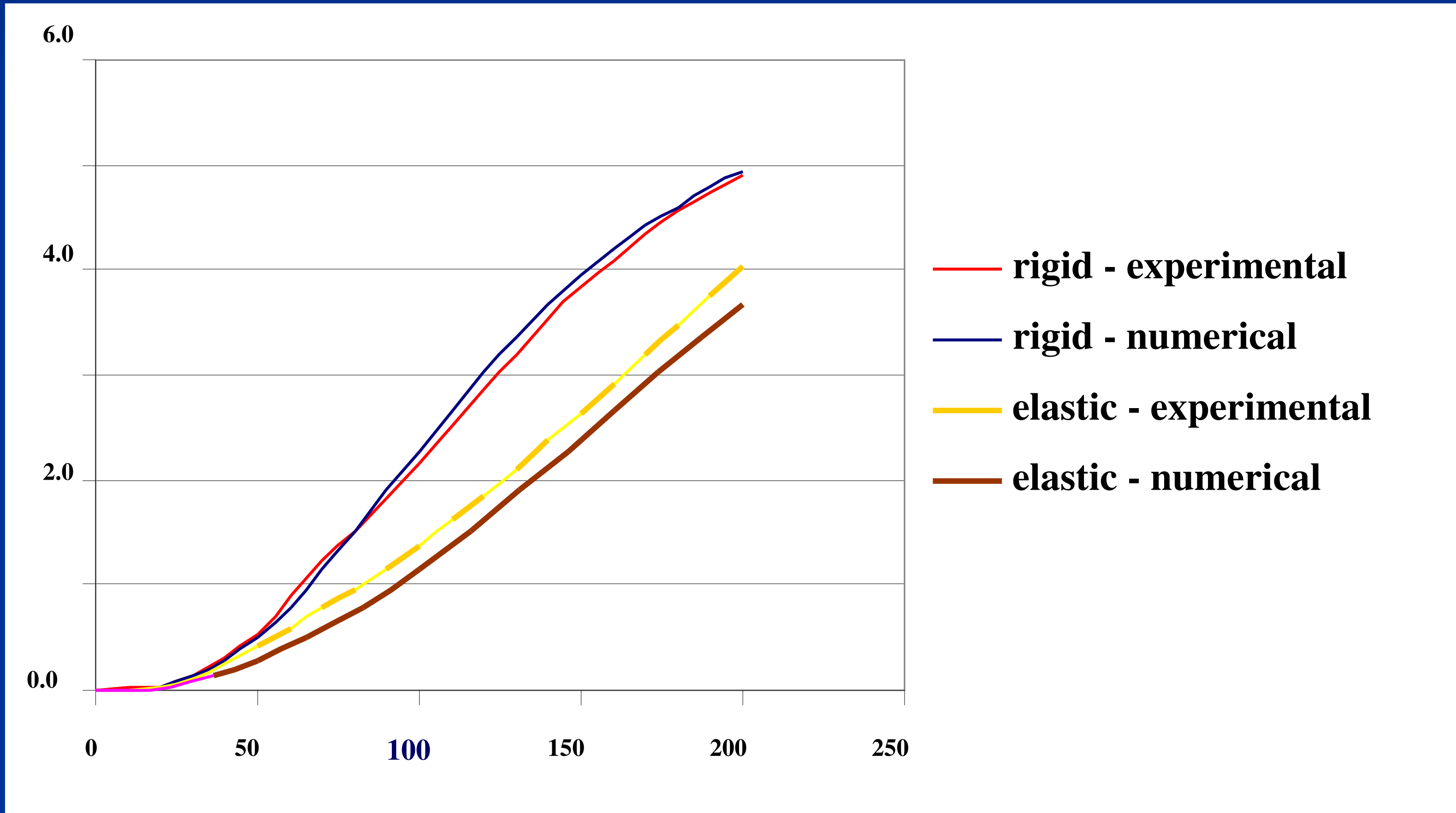
Experimental Results

volume (ml) x pressure (kPa)



Comparison Between Numerical and Experimental Results

internal volume (ml) x internal pressure kPa



Conclusions

The formulation of separating the problem in two parts summing the configurations for the same pressure of both problems is quite original.

The comparison of the experimental results between the two foundations shows that the pressure inside the expander with the same internal volume <u>can reach 52% more for the rigid foundation</u>, as expected.

For the numerical results this difference can reach 68 %.

We can conclude that the numerical and experimental results are in good agreement. Both results present a low pressure in the beginning of the expansion, increasing in a nonlinear way as the volume is infiltrated.

Acknowledgement

- **We are grateful to CAPES and CNPq for the support with research projects.**

- **Special thanks to Professor Ivo Pitanguy and his staff for supporting our work.**

- **Last but not least thanks to the person who is presenting this talk.**