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Exact formulations for the minimum interference problem in *k*-connected *ad hoc* wireless networks

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Abstract

Energy consumption is one of the most critical issues in wireless *ad hoc* and sensor networks. A considerable amount of energy is dissipated due to radio transmission power and interference (message collisions). A typical topology control technique aims at reducing energy consumption while ensuring specific desired properties to the established wireless network (such as biconnectivity). Energy minimization can be achieved by reducing the transmission power and selecting edges that suffer or cause less interference. We propose four integer programming formulations for the *k*-connected minimum wireless *ad hoc* interference problem, which consists in a topology control technique to find a power assignment to the nodes of an *ad hoc* wireless network such that the resulting network topology is *k*-vertex connected and the radio interference is minimum. Interference is measured by three different models: Boolean, protocol, and physical. We report computational experiments comparing the formulations and interference models. Optimal solutions for moderately sized networks are obtained using a commercial solver.

Keywords: ad hoc networks; wireless sensor networks; topology control; energy consumption optimization; interference minimization; exact formulations

1. Introduction

Ad hoc networks are wireless mobile networks that can be set up anywhere and anytime, even when access to the Internet or another preexisting network infrastructure is unavailable. *Ad hoc* networks allow mobile computer users with (compatible) wireless communication devices to set up a possibly short-lived network just for the communication needs of the moment (Perkins, 2001).

Ad hoc network nodes communicate with each other using multihop wireless links. There is no stationary infrastructure such as base stations. Each node in the network also acts as a router, forwarding data packets to other nodes. These infrastructureless networks have many potential applications, ranging from personal area networks to search and rescue operations, to massive networks with millions of sensors (Shorey et al., 2006).

Wireless sensor networks (WSNs) are a particular type of *ad hoc* networks in which the nodes are “smart sensors”, that is, small devices equipped with advanced sensing functionalities (thermal, pressure, and acoustic are some examples of such sensing abilities), a small processor, and a short-range wireless transceiver. Different from the case of generic *ad hoc* networks, nodes of a WSN are typically stationary, or at most slowly moving (Santi, 2005).

One of the most important constraints on wireless *ad hoc* and sensor networks is the low energy consumption requirement. Wireless nodes usually carry limited, generally irreplaceable, power sources. Therefore, while traditional networks aim to achieve high quality of service provisions, wireless network algorithms must focus primarily on energy conservation (Rickenbach et al., 2009). Radios tend to be the major source of power dissipation in wireless networks (Xing et al., 2007). Since a communication link between two wireless nodes exists only when each node is in the radio transmission range of the other, the general approach of an energy-aware algorithm is to remove longer links from the wireless network, in order to force the nodes to use several shorter hops instead, allowing the nodes to reduce their transmission powers and thereby using a smaller amount of energy. Topology control is one of the main techniques used to save energy and extend the lifetime of *ad hoc* wireless networks.

Topology control may be seen as the task of adjusting the transmission powers at node level, in order to decrease the energy consumption of a given wireless *ad hoc* or sensor network. Topology control refers to selecting a subset of the available communication links to enforce data transmission while computing a sparse network topology with specific desired properties, such as connectivity, fault tolerance, short stretches, sparsity, or low node degrees.

The main goal of topology control algorithms is to eliminate inefficient links that are not used for communication. If nodes are far from each other, communicating directly over large distances would require very strong transmission powers. Large transmission powers may interfere with the communication between other nodes within reach. Interference causes errors when a receiver is not able to decode the messages from its legitimate sender and, as a consequence, these messages have to be sent again. Therefore, it is advisable to eliminate inefficient communication, and direct communication is restricted to pairs of nodes that can reach each other with relatively weak transmission powers that will interfere with a small number of nodes. Reducing interference is one of the main challenges in wireless *ad hoc* communication (Tan et al., 2011; Lou et al., 2012; Khabbazian et al., 2015). In fact, if too many links (or a wrong selection of links) are removed, the network becomes more susceptible to node failure. For instance, some sensor nodes may fail or be blocked due to the lack of power, and suffer physical damage or environmental interference. The failure of sensor nodes should not affect the overall task of the network. This is the reliability or fault tolerance issue, which is the ability to sustain sensor network functionalities without any interruption due to sensor node failures (Akyildiz et al., 2002; Geeta et al., 2013). Since WSNs are more susceptible to failures (Kakamanshadi et al., 2015), fault tolerance becomes an important requirement for many applications.

In this work, we propose topology control approaches to minimize the radio interference in wireless nodes while designing a fault-tolerant network. More precisely, we propose exact formulations to solve the minimum interference problem in k-connected *ad hoc* wireless networks, which consists in finding a power assignment to the nodes of a wireless *ad hoc* network such that the resulting topology is k-vertex connected and the radio interference is minimum. A k-vertex-connected network can tolerate the simultaneous failure of up to $k-1$ nodes (or links), that is, the failure of any $k-1$ nodes will not disconnect the network.

We consider the three most studied interference models for *ad hoc* wireless networks: the Boolean interference model (Rickenbach et al., 2005), the protocol interference model (Gupta and Kumar, 2000), and the physical interference model (Gupta and Kumar, 2000; Goussevskaia et al., 2007). Each interference model defines an interference-free condition for successful communication over a given link. We use a commercial solver to compute optimal solutions for the exact formulations. We study and compare the quality of the optimal solutions of the interference models.

The proposed formulations to solve the minimum interference problem are also compared with an energy-efficient topology control method that minimizes the total power assignment ensuring a k-connected network (Moraes et al., 2009).

In the following, we first provide an overview of wireless *ad hoc* interference models and topology control algorithms considering interference in Section 2, where related work is also reviewed. In Section 3, we formally define the minimum interference problem in k-connected *ad hoc* wireless networks and present the integer programming formulations to solve it. Computational results are reported and discussed in Section 4. Concluding remarks are made in the last section.

2. Wireless *ad hoc* interference models

An interference model defines a condition for successful communication over a given link. We consider three wireless *ad hoc* interference models: the Boolean interference model, the protocol interference model, and the physical interference model.

The Boolean interference model assumes that interference is an “all-or-nothing” phenomenon, that is, all messages are lost if a node receives two or more messages at the same time, regardless of their transmission power. Under the protocol interference model, the condition for a message to be correctly received is that its transmission power be stronger than those of the contending messages. Finally, in the physical interference model, a message is successfully received if its transmission power exceeds the power of the aggregate signal composed by the sum of all contending messages.

The protocol interference model is more realistic than the Boolean interference model. It is widely accepted that the physical interference model is more accurate than the protocol interference model (Cardieri, 2010). In this section, we describe in detail the Boolean, protocol, and physical interference models.

2.1. Boolean interference model

Let us consider the set $V = \{0, 1, \ldots, |V| - 1\}$ of transceivers (nodes) of a wireless network, together with their locations (or the distances between them). A transmission power p_u is associated with each node $u \in V$. A transmitter–receiver pair (or link) is defined by an ordered pair $(u, v) : u, v \in V$, where u and v denote a transmitting node and a receiving node, respectively.

We consider that each node is equipped with an omnidirectional antenna. Each node adjusts its transmission power based on the distance to the receiving nodes and background noise. In the most common power attenuation model (Rappaport, 2001), the signal power falls with $1/d^{\theta}$, where

d is the distance from the transmitter and θ is the path loss exponent (typical values of θ are between 2 and 4). Under this model, the transmission power requirement at node u for supporting a communication to v is given by

$$p_u \geq d_{uv}^{\theta} q_v, \tag{1}$$

where q_v is the receiver's power threshold for signal detection (which is usually normalized to 1), and d_{uv} is the distance between u and v. Assuming a deterministic path loss model and $q_v = 1$, p_u should be greater than or equal to d_{uv}^{θ}. In this case, we say that node v is covered by node u.

The Boolean interference model is defined based on the concept of a communication graph, whose nodes correspond to the set of devices in V. Given power assignments $p_0, \ldots, p_{|V|-1}$ to the nodes in V, there is an arc (u, v) with a nonnegative arc weight d_{uv}^{θ} between a pair of nodes $u, v \in V$ whenever a signal transmitted by node u can be received at node v, that is, if and only if $p_u \geq d_{uv}^{\theta}$.

Communication graphs neglect the effects of interference from concurrent transmissions and do not reflect the real-world behavior. In fact, given two transmitters u and w, and a receiver v, successful communication over links (u, v) and (w, v) of the communication graph are guaranteed if $p_u \geq d_{uv}^{\theta}$ and $p_w \geq d_{wv}^{\theta}$, even if both transmitters u and w send messages to v simultaneously. However, in real situations, transmissions can be unsuccessful because communication may become corrupted due to interference in the receiver node if it receives two or more messages at the same time.

The Boolean interference model takes into account the interference from concurrent transmissions in communication graphs from two different perspectives: the sender-centric perspective (interference is considered at the sender extremity and is based on the number of nodes affected by communication over a given link) and the receiver-centric perspective (interference is considered at the receiver extremity and is defined as the number of nodes whose transmission power interferes with the receiver).

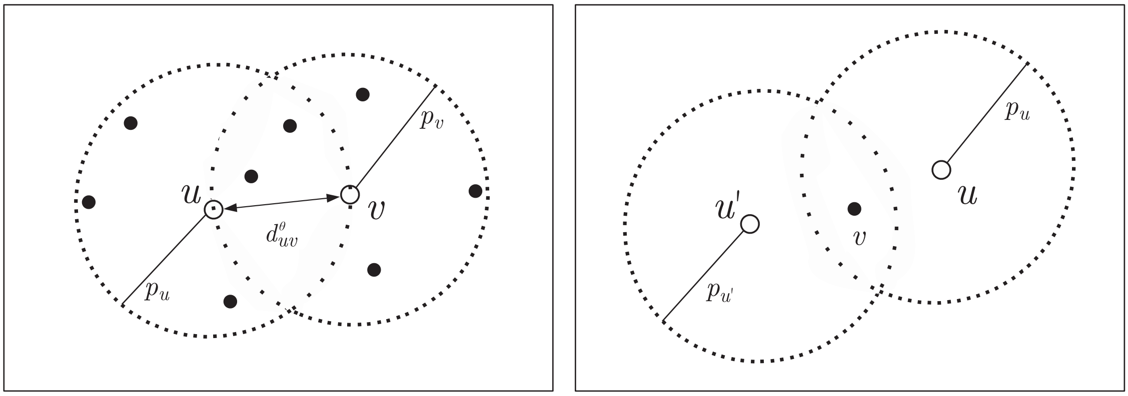
Sender-centric perspective

The sender-centric perspective is a Boolean interference model that considers how many nodes are affected by communication over a given link (Rickenbach et al., 2009). Let $G_E(p) = (V, E(p))$ be an undirected communication graph, where $E(p) = \{[u, v] : u, v \in V, p_u \geq d_{uv}^{\theta}, p_v \geq d_{vu}^{\theta}\}$. Burkhart et al. (2004) proposed the sender-centric Boolean interference model based on the concept of link coverage, defined as the number of nodes covered by any of the extremities u or v of any bidirectional link $[u, v]$, as illustrated in Fig. 1(a). The link coverage of an edge $[u, v]$ is defined as

$$\mathrm{CovI}(u, v) = |\{w \in V : w \text{ is covered by } u\} \cup \{w \in V : w \text{ is covered by } v\}|. \tag{2}$$

If $\mathrm{CovI}(u, v) = 2$, then the bidirectional link $[u, v]$ does not interfere with any node.

Adopting the sender-centric perspective, Burkhart et al. (2004) proposed optimal polynomial-time algorithms to find node power assignments that minimize the maximum link interference $I(G_E(p)) = \max_{[u,v]\in E(p)} \mathrm{CovI}(u, v)$ over all power assignments p such that the resulting graph $G_E(p) = (V, E(p))$ is connected or has a spanner property (Gao et al., 2005; Wang and Li, 2006; Rickenbach et al., 2009). Li et al. (2005) extended the coverage definition for the sender-centric model, proposed a new node interference model, and defined the average interference for the sender-centric model as the result of the division of the sum of all edge coverages by the number of



(a) Sender-centric interference model (link coverage): $\mathrm{CovI}(u, v) = 10$, with $p_u = d^{\theta}_{uv}$ and $p_v = d^{\theta}_{vu}$.

(b) Receiver-centric interference model: $\mathrm{rcI}(v) = 2$ with $p_u \geq d^{\theta}_{uv}$ and $p_{u'} \geq d^{\theta}_{u'v}$.

Fig. 1. Boolean interference model.

edges in the resulting graph $G_E(p) = (V, E(p))$. They also proposed a 2-approximation algorithm to solve the problem of computing a connected graph with minimum average interference.

Haque and Rahman (2011) also proposed centralized and local algorithms to solve the minimum interference problem with a biconnected topology using the sender-centric perspective.

Receiver-centric perspective

The receiver-centric interference perspective (Fussen et al., 2005; Rickenbach et al., 2005) associates each node v with the number of nodes whose transmissions interfere with the reception at v, see Fig. 1(b). Let $G_A(p) = (V, A(p))$ be a directed communication graph, whose arc set is $A(p) = \{(u, v) : u, v \in V, p_u \geq d^{\theta}_{uv}\}$. The receiver-centric interference value of a single node v is defined as

$$\mathrm{rcI}(v) = |\{u \in V \setminus \{v\} : v \text{ is covered by } u\}|. \tag{3}$$

Considering the receiver-centric perspective and minimization of the maximum node interference, Rickenbach et al. (2005) described an $\sqrt[4]{\delta}$-approximation algorithm for the optimal connectivity-preserving topology in the general highway model, where δ is the maximum node degree. The one-dimensional (1D) highway case (Rickenbach et al., 2005) was generalized to the 2D case in Halldórsson and Tokuyama (2008). Buchin (2011) proved that minimizing the maximum interference is NP-hard in the 2D receiver-centric case. Also considering the receiver-centric model, Fussen et al. (2005) proposed the nearest component connector algorithm, which constructs a tree rooted at the sink node in order to minimize the maximum interference.

For the problem of computing the minimum average interference in the receiver-centric perspective, Moscibroda and Wattenhofer (2005) developed an asymptotically optimal algorithm with an approximation ratio of $O(\log n)$ for minimizing the average interference in 2D networks. Tan et al. (2011) studied the minimization of the average and maximum receiver-centric interference for the highway model. Among other results, they proposed a polynomial-time exact algorithm that

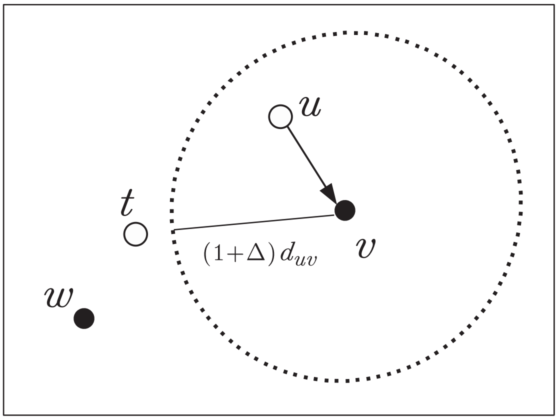


Fig. 2. Transmitter t is outside of the circular guard zone around receiver node v, with node u being the transmitter to v.

constructs a connected topology with the minimum average interference. Lou et al. (2012) improved the results in Tan et al. (2011) for the minimum average interference problem.

Khabbazian et al. (2015) studied the problem of building a connected communication graph (assigning a transmission radius to each node) while minimizing the maximum receiver-centric interference. They noticed that, given a set of wireless nodes together with their locations and a fixed integer k, the problem of connecting them by a k-connected graph that minimizes the maximum interference is still open. They also identified the need to investigate the minimization of interference in *ad hoc* wireless networks considering physical interference models. These issues are dealt within the present work. Other topology control algorithms using the Boolean interference model can be found in the literature (Meyer auf de Heide et al., 2002; Johansson and Carr-Motyčková, 2005; Blough et al., 2007; Benkert et al., 2008; Bilò and Proietti, 2008; Wu and Liao, 2008; Nguyen et al., 2010; Haque and Rahman, 2011; Yilmaz et al., 2011; Agrawal and Das, 2013; Sun et al., 2015).

2.2. Protocol interference model

In the protocol interference model, communication over an unidirectional link (u, v) is successful if the attenuated transmission power p_u/d_{uv}^{θ} sent from the transmitter u and observed by the receiver v is greater than or equal to the attenuated transmission power from any other simultaneously transmitting node $t \in V \setminus \{u, v\}$ by a factor $(1 + \Delta)$, that is,

$$\frac{p_u}{d_{uv}^{\theta}} \geq (1 + \Delta)\frac{p_t}{d_{tv}^{\theta}}, \qquad \forall t \in V \setminus \{u, v\}, \tag{4}$$

where $\Delta > 0$ is a parameter representing the spatial protection margin. This is used to model situations where a guard zone is specified by the protocol interference model to prevent any neighboring node t from interfering with the transmission from u, see Fig. 2. It also allows for imprecision in the achieved range of transmissions. If all transmission powers are equal (see Gupta and Kumar, 2000), then Expression (4) becomes

$$d_{tv} \geq (1 + \Delta)d_{uv}, \qquad \forall t \in V \setminus \{u, v\}. \tag{5}$$

In this case, a transmission from node u is successfully received by node v, if the distance between any other transmitting node t and receiver v is greater than or equal to the distance between u and v by a factor of $(1 + \Delta)$, see Fig. 2. The protocol interference model requires circular guard zones around the receiver nodes to guarantee successful transmissions.

The protocol interference model has been extensively adopted in the design and evaluation of communication protocols (Moscibroda et al., 2006a; Chafekar et al., 2007). It has also been used in scheduling algorithms for the STDMA (*spatial time division multiple access*) protocol (Behzad and Rubin, 2002), which is a prevalent medium access scheme for channel spatial reuse. For instance, the packet transmission scheduling problem using the protocol interference model has been studied by Behzad and Rubin (2002), Jain et al. (2005), and Gore et al. (2007). In this context, one would attempt to maximize the number of concurrent transmissions in order to maximize the network capacity.

It is a common observation (Cardieri, 2010) that solutions based on the protocol interference model may lead to communication graphs with degraded performance, since it does not take the aggregated effect of interference into consideration. The aggregate nature of interference in wireless communication networks (due to all nodes transmitting simultaneously) degrades the performance of the communication links. The physical interference model discussed in the next section takes into account the aggregate interference.

2.3. *Physical interference model*

The physical interference model focuses on the effects of the aggregate interference observed by the receiver. In this model, a message transmitted from node u is successfully received by node v of the unidirectional link (u, v) if the attenuated transmission power p_u/d_{uv}^{θ} from node u exceeds the aggregate signal composed by the sum of all contending messages.

According to this model, the transmission from node u is successfully received by node v if the signal-to-interference plus noise ratio at node v is greater than or equal to a given threshold β, that is,

$$\frac{\dfrac{p_u}{d_{uv}^{\theta}}}{\sigma^2 + \displaystyle\sum_{t \in V \setminus \{u,v\}} \dfrac{p_t}{d_{tv}^{\theta}}} \geq \beta, \tag{6}$$

where σ^2 is the noise floor. The noise floor can be obtained by the well-known thermal noise equation (Rappaport, 2001; Zamalloa and Krishnamachari, 2007) and the threshold β value is a hardware-dependent constant that can be approximated from empirical experiments (Son et al., 2006; Zamalloa and Krishnamachari, 2007). The physical interference model has been used, for example, for wireless link scheduling (Jain et al., 2005; Goussevskaia et al., 2007), wireless link scheduling with power control (Fu et al., 2010; Katz et al., 2010; Gogu et al., 2013; Gong and Yang, 2013; Charalambous et al., 2015), and topology control (Moscibroda and Wattenhofer, 2006; Moscibroda et al., 2006b; Gao et al., 2008; Zhang et al., 2011). The three interference models discussed in this section will be used in the next section to build exact formulations to solve the minimum interference problem.

3. Exact formulations

The *k-connected minimum interference problem in ad hoc wireless networks* (k-CMI) is the topology control problem consisting in finding a power assignment to the nodes of a wireless network such that the resulting network topology is k-vertex connected (Moraes et al., 2009; Moraes and Ribeiro, 2013) and radio interference is minimized. Formally, given the node set V, the distance d_{uv} between any pair of nodes $u, v \in V$, the path loss exponent θ, and a parameter $k \geq 2$, k-CMI consists in finding an assignment of transmission powers $p : V \rightarrow R+$ to every node $u \in V$, such that the interference is minimized and the resulting undirected communication graph $G_E(p) = (V, E(p))$ is k-connected (i.e., it remains connected if any $k - 1$ of its nodes are removed).

We propose integer programming models for k-CMI. They are based on the incremental power formulation proposed by Moraes et al. (2009) to solve the k-connected minimum power consumption problem, which consists in finding a power assignment to the nodes of a wireless network such that the resulting network topology is k-vertex connected and the total power consumption is minimum.

In the following, we assume that the distances are asymmetric and the use of a bidirectional topology. In the asymmetric input case, there may be pairs of nodes $u, v \in V$ such that the attenuated transmissions powers $p_u/d_{uv}^{\theta} \neq p_v/d_{vu}^{\theta}$, even if $p_u = p_v$. In a bidirectional topology, only bidirectional edges $[u, v]$ are used as communication links. Moraes et al. (2009) showed that this problem variant is more realistic, but they did not explicitly consider the effects of interference from concurrent transmissions. However, the authors conjectured (Moraes and Ribeiro, 2013) that solutions with low total power consumption may give small interference values. The computational experiments reported in Section 4 show that the formulation presented in Section 3.1 to solve the minimum power consumption problem gives low interference communication graphs when compared with the optimal values given by the formulations proposed in Sections 3.2, 3.3, and 3.4 to solve the k-CMI problem, even though it does not explicitly consider interference. In this section, we incorporate the effects of interference using the wireless *ad hoc* interference models presented in Section 2, starting by the description of the incremental power model proposed in Moraes et al. (2009).

3.1. Incremental power model

The formulations of both k-CMI and the k-connected minimum power consumption problem consider a set C of $\lceil k|V|/2 \rceil$ commodities with a demand of one unit each (Moraes et al., 2009), as originally proposed in Raghavan (1995). For each commodity $c \in C$, we represent by $o(c)$ its origin and by $d(c)$ its destination. For any node $u \in V$ and any commodity $c \in C$, let $D_c(u) = -k$ if $i = o(c)$, $D_c(u) = +k$ if $u = d(c)$, and $D_c(u) = 0$ otherwise. The discrete variable f_{uv}^c represents the

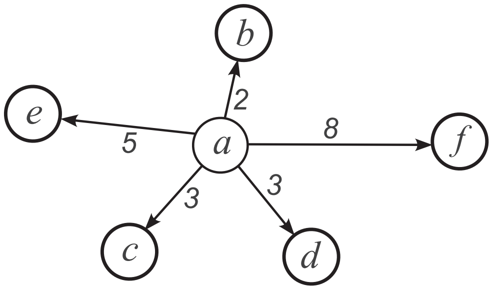


Fig. 3. Increasing discrete power levels for node a: $P_a = [2, 3, 5, 8]$; reachable nodes from node a: $S_a^1 = \{b\}$, $S_a^2 = \{b, c, d\}$, $S_a^3 = \{b, c, d, e\}$, and $S_a^4 = \{b, c, d, e, f\}$; successive cumulative power increments to node a: $Q_a = [2, 1, 2, 3]$; and incrementally reachable nodes from node a: $T_a^1 = \{b\}$, $T_a^2 = \{c, d\}$, $T_a^3 = \{e\}$, $T_a^4 = \{f\}$.

flow of commodity c through arc (u, v). The binary variable f_{uv}^c is equal to 1 if arc (u, v) is used by commodity c for communication from node u to v, 0 otherwise.

Let $P_u = [p_u^1, \ldots, p_u^{\phi(u)}]$ be a finite list of increasing discrete power levels that can be assigned to node $u \in V$. We denote by p_u^1 the minimum power assignment p_u such that transmissions from node u reach at least one node in $V \setminus \{u\}$. Furthermore, $\phi(u) \leq |V| - 1$ and $p_u^{\ell+1} > p_u^\ell$ for any $\ell = 1, \ldots, \phi(u) - 1$. We define S_u^ℓ as the set of nodes reachable from node u with the power assignment $p_u = p_u^\ell$, for any $\ell = 1, \ldots, \phi(u)$, as illustrated in Fig. 3. For ease of notation, we define $S_0 = \emptyset$.

Let $Q_u = [q_u^1, \ldots, q_u^{\phi(u)}]$ be a finite list of successive cumulative increments in the power that can be assigned to node u, for any $u \in V$. Furthermore, let T_u^ℓ be the set of new nodes reachable from node u if an additional increment q_u^ℓ is added to its current power assignment. Considering each list P_u and the sets S_u^ℓ, for $u \in V$ and $\ell = 1, \ldots, \phi(u) - 1$, we have $q_u^1 = p_u^1$, $T_u^1 = S_u^1$, $q_u^\ell = p_u^\ell - p_u^{\ell-1}$ and $T_u^\ell = S_u^\ell - S_u^{\ell-1}$ for any $\ell = 2, \ldots, \phi(u)$, once again as illustrated in Fig. 3.

The binary variable x_u^ℓ takes the value 1 if there is a node $v \in T_u^\ell$ such that arc (u, v) is used for communication from node u to v, 0 otherwise. We also define $\bar{\ell}(u) \in \{1, \ldots, \phi(u)\}$ such that $|S_u^{\bar{\ell}(u)-1}| < k \leq |S_u^{\bar{\ell}(u)}|$. Then, for any node u, $|S_u^{\bar{\ell}(u)}|$ gives the minimum number of nodes needed to establish the k-connectivity requirement from node u.

The integer program defined by the objective function (7) and constraints (8)–(14) presented in Fig. 4 is a valid formulation for the asymmetric version of the k-connected minimum power consumption problem with bidirectional topology.

Constraints (8) are flow conservation equations. Inequalities (9) ensure node disjointness. Inequalities (10) state that x_u^ℓ must be set to 1 if there is a node $v \in T_u^\ell$ such that arc (u, v) (resp. arc (v, u)) is used for communication from node u to v (resp. from node v to u) by commodity c. In other words, they ensure that a bidirectional edge $[u, v]$ is used if there is flow from u to v or from v to u. Constraints (11) enforce $x_u^{\ell+1}$ to be equal to 0 if the previous increment was not used, that is, if $x_u^\ell = 0$. Constraints (12) set to 1 the incremental powers that are necessary to reach at least the k closest nodes of each node u. Constraints (13) and (14) express the integrality requirements. The power assignments to the transmitter nodes are given by $p_u = \sum_{\ell=1}^{\phi(u)} q_u^\ell x_u^\ell$, for any $u \in V$.

This formulation builds a k-connected communication graph minimizing the transmission power without taking interference into account. The formulations proposed in the following build a k-connected communication graph with minimum interference.

$$\text{minimize} \quad \sum_{u\in V}\sum_{\ell=1}^{\phi(u)} q_u^\ell x_u^\ell \tag{7}$$

subject to

$$\sum_{v\in V} f_{vu}^c - \sum_{w\in V} f_{uw}^c = D_c(u) \qquad \forall c \in C, \forall u \in V \tag{8}$$

$$\sum_{v\in V} f_{uv}^c \le 1 \qquad \forall c \in C, \forall u \in V : u \neq o(c), u \neq d(c) \tag{9}$$

$$x_u^\ell \ge f_{uv}^c + f_{vu}^c \qquad \forall u \in V,\ c \in C,\ v \in T_u^\ell \text{ and } \ell = 1,\dots,\phi(u) \tag{10}$$

$$x_u^{\ell+1} \le x_u^\ell \qquad \forall u \in V \text{ and } \ell = 1,\dots,\phi(u)-1 \tag{11}$$

$$x_u^\ell = 1 \qquad \forall u \in V \text{ and } \ell = 1,\dots,\bar{\ell}(u) \tag{12}$$

$$f_{uv}^c \in \{0,1\} \qquad \forall u, v \in V, \forall c \in C \tag{13}$$

$$x_u^\ell \in \{0,1\} \qquad \forall u \in V \text{ and } \ell = 1,\dots,\phi(u). \tag{14}$$

Fig. 4. Incremental power model (IP).

3.2. Exact formulation with the Boolean interference model

Given a power assignment p, we used in the previous section the set of bidirectional edges $E(p) = \{[u, v] : u, v \in V, p_u \ge d_{uv}^\theta, p_v \ge d_{vu}^\theta\}$ to enforce that the resulting undirected communication graph $G_E(p) = (V, E(p))$ be k-connected. As discussed, the k-connected bidirectional topology is more useful, more realistic, and provides better performance (Marina and Das, 2002) when compared to the unidirectional topology.

The same power assignment p also determines the set of unidirectional arcs $A(p) = \{(u, v) : u, v \in V, p_u \ge d_{uv}^\theta\}$ such that edge $[u, v] \in E(p)$ if and only if arc $(u, v) \in A(p)$ and arc $(v, u) \in A(p)$. Since unidirectional links are used in the definitions of the receiver-centric perspective (Section 2.1) of the Boolean interference model, as well as in the definitions of the protocol (Section 2.2) and physical (Section 2.3) interference models, they allow for comparisons between the three interference models (Boolean, protocol, and physical) while solving the k-CMI problem.

Given an unidirectional link (u, v), the protocol and physical interference models define the interference observed by the receiver v as a relation between the attenuated transmission power p_u/d_{uv}^θ sent from the transmitter u (whose message must be successfully decoded by v) and the attenuated transmission power from any other simultaneously transmitting node $t \in V \setminus \{u, v\}$. The receiver-centric interference perspective of the Boolean model is defined as the number of nodes potentially disturbing reception of a message at the receiver v of the unidirectional link (u, v).

Let $G_A(p) = (V, A(p))$ and $G_E(p) = (V, E(p))$ be, respectively, the directed and undirected communication graphs defined by power assignment p. According to the Boolean receiver-centric interference model, interference occurs at node v of a link (u, v) if there is a node $t \in V \setminus \{u, v\}$ whose transmitting power satisfies $p_t \ge d_{tv}^\theta$. In other words, simultaneous communication without interference over receiver v of link (u, v) is possible when $p_u \ge d_{uv}^\theta$ and $p_t < d_{tv}^\theta$ for any node $t \in V \setminus \{u, v\}$.

Under this model, the Boolean interference from transmitter t over the receiver node v of link (u, v) is defined as

$$\text{boI}^{t,v}_{(u,v)} = \begin{cases} 1, & \text{if } p_t \geq d^{\theta}_{tv}, \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

for all $t \in V \setminus \{u, v\}$, and the total interference over receiver v of link (u, v) is formally defined as

$$\text{boI}^{v}_{(u,v)} = \sum_{t \in V \setminus \{u,v\}} \text{boI}^{t,v}_{(u,v)}. \tag{16}$$

The communication over a link (u, v) is successful when $\text{boI}^{v}_{(u,v)} = 0$. If the k-connectivity property is achieved and $\text{boI}^{v}_{(u,v)} = k - 1$, then receiver v is covered by the minimum necessary communication links needed to ensure the k-connectivity property. The exact formulation consists in minimizing the maximum receiver interference value over all power assignments:

$$\min \quad \text{boI} = \max_{u,v \in V} \text{boI}^{v}_{(u,v)}. \tag{17}$$

Since we are considering the bidirectional edges $[u, v] \in E(p)$ to enforce the k-connectivity property, we are interested in minimizing the interference over a receiver v that belongs to a nondummy unidirectional link $(u, v) \in A(p)$. A link $(u, v) \in A(p)$ is said to be nondummy if the bidirectional edge $[u, v] \in E(p)$ is used as a communication link to set the k-connectivity property.

Let the binary variables z^t_{uv} take the value 1 if link (u, v) is nondummy and the transmission from transmitter t interferes in the transmission from u to v as defined in Equation (15), that is, $p_u \geq d^{\theta}_{uv}$, $p_v \geq d^{\theta}_{vu}$ and $p_t \geq d^{\theta}_{tv}$. The integer program BO defined by the objective function (18) and constraints (8)–(23) presented in Fig. 5 is a valid formulation for the k-CMI problem assuming the receiver-centric perspective of the Boolean interference model.

Constraints (8)–(14) enforce the k-connectivity restrictions, as discussed in Section 3.1 (see Fig. 4). Constraints (19) give the maximum receiver interference value, which is minimized by the objective function (18). In constraints (19), (20), and (21), variables z^t_{uv} denote the Boolean interference from transmitter t over the receiver node v of a nondummy link (u, v), as defined in Equation (15). Constraints (20) and (21) set z^t_{uv} to 1 if (u, v) is nondummy and the transmission from transmitter t interferes in the transmission from u to v, that is, $z^t_{uv} = 1$, if and only if variables $x^{\ell}_u = x^m_v = x^n_t = 1$: variable x^{ℓ}_u (resp. x^m_v) takes the value 1 when the power assignment p_u (resp. p_v) is greater than or equal to the power level p^{ℓ}_u (resp. p^m_v) needed to set the communication link (u, v) (resp. (v, u)), that is, $p_u \geq p^{\ell}_u$ and $\ell : v \in T^{\ell}_u$ (resp. $p_v \geq p^m_v$ and $m : u \in T^m_v$). Thus, if both variables x^{ℓ}_u and x^m_v are set to 1, then the bidirectional edge $[u, v] \in E(p)$ is used as a communication link and, consequently, we have a nondummy link (u, v). Any transmitter $t \in V \setminus \{u, v\}$ interferes in the transmission from u to v if its power assignment p_t reaches v, that is, if $p_t \geq p^n_t$ and $n : v \in T^n_t$, then $x^n_t = 1$. Constraints (22) and (23) express the integrality and nonnegativity requirements on the variables.

$$\text{minimize} \quad \text{boI} \tag{18}$$

subject to

$$\sum_{v \in V} f^c_{vu} - \sum_{w \in V} f^c_{uw} = D_c(u) \qquad \forall c \in C, u \in V \tag{8}$$

$$\sum_{v \in V} f^c_{uv} \leq 1 \qquad \forall c \in C, u \in V : u \neq o(c), u \neq d(c) \tag{9}$$

$$x^\ell_u \geq f^c_{uv} + f^c_{vu} \qquad \forall u \in V, c \in C, v \in T^\ell_u \text{ and } \ell = 1, \ldots, \phi(u) \tag{10}$$

$$x^{\ell+1}_u \leq x^\ell_u \qquad \forall u \in V \text{ and } \ell = 1, \ldots, \phi(u) - 1 \tag{11}$$

$$x^\ell_u = 1 \qquad \forall u \in V \ell = 1, \ldots, \bar{\ell}(u) \tag{12}$$

$$f^c_{uv} \in \{0, 1\} \qquad \forall u, v \in V, c \in C \tag{13}$$

$$x^\ell_u \in \{0, 1\} \qquad \forall u \in V \text{ and } \ell = 1, \ldots, \phi(u). \tag{14}$$

$$\text{boI} \geq \sum_{t \in V \setminus \{u,v\}} z^t_{uv} \quad \forall u, v \in V \tag{19}$$

$$z^t_{uv} \geq (x^\ell_u + x^m_v + x^n_t) - 2 \qquad \forall u, v \in V, t \in V \setminus \{u, v\}, \ell : v \in T^\ell_u, m : u \in T^m_v, n : v \in T^n_t \tag{20}$$

$$z^t_{uv} \leq (x^\ell_u + x^m_v + x^n_t)/3 \qquad \forall u, v \in V, t \in V \setminus \{u, v\}, \ell : v \in T^\ell_u, m : u \in T^m_v, n : v \in T^n_t \tag{21}$$

$$z^t_{uv} \in \{0, 1\} \qquad u, v \in V, t \in V \setminus \{u, v\} \tag{22}$$

$$\text{boI} \geq 0 \tag{23}$$

Fig. 5. Boolean interference (BO) model.

3.3. Exact formulation with the protocol interference model

Let $G_A(p) = (V, A(p))$ and $G_E(p) = (V, E(p))$ be, respectively, the directed and undirected communication graphs defined by a power assignment p. According to the protocol interference model (4), interference occurs at node v of a link (u, v) if there is at least one node $t \in V \setminus \{u, v\}$ such that

$$(1 + \Delta)\frac{p_t}{d^\theta_{tv}} - \frac{p_u}{d^\theta_{uv}} > 0. \tag{24}$$

Therefore, the observed interference from the transmitter t over the receiver node v of the unidirectional link (u, v) is defined as

$$\text{prI}^{t,v}_{(u,v)} = \begin{cases} 1, & \text{if } \dfrac{(1+\Delta)p_t}{d^\theta_{tv}} - \dfrac{p_u}{d^\theta_{uv}} > 0; \\ 0, & \text{otherwise,} \end{cases} \tag{25}$$

for all $t \in V \setminus \{u, v\}$, and the total interference over receiver v of link (u, v) is formally defined as

$$\text{prI}^v_{(u,v)} = \sum_{t \in V \setminus \{u,v\}} \text{prI}^{t,v}_{(u,v)}. \tag{26}$$

The communication over a link (u, v) is successful if $\mathrm{prI}^{t,v}_{(u,v)} = 0$, for any $t \in V \setminus \{u, v\}$. The exact formulation consists in minimizing the maximum receiver interference value over all power assignments:

$$\min \quad \mathrm{prI} = \max_{u,v \in V} \mathrm{prI}^{v}_{(u,v)}. \tag{27}$$

The integer program PR defined by the objective function (28) and constraints (8)–(37) presented in Fig. 6 is a valid formulation for the k-CMI problem assuming the protocol interference model. Constraints (8)–(14) were discussed in Section 3.1 (see Fig. 4), while constraints (22) were discussed in Section 3.2 (see Fig. 5). Inequalities (29) calculate the observed interference over the receiver v of link (u, v), as defined for the protocol interference model (see Equation (25)). Constant M_1 is large enough to allow setting variable $\mathrm{prI}^{t,v}_{(u,v)}$ to 1 whenever node t interferes over the receiver v of link

$$\text{minimize} \quad \mathrm{prI} \tag{28}$$

subject to

$$\sum_{v \in V} f^c_{vu} - \sum_{w \in V} f^c_{uw} = D_c(u) \qquad \forall c \in C, u \in V \tag{8}$$

$$\sum_{v \in V} f^c_{uv} \le 1 \qquad \forall c \in C, u \in V : u \neq o(c), u \neq d(c) \tag{9}$$

$$x^\ell_u \ge f^c_{uv} + f^c_{vu} \qquad \forall u \in V, c \in C, v \in T^\ell_u \text{ and } \ell = 1, \dots, \phi(u) \tag{10}$$

$$x^{\ell+1}_u \le x^\ell_u \qquad \forall u \in V \text{ and } \ell = 1, \dots, \phi(u) - 1 \tag{11}$$

$$x^\ell_u = 1 \qquad \forall u \in V \ell = 1, \dots, \bar{\ell}(u) \tag{12}$$

$$f^c_{uv} \in \{0,1\} \qquad \forall u, v \in V, c \in C \tag{13}$$

$$x^\ell_u \in \{0,1\} \qquad \forall u \in V \text{ and } \ell = 1, \dots, \phi(u). \tag{14}$$

$$z^t_{uv} \in \{0,1\} \qquad u, v \in V, t \in V \setminus \{u, v\} \tag{22}$$

$$M_1 \cdot \mathrm{prI}^{t,v}_{(u,v)} \ge \frac{(1+\Delta)p_t}{d^\theta_{tv}} - \frac{p_u}{d^\theta_{uv}} \qquad \forall u, v \in V, t \in V \setminus \{u, v\} \tag{29}$$

$$\mathrm{prI} \ge \sum_{t \in V \setminus \{u,v\}} z^t_{uv} \qquad \forall u, v \in V \tag{30}$$

$$z^t_{uv} \ge (x^\ell_u + x^m_v + \mathrm{prI}^{t,v}_{(u,v)}) - 2 \qquad \forall u, v \in V, t \in V \setminus \{u, v\}, \ell : v \in T^\ell_u, m : u \in T^m_v \tag{31}$$

$$z^t_{uv} \le (x^\ell_u + x^m_v + \mathrm{prI}^{t,v}_{(u,v)})/3 \qquad \forall u, v \in V, t \in V \setminus \{u, v\}, \ell : v \in T^\ell_u, m : u \in T^m_v \tag{32}$$

$$p_u \ge \sum_{\ell=1}^{\phi(u)} q^\ell_u x^\ell_u \qquad \forall u \in V \tag{33}$$

$$M_2 \cdot x^\ell_u \ge p_u - d^\theta_{uv} \qquad \forall u, v \in V, \ell : v \in T^\ell_u \tag{34}$$

$$0 \le p_u \le (1+\alpha) \max_{v \in V} \{d^\theta_{uv}\} \qquad \forall u \in V \tag{35}$$

$$\mathrm{prI}^{t,v}_{(u,v)} \in \{0,1\} \qquad \forall u, v \in V, t \in V \setminus \{u, v\} \tag{36}$$

$$0 \le \mathrm{prI} \le |V| \tag{37}$$

Fig. 6. Protocol interference model (PR).

(u, v). Constraints (30), (31), and (32) are equivalent, respectively, to constraints (19), (20), and (21) of the previous formulation using the Boolean interference model (Fig. 5).

In the IP formulation presented in Fig. 4, the power assignment of a transmitter u is defined only by the binary variable x_u^ℓ, which takes the value 1 to enforce the k-connectivity restrictions. Consequently, the power assignment to transmitter u is given by $p_u = \sum_{\ell=1}^{\phi(u)} q_u^\ell x_u^\ell$, for any $u \in V$. In the PR formulation presented in Fig. 6, the power assignment of a transmitter u can also be defined by the interference constraints (29), which allow any transmitter u to increase or decrease its power p_u to avoid interference. Consequently, the power assignment to transmitter u must be bounded by $\sum_{\ell=1}^{\phi(u)} q_u^\ell x_u^\ell \leq p_u \leq (1+\alpha)\max_{v\in V}\{d_{uv}^\theta\}$, for any $u \in V$, as enforced by constraints (33) and (35). A new node v may be added to the set S_u^ℓ of nodes reachable from u (see Fig. 3) whenever the power p_u is increased to avoid interference. In other words, we have $p_u \geq p_u^\ell$, $\ell : v \in T_u^\ell$ and $v \notin T_u^{\ell-1}$, and $x_u^\ell = 0$. In this case, however, to indicate the existence of arc (u, v), variable x_u^ℓ must be set to 1. This is enforced by constraints (34), where constant M_2 is large enough to allow setting variable x_u^ℓ to 1 when $p_u \geq p_u^\ell$, where $p_u^\ell = d_{uv}^\theta$ and $\ell : v \in T_u^\ell$.

In the previous formulations, the maximum power assigned to a transmitter is the power needed to cover the farthest node from it. In the PR formulation presented in Fig. 6, any transmitter can increase its power beyond the power that is needed to cover the farthest node from it, with the purpose of avoiding interference. Thus, constraints (35) allow each node to overpass the power needed to cover the farthest node from it by a factor $\alpha \in [0, 1]$, which is defined considering the transmitter's hardware limitation. Constraints (36) and (37) express the integrality and nonnegativity requirements on the variables.

3.4. Exact formulation with physical interference model

Let $G_A(p) = (V, A(p))$ and $G_E(p) = (V, E(p))$ be, respectively, the directed and undirected communication graphs defined by the power assignment p. According to the physical interference model (see Expression (6)), interference occurs at a node v of link (u, v) whenever

$$\sum_{t\in V\setminus\{u,v\}} \frac{p_t}{d_{tv}^\theta} > \left(\frac{1}{\beta} \times \frac{p_u}{d_{uv}^\theta}\right) - \sigma^2. \tag{38}$$

Thus, the observed interference from the transmitter t over the receiver node v of the unidirectional nondummy link (u, v) is defined as

$$\mathrm{phI}_{(u,v)}^{t,v} = \frac{p_t}{d_{tv}^\theta}, \tag{39}$$

for all $t \in V \setminus \{u, v\}$, and the interference over the receiver node v of the nondummy link (u, v) is defined as

$$\mathrm{phI}_{(u,v)}^{v} = \begin{cases} 1 & \text{if } \sum_{t\in V\setminus\{u,v\}} \mathrm{phI}_{(u,v)}^{t,v} > \left(\frac{1}{\beta} \times \frac{p_u}{d_{uv}^\theta}\right) - \sigma^2 \,. \\ 0 & \text{otherwise.} \end{cases} \tag{40}$$

$$\text{minimize} \quad \sum_{u \in V} \sum_{v \in V} \text{phI}^{v}_{(u,v)} \tag{41}$$

subject to

$$\sum_{v \in V} f^c_{vu} - \sum_{w \in V} f^c_{uw} = D_c(u) \qquad \forall c \in C, u \in V \tag{8}$$

$$\sum_{v \in V} f^c_{uv} \leq 1 \qquad \forall c \in C, u \in V : u \neq o(c), u \neq d(c) \tag{9}$$

$$x^\ell_u \geq f^c_{uv} + f^c_{vu} \qquad \forall u \in V, c \in C, v \in T^\ell_u \text{ and } \ell = 1, \ldots, \phi(u) \tag{10}$$

$$x^{\ell+1}_u \leq x^\ell_u \qquad \forall u \in V \text{ and } \ell = 1, \ldots, \phi(u) - 1 \tag{11}$$

$$x^\ell_u = 1 \qquad \forall u \in V \text{ and } \ell = 1, \ldots, \bar{\ell}(u) \tag{12}$$

$$f^c_{uv} \in \{0, 1\} \qquad \forall u, v \in V, c \in C \tag{13}$$

$$x^\ell_u \in \{0, 1\} \qquad \forall u \in V \text{ and } \ell = 1, \ldots, \phi(u). \tag{14}$$

$$p_u \geq \sum_{\ell=1}^{\phi(u)} q^\ell_u x^\ell_u \qquad \forall u \in V \tag{33}$$

$$M_2 \cdot x^\ell_u \geq p_u - d^\theta_{uv} \qquad \forall u, v \in V, \ell : v \in T^\ell_u \tag{34}$$

$$0 \leq p_u \leq (1 + \alpha) \max_{v \in V} \{d^\theta_{uv}\} \qquad \forall u \in V \tag{35}$$

$$M_4 \cdot z_{uv} \geq \sum_{t \in V \setminus \{u,v\}} \frac{p_t}{d^\theta_{tv}} - \left(\frac{p_u}{\beta d^\theta_{uv}} - \sigma^2 \right) \qquad \forall u, v \in V \tag{42}$$

$$\text{phI}^{v}_{(u,v)} \geq x^\ell_u + x^m_v + z_{uv} - 2 \qquad \forall u, v \in V, \ell : v \in T^\ell_u, m : u \in T^m_v \tag{43}$$

$$\text{phI}^{v}_{(u,v)} \leq (x^\ell_u + x^m_v + z_{uv})/3 \qquad \forall u, v \in V, \ell : v \in T^\ell_u, m : u \in T^m_v \tag{44}$$

$$z_{uv} \in \{0, 1\} \qquad \forall u, v \in V \tag{45}$$

$$\text{phI}^{v}_{(u,v)} \in \{0, 1\} \qquad \forall u, v \in V \tag{46}$$

Fig. 7. Physical interference model (PH).

If $\text{phI}^{v}_{(u,v)} = 1$, then the aggregate interference makes communication from u to v impossible. The exact formulation consists in minimizing the number of receiver nodes v of nondummy links (u, v) to suffer interference over all power assignments:

$$\min \quad \text{phI} = \sum_{u \in V} \sum_{v \in V} \text{phI}^{v}_{(u,v)}. \tag{41}$$

The integer program PH defined by the objective function (41) and constraints (8)–(46) is a valid formulation for the k-CMI problem, assuming the physical interference model. Constraints (8)–(14) were discussed in Section 3.1 (see Fig. 4). Constraints (33)–(35) were discussed in Section 3.3 (see Fig. 6). Constraints (42) give the aggregate interference over the receiver node v of link (u, v) as defined in the physical interference model using Equation (38). Constant M_4 is large enough to allow for setting variable z_{uv} to 1 whenever the aggregate interference makes communication over

$$\text{minimize} \quad \text{phcI} \tag{47}$$

subject to

$$\sum_{v \in V} f^c_{vu} - \sum_{w \in V} f^c_{uw} = D_c(u) \qquad \forall c \in C, u \in V \tag{8}$$

$$\sum_{v \in V} f^c_{uv} \leq 1 \qquad \forall c \in C, u \in V : u \neq o(c), u \neq d(c) \tag{9}$$

$$x^\ell_u \geq f^c_{uv} + f^c_{vu} \qquad \forall u \in V, c \in C, v \in T^\ell_u \text{ and } \ell = 1, \ldots, \phi(u) \tag{10}$$

$$x^{\ell+1}_u \leq x^\ell_u \qquad \forall u \in V \text{ and } \ell = 1, \ldots, \phi(u) - 1 \tag{11}$$

$$x^\ell_u = 1 \qquad \forall u \in V \text{ and } \ell = 1, \ldots, \bar{\ell}(u) \tag{12}$$

$$f^c_{uv} \in \{0, 1\} \qquad \forall u, v \in V, c \in C \tag{13}$$

$$x^\ell_u \in \{0, 1\} \qquad \forall u \in V \text{ and } \ell = 1, \ldots, \phi(u). \tag{14}$$

$$p_u \geq \sum_{\ell=1}^{\phi(u)} q^\ell_u x^\ell_u \qquad \forall u \in V \tag{33}$$

$$M_2 \cdot x^\ell_u \geq p_u - d^\theta_{uv} \qquad \forall u, v \in V, \ell : v \in T^\ell_u \tag{34}$$

$$0 \leq p_u \leq (1 + \alpha) \max_{v \in V} \{d^\theta_{uv}\} \qquad \forall u \in V \tag{35}$$

$$\text{phcI}^v_{(u,v)} \geq \sum_{t \in V \setminus \{u,v\}} \frac{p_t}{d^\theta_{tv}} - \left(\frac{p_u}{\beta d^\theta_{uv}} - \sigma^2 \right) - (1 - y_{uv}) \cdot M_5 \qquad \forall u, v \in V \tag{48}$$

$$\text{phcI} \geq \text{phcI}^v_{(u,v)} \qquad \forall u, v \in V \tag{49}$$

$$y_{uv} \geq x^\ell_u + x^m_v - 1 \qquad \forall u, v \in V, \ell : v \in T^\ell_u, m : u \in T^m_v \tag{50}$$

$$y_{uv} \leq (x^\ell_u + x^m_v)/2 \qquad \forall u, v \in V, \ell : v \in T^\ell_u, m : u \in T^m_v \tag{51}$$

$$y_{uv} \in \{0, 1\} \qquad \forall u, v \in V \tag{52}$$

$$\text{phcI}^v_{(u,v)} \geq 0 \qquad \forall u, v \in V \tag{53}$$

Fig. 8. Physical interference model with continuous interference (PHci).

link (u, v) impossible. Constraints (43) and (44) state that $\text{phI}^v_{(u,v)}$ must be set to 1 if the link (u, v) is nondummy and the communication from u to v suffers interference. Constraints (45) and (46) define the domain of the variables.

We use the fact that interference is intrinsically continuous in the physical interference model to propose another exact formulation, named PHci. In this formulation, we interpret the physical interference over the receiver node v of a link (u, v) as a continuous variable:

$$\text{phcI}^v_{(u,v)} = \sum_{t \in V \setminus \{u,v\}} \text{phI}^{t,v}_{(u,v)} - \left(\left(\frac{1}{\beta} \times \frac{p_u}{d^\theta_{uv}} \right) - \sigma^2 \right). \tag{54}$$

While formulation PH minimizes the number of receiver nodes v of nondummy links (u, v) to suffer interference, the exact formulation PHci proposed in Fig. 8 minimizes the maximum interference value over all receiver nodes v of all nondummy links (u, v), using the continuous interference interpretation:

$$\min \quad \text{phcI} = \max_{u,v \in V} \text{phcI}^{v}_{(u,v)}. \tag{55}$$

Let the binary variable y_{uv} take the value 1 if link (u, v) is nondummy, that is, $p_u \geq d^{\theta}_{uv}$ and $p_v \geq d^{\theta}_{vu}$. The integer program PHci defined by the objective function (47) and constraints (8)–(53) presented in Fig. 8 is a valid formulation for the k-CMI problem assuming the physical interference model with continuous interference. Constraints (8)–(14) were discussed in Section 3.1 (see Fig. 4). Constraints (33)–(35) were discussed in Section 3.3 (see Fig. 6).

Inequalities (48) calculate the observed link interference over the receiver node v of a nondummy link (u, v). Constant M_5 is large enough to allow setting variable $\text{phcI}^{v}_{(u,v)}$ to 0 whenever the aggregate interference does not interfere over the receiver node v of link (u, v). Constraints (49) identify the maximum interference value over the receiver node v of all nondummy links (u, v), which is minimized by the objective function (47). Constraints (50) and (51) state that y_{uv} must be set to 1 when the link (u, v) is nondummy. Constraints (52) and (53) express the integrality and nonnegativity requirements on the variables.

4. Experimental results

Computational experiments have been carried out on a set of moderate size random asymmetric instances with $10 \leq |V| \leq 40$ nodes uniformly distributed in the unit square grid. The weight of the arc between nodes u and v is given by $F \times d^{\theta}_{uv}$, where d_{uv} is the Euclidean distance between nodes u and v, θ is the path loss exponent set at 2, and F is a random uniform perturbation in the interval [0.8, 1.2]. We allow a node to overpass the power needed to cover the farthest node from it by a factor of at most 20%, that is, $\alpha = 0.2$ (see Fig. 6). For the sake of simplicity, we also assume that $\sigma^2 = 0$ (i.e., there is no ambient noise), $\beta = 1$, and $\Delta = 0$ (i.e., there is no spatial protection margin), as in Halldórsson (2012) and Tonoyan (2011). These settings reduce the power differences between the Boolean, protocol, and physical interference models.

Fifteen randomly generated instances have been created for each problem size $|V|$. Each instance is represented as a complete graph. We focus our analysis into the biconnected case ($k = 2$), since it gives the most useful fault-tolerant properties.

An Intel Xeon machine with a 2.40-GHz clock and 8-GB RAM memory running under GNU/Linux 2.6.24 was used in the experiments. ILOG CPLEX 12.4 was used as the linear and mixed-integer programming solver, with parallel features disabled. CPLEX was run once for each formulation and for each test instance.

For each problem size $|V|$ and each formulation, Table 1 shows the total number of instances solved to optimality and the total number of instances for which a feasible solution was found (but optimality was not necessarily proven) within eight hours of CPU time. All formulations found optimal solutions for $|V| \leq 20$ and feasible solutions for $|V| \leq 35$ within eight hours. All formulations have $O(|V|^3)$ constraints and variables. Model IP found the largest number of optimal solutions. Formulation IP is enhanced with interference considerations to form formulations BO,

Table 1
Number of instances solved to optimality and to feasibility within eight hours of running time

	IP		BO		PR		PH		PHci	
$\|V\|$	#opt.	#feas.	#opt.	#feas.	#opt.	#feas.	#opt.	#feas.	#opt.	#feas.
10	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	10	15	15	15	15	15
20	15	15	15	15	3	15	13	15	15	15
25	15	15	15	15	0	15	3	15	15	15
30	15	15	15	15	0	13	0	15	15	15
35	14	15	11	13	0	5	0	15	11	15
40	12	15	1	1	0	0	0	11	11	15

#opt., number of instances solved to optimality; #feas., number of instances solved to feasibility.

Table 2
Average running times in seconds to find the optimal solutions

$\|V\|$	IP	BO	PR	PH	PHci
10 (15)	0.32	0.38	75.49	2.94	1.80
15 (10)	1.54	10.70	5153.74	140.29	7.21
20 (13)	15.61	332.15	–	7815.69	77.32
25 (15)	111.51	3300.99	–	–	433.74
30 (15)	813.89	5119.48	–	–	3019.51
35 (7)	1455.28	11,542.42	–	–	12,620.93
40 (10)	9317.74	–	–	–	15,589.75

PR, PH, and PHci. The results in Table 1 show that formulations BO and PHci found the maximum number of optimal solutions among those considering interference.

Since some formulations have not solved all instances to optimality within eight hours of computation, we present in Table 2 the average running times in seconds over a common set of at least seven instances, exclusively for the formulations that achieved optimality for all of them. The first column of Table 2 displays the problem size and the number of instances exactly solved by all formulations considered in this experiment. Formulations with blank cells have not been able to solve the common set of at least seven instances. Interference formulations BO and PHci presented smaller computation times than the other interference formulations.

Tables 1 and 2 also show that formulations PR and PH are more difficult to be solved, finding only three optimal solutions within the time limit of eight hours for $|V| = 20$ and $|V| = 25$, respectively. Considering the physical interference model, the continuous interference variables in formulation PHci make it easier to be solved by CPLEX than the noncontinuous formulation PH.

Among the formulations with noncontinuous interference values, the Boolean interference model BO leads CPLEX to find the largest number of optimal solutions, obtaining optimal solutions even for $|V| = 40$. The other formulations with noncontinuous interference values (PR and PH) evaluate interference differently from formulation BO and make use of power constraints, becoming harder to solve.

In the following experiments, we focus our analysis into the average values over the same set of instances exactly solved by all formulations considered in Table 2. As for Table 2, the first column

Table 3
Average total power consumption for each formulation

	IP		BO		PR		PH		PHci	
$\|V\|$	Power	Degradation (%)	Power	Degradation (%)	Power	Degradation (%)	Power	Degradation (%)	Power	Degradation (%)
10 (15)	**1.66383**	0.00	2.14030	28.63	2.73426	64.33	3.20268	92.48	2.00917	20.75
15 (10)	**1.43201**	0.00	1.80642	26.14	2.29552	60.30	3.68530	157.35	1.68469	17.64
20 (13)	**1.41856**	0.00	1.79492	26.53	–	–	3.22641	127.44	1.78873	26.09
25 (15)	**1.37880**	0.00	1.92755	39.79	–	–	–	–	1.64632	19.40
30 (15)	**1.31779**	0.00	1.69455	28.59	–	–	–	–	1.54352	17.13
35 (7)	**1.24709**	0.00	1.77801	42.57	–	–	–	–	1.40274	12.48
40 (10)	**1.22817**	0.00	–	–	–	–	–	–	1.37696	12.11

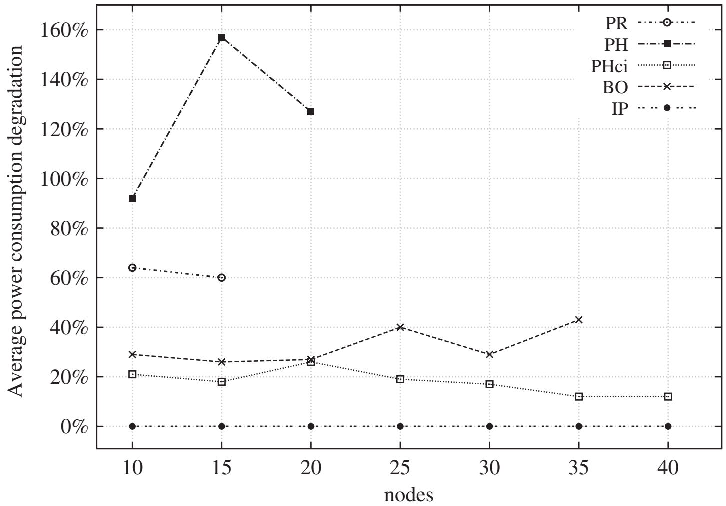


Fig. 9. Average total power consumption degradation.

of all forthcoming tables displays the problem size and the number of instances exactly solved by all formulations. Formulations with blank cells have not been able to solve a common set of at least seven instances.

Next, we assess the quality of the solutions produced by the different formulations, in terms of their power consumption $\sum_{u \in V} p_u$. Table 3 presents the average power consumption over the same set of instances exactly solved by all formulations considered in Table 2. Table 3 and Fig. 9 show the average power consumption degradation in percent obtained by each formulation, with respect to the best average power consumption. The best average power consumption values are those depicted in bold in Table 3. Formulation IP is necessarily the formulation that finds the minimum average power consumption values for all problem sizes but, of course, it does not take into account the minimization of the interference.

Figure 9 shows that formulation PHci leads to the lowest power consumption values when interference is minimized. This formulation obtains solutions whose average power consumption is at most 26.09% from the minimum, while minimizing the continuous physical interference. The

Boolean model BO found the smallest average power consumption among the formulations considering noncontinuous interference, exceeding the minimum by at most 42.57%. Low power settings reduce energy consumption and increase network lifetime. Although formulations PR and PH minimize interference, they lead to solutions with average power consumption up to 157.35% greater than the minimum corresponding to the case where interference is not considered.

Tables 4 and 5 show the average noncontinuous interference values and the continuous interference value, respectively. Table 4(a) and Fig. 10(a) present the average noncontinuous interference $\text{boI} = \max_{u,v \in V} \text{boI}^v_{(u,v)}$ (17) and its degradation in percent with respect to the best average (in bold), for each formulation. Formulation BO was able to provide optimal (minimum) boI values for instances with $|V| \leq 35$. Similarly, Table 4(b) and Fig. 10(b) show the average noncontinuous interference $\text{prI} = \max_{u,v \in V} \text{prI}^v_{(u,v)}$ (27) and its degradation in percentage with respect to the best average (in bold), for each formulation. Formulation PR found optimal (minimum) prI values only for instances with $|V| \leq 15$. Table 4(c) and Fig. 10(c) show the average noncontinuous interference $\text{phI} = \sum_{u \in V} \sum_{v \in V} \text{phI}^v_{(u,v)}$ (41) and its degradation in percentage with respect to the best average (in bold), for each formulation. Formulation PH provides optimal (minimum) phI values for instances with $|V| \leq 20$.

Table 5 and Fig. 11 show the average continuous interference $\text{phcI} = \max_{u,v \in V} \text{phcI}^v_{(u,v)}$ (55) and its degradation in percentage with respect to the best average (in bold), for each formulation. Regarding the interference formulations, results in Tables 4 and 5 and Figs. 10 and 11 show that formulations BO and PHci lead to the smallest interference values in most cases, independently of how interference is calculated.

If interference is not considered explicitly in the objective function, then formulation IP presents the lowest boI, prI, phI, and phcI interference degradations after the formulation that considers explicitly the optimization of each interference measure. In particular, optimal solutions are known for all cells in Table 5 and formulation IP is clearly the second best in Fig. 11, with phcI values at most 70.63% from the optimal. Therefore, the optimal solutions minimizing the total power consumption given by formulation IP give low interference values, as conjectured in Moraes and Ribeiro (2013).

Each interference model defines an interference-free condition over each receiver node. Assuming the Boolean model, the receiver node v of link (u, v) is interference-free if $\text{boI}^v_{(u,v)} = 0$ (see Equation (16)). However, to ensure the k-connectivity property, receiver v must be covered by the minimum necessary communication links needed to achieve the k-connectivity, that is, $\text{boI}^v_{(u,v)} \geq k - 1$. Since in our analysis we set $k = 2$, the Boolean interference-free condition becomes $\text{boI}^v_{(u,v)} = 1$. In the protocol model, the receiver node v of link (u, v) is interference-free if $\frac{(1+\Delta)p_t}{d^\theta_{tv}} - \frac{p_u}{d^\theta_{uv}} \leq 0$, $\forall t \in V \setminus \{u, v\}$ (see Equations (4) and (24)). In the physical model, the receiver node v of link (u, v) is interference-free if $\sum_{t \in V \setminus \{u,v\}} \frac{p_t}{d^\theta_{tv}} \leq (\frac{1}{\beta} \times \frac{p_u}{d^\theta_{uv}}) - \sigma^2$ (see Equations (6) and (38)).

In the experiment reported next, each test set is characterized by a different combination of an interference-free condition (Boolean, protocol, and physical) with a problem size $|V| \in \{10, 15, 20, 25, 30, 35, 40\}$. We used 21 test sets for formulations IP and PHci, and 18, 9, and 6 test sets for formulations BO, PH, and PR, respectively. Table 6 shows, for each formulation, the average total number of receivers of the links used to set the biconnectivity property and, for each interference-free condition (Boolean, protocol, and physical), the average number of interference-free receivers of these links and the average relative differences B, P, and H in percentage between

Table 4
Average noncontinuous interference for each formulation

(a) Average boI = $\max_{u,v \in V} \text{boI}^{v}_{(u,v)}$ interference for each formulation

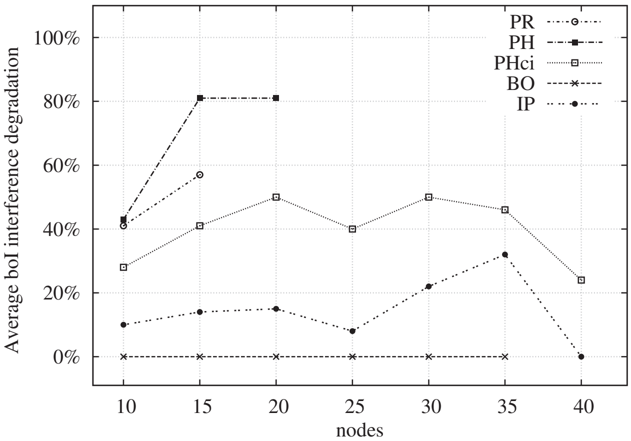
	IP		BO		PR		PH		PHci	
$\|V\|$	boI	Degradation (%)	boI	Degradation (%)	boI	Degradation (%)	boI	Degradation (%)	boI	Degradation (%)
10 (15)	4.46667	9.83	**4.06667**	0.00	5.73333	40.98	5.80000	42.62	5.20000	27.86
15 (10)	4.20000	13.51	**3.70000**	0.00	5.80000	56.75	6.70000	81.08	5.20000	40.54
20 (13)	4.61538	15.38	**4.00000**	0.00	–	–	7.23077	80.76	6.00000	50.00
25 (15)	4.66667	7.69	**4.33333**	0.00	–	–	–	–	6.06667	40.00
30 (15)	4.86667	21.66	**4.00000**	0.00	–	–	–	–	6.00000	50.00
35 (7)	5.28571	32.14	**4.00000**	0.00	–	–	–	–	5.85714	46.42
40 (10)	**5.10000**	0.00	–	–	–	–	–	–	6.30000	23.52

(b) Average prI = $\max_{u,v \in V} \text{prI}^{v}_{(u,v)}$ interference for each formulation

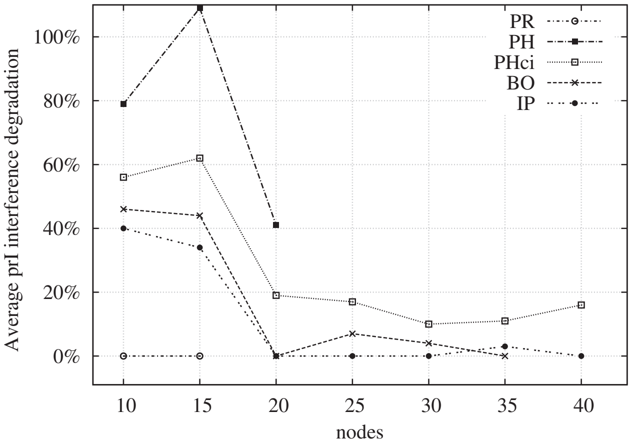
	IP		BO		PR		PH		PHci	
$\|V\|$	prI	Degradation (%)	prI	Degradation (%)	prI	Degradation (%)	prI	Degradation (%)	prI	Degradation (%)
10 (15)	4.46667	39.58	4.66667	45.83	**3.20000**	0.00	5.73333	79.16	5.00000	56.25
15 (10)	4.30000	34.37	4.60000	43.75	**3.20000**	0.00	6.70000	109.37	5.20000	62.50
20 (13)	**4.92308**	0.00	**4.92308**	0.00	–	–	6.92308	40.62	5.84615	18.75
25 (15)	**5.00000**	0.00	5.33333	6.66	–	–	–	–	5.86667	17.33
30 (15)	**5.26667**	0.00	5.46667	3.79	–	–	–	–	5.80000	10.12
35 (7)	5.42857	2.70	**5.28571**	0.00	–	–	–	–	5.85714	10.81
40 (10)	**5.10000**	0.00	–	–	–	–	–	–	5.90000	15.68

(c) Average phI interference for each formulation

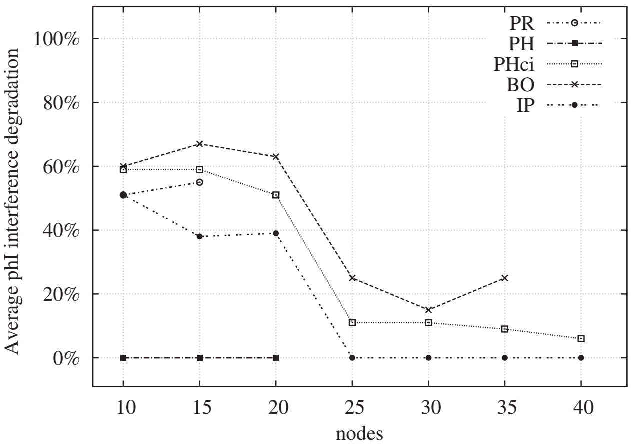
	IP		BO		PR		PH		PHci	
$\|V\|$	phI	Degradation (%)	phI	Degradation (%)	phI	Degradation (%)	phI	Degradation (%)	phI	Degradation (%)
10 (15)	12.66667	50.79	13.46667	60.31	12.66667	50.79	**8.40000**	0.00	13.33333	58.73
15 (10)	17.00000	38.21	20.50000	66.66	19.10000	55.28	**12.30000**	0.00	19.50000	58.53
20 (13)	24.53846	38.69	28.76923	62.60	–	–	**17.69231**	0.00	26.76923	51.30
25 (15)	**31.33333**	0.00	39.06667	24.68	–	–	–	–	34.80000	11.06
30 (15)	**36.93333**	0.00	42.60000	15.34	–	–	–	–	41.00000	11.01
35 (7)	**43.71429**	0.00	54.71429	25.16	–	–	–	–	47.71429	9.15
40 (10)	**51.80000**	0.00	–	–	–	–	–	–	54.80000	5.79



(a) Average boI interference degradation.



(b) Average prI interference degradation.



(c) Average phI interference degradation.

Fig. 10. Average noncontinuous interference degradation.

Table 5
Average phcI continuous interference for each formulation

$\|V\|$	IP phciI	IP Degradation (%)	BO phciI	BO Degradation (%)	PR phciI	PR Degradation (%)	PH phciI	PH Degradation (%)	PHci phciI	PHci Degradation (%)
10 (15)	29.92828	70.63	50.12387	185.77	41.83287	138.50	51.70619	194.80	**17.53943**	0.00
15 (10)	25.18343	55.77	34.57978	113.89	44.90264	177.74	79.04628	388.93	**16.16691**	0.00
20 (13)	26.66332	68.94	34.45930	118.34	–	–	93.94636	495.27	**15.78193**	0.00
25 (15)	24.41576	49.26	36.56715	123.55	–	–	–	–	**16.35750**	0.00
30 (15)	21.27237	24.92	32.06310	88.29	–	–	–	–	**17.02839**	0.00
35 (7)	19.21469	30.01	27.34241	85.01	–	–	–	–	**14.77891**	0.00
40 (10)	18.72114	23.91	–	–	–	–	–	–	**15.10825**	0.00

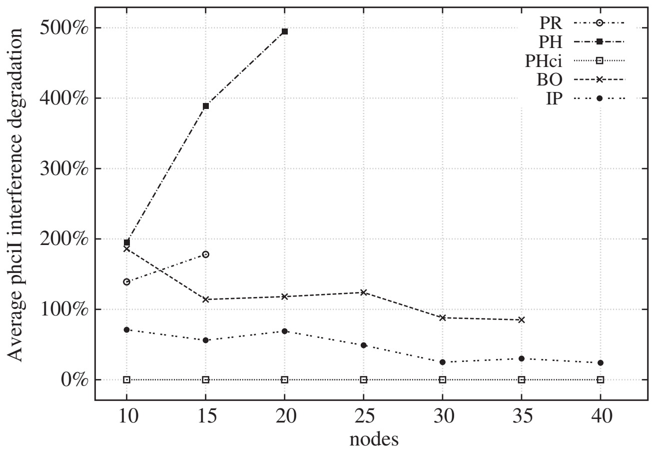


Fig. 11. Average continuous phciI interference degradation.

the total number of receivers and the number of interference-free receivers. The table depicts, in bold, the formulation that found the largest average relative difference for each test set. For instance, considering the test set with $|V| = 10$ and the protocol interference-free condition, the solutions obtained by formulation PH have the largest average relative difference (34.03%), that is, they have an average of 34.03% of receivers with no interference, which is greater than the average results found by the other formulations for the same test set. Still considering the protocol interference-free condition, the largest relative differences for $|V| = 15$ and $|V| = 25$ are obtained by formulations PR (30.75%) and IP (28.18%), respectively.

Table 6 shows that the solutions provided by formulation IP lead to the largest average relative differences for all test sets using the Boolean interference-free condition. For instance, solutions provided by formulation IP for $|V| = 40$ show an average of 22.81% of receivers with no interference. For $|V| \geq 25$, formulation IP leads to the largest B, P, and H differences for almost all test sets, with the exception of that defined by $|V| = 30$ and the interference-free condition, which has its largest value found by formulation BO. As for the interference degradation indices considered in Figs. 10 and 11, formulation IP obtains solutions with a large number of interference-free receivers, even though it does not explicitly consider interference.

Table 6
Average relative differences between the total number of receivers and the number of interference-free receivers for each formulation, considering the Boolean (B), protocol (P), and physical (H) interference-free conditions

	IP							BO						
$\|V\|$	Receivers	Free	B (%)	Free	P (%)	Free	H (%)	Receivers	Free	B (%)	Free	P (%)	Free	H (%)
10 (15)	29.47	6.27	**21.27**	8.53	28.96	2.07	7.01	30.67	3.60	11.74	8.20	26.74	1.87	6.09
15 (10)	37.80	10.20	**26.98**	10.80	28.57	1.90	5.03	44.00	6.40	14.55	11.60	26.36	1.50	3.41
20 (13)	53.23	12.62	**23.70**	14.77	27.75	2.08	3.90	61.08	6.62	10.83	16.38	26.83	1.77	2.90
25 (15)	66.00	14.13	**21.41**	18.60	**28.18**	1.67	**2.53**	82.27	4.00	4.86	19.93	24.23	2.07	2.51
30 (15)	76.93	17.47	**22.70**	22.33	**29.03**	1.53	1.99	89.20	7.47	8.37	23.80	26.68	2.00	**2.24**
35 (7)	90.00	23.14	**25.71**	25.71	**28.57**	1.29	**1.43**	112.29	6.00	5.34	28.71	25.57	1.43	1.27
40 (10)	105.20	24.00	**22.81**	30.00	**28.52**	0.80	**0.76**	–	–	–	–	–	–	–
	PR							PH						
$\|V\|$	Receivers	Free	B (%)	Free	P (%)	Free	H (%)	Receivers	Free	B (%)	Free	P (%)	Free	H (%)
10 (15)	28.80	3.33	11.57	9.00	31.25	1.73	6.02	25.47	2.67	10.47	8.67	**34.03**	4.33	**17.02**
15 (10)	42.60	5.60	13.15	13.10	**30.75**	2.20	5.16	36.60	2.40	6.56	10.60	28.96	6.00	**16.39**
20 (13)	–	–	–	–	–	–	–	52.00	2.77	5.33	15.23	**29.29**	8.31	**15.98**
	PHci													
$\|V\|$	Receivers	Free	B (%)	Free	P (%)	Free	H (%)							
10 (15)	30.13	4.27	14.16	8.60	28.54	1.73	5.75							
15 (10)	42.00	6.60	15.71	11.60	27.62	1.50	3.57							
20 (13)	56.31	8.46	15.03	15.38	27.32	1.38	2.46							
25 (15)	72.13	8.27	11.46	19.27	26.71	1.27	1.76							
30 (15)	84.40	11.20	13.27	23.27	27.57	1.20	1.42							
35 (7)	96.57	16.57	17.16	27.29	28.25	0.57	0.59							
40 (10)	110.60	17.00	15.37	29.30	26.49	0.50	0.45							

Table 6 also shows that the interference-free condition H is the hardest to be satisfied. While all formulations obtain at most 26.98% and 34.03% for the average relative differences B and P, respectively, the largest average relative difference H is 17.02%, obtained by formulation PH that considers the physical interference-free condition explicitly in the objective function. Formulations that do not explicitly consider the physical interference-free condition obtain at most 7.01% for the average relative difference H.

5. Concluding remarks

In this work, we considered three interference models (Boolean, protocol, and physical) for *ad hoc* wireless networks through exact formulations for topology control algorithms. We proposed integer programming formulations for each interference model. All formulations find a power assignment to the nodes of a wireless network such as the resulting topology is k-vertex connected and radio interference is minimized. The four newly proposed integer formulations (BO, PR, PH, and PHci) have also been compared with an integer programming formulation that minimizes the total power consumption that is needed to establish k-connectedness without taking interference into account explicitly (Moraes et al., 2009; see also Moraes and Ribeiro, 2013).

Extensive computational experiments have been performed comparing all formulations. Considering the noncontinuous formulations, the numerical results showed that the Boolean interference model is the easiest to be solved by an integer programming solver, while the protocol interference model is the hardest. Although the noncontinuous formulation of the physical interference model is hard to be solved, its continuous formulation gives solutions in reasonable computational time with competitive transmission powers and interference results when compared with the other proposed formulations. The physical interference model has the advantage of being more realistic.

The approaches proposed in this work are able to handle the open issues discussed by Khabbazian et al. (2015). The numerical results reported in the previous section also showed that the exact formulation proposed by Moraes et al. (2009) to solve the k-connected minimum power consumption problem gives low interference communication graphs, even though it does not explicitly consider interference-free conditions in its objective function.

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