Improved solutions for the freight consolidation and containerization problem using aggregation and symmetry breaking

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Abstract

We consider the freight consolidation and containerization problem, which consists of loading items into containers and then shipping these containers to different warehouses from where they are delivered to their final destinations. We show through computational experiments that very good solutions can be obtained by heuristically aggregating the items and then using MIP approaches to deal with the aggregated problem. We have been able to find a solution as good as the best known in the literature for 100% of the instances with small items, encountering strictly better solutions for 40.6% of them. Our approach found solutions as good as the best known in the literature for 88.9% of the instances with large items, obtaining strictly better in 59.4% of the cases.

Keywords: Logistics, Freight consolidation, Containerization, Integer programming, Heuristics.

1 Introduction

The management of containers is an important problem in global logistics networks. The freight consolidation and containerization problem (FCCP) deals with the loading of items into containers that are shipped to different possible locations, from where they are sent to their final destinations. It is assumed that third-party logistics providers will take care of the transportation. This problem was introduced in Qin et al. [11] in the context of the transportation of textile products for children.

Some important aspects to be considered in global logistics networks are the loading followed by the transportation of the containers. The complexity of the loading varies depending on the type of items transported. A simple type of containerization corresponds to the NP-hard bin packing problem [7], which consists in packing one-dimensional items into the minimum amount of bins. The more general multi-capacity one-dimensional bin packing

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is a special case of the FCCP, in which there is only one location to which the bins can be shipped. Recent work on the multi-capacity bin packing can be found in Masson et al. [10].

The transportation of containers or goods can be categorized as long or short transportation. Long transportation modes include maritime and rail transport, see Christiansen et al. [3] for a review on maritime routing. Short transportation usually occurs after (or before) a long transportation between the port or rail station and a warehouse, usually giving rise to pick-up and delivery problems [1]. A short transportation was studied in Escudero et al. [5].

In some situations, as in the context of the problem we are considering in this paper, a long transportation of containers (e.g. from one country to another), the goods still have to be delivered to their final destination using a short transportation mode after the containers are unloaded.

The symmetric characteristic of certain problems (with bin packing among them), has challenged the performance of optimization approaches. Some authors have coped with symmetry by providing problem specific alternatives such as Campêlo et al. [2] for the vertex coloring problem, Valério de Carvalho [4] for the bin packing and cutting stock problems, Frota et al. [6] for the partition coloring problem and Jans and Desrosiers [8] for the job grouping problem. For the interested reader, a survey on symmetry in integer programming appeared in Margot [9].

Qin et al. [11] studied the FCCP and proposed a standard formulation and two evolutionary metaheuristics for the problem. In this paper, we investigate compact formulations (i.e., with a polynomial number of variables and constraints) for a simplified version of the FCCP, exploring the idea of asymmetric representative formulations, which serves as a heuristic solution to the problem. We analyze the problem structure in order to devise methods providing good solutions, in particular for instances in which the items are small. Our approach includes the use of symmetry breaking formulations together with improved solver settings and a restricted formulation that limits the number of possible bins and make it possible to find high quality solutions in small running times. This manuscript is organized as follows. In Section 2 we formally describe the problem. In Section 3 we present the problem obtained by aggregating the FCCP according to the item shipments, together with some MIP formulations. Computational results are presented in Section 5. Final remarks are made in the last section.

2 The freight consolidation and containerization problem

The freight consolidation and containerization problem was introduced in Qin et al. [11] and can be formally defined as follows. There is a set I of items to be loaded into containers and sent along some possible routes to warehouses, from where they are delivered to their final destinations. The items are organized in shipments, with each shipment being composed by one or more items. The set of shipments is denoted by S, while R denotes the set of possible routes along which the shipments are sent. Each item $i \in I$ has a size v_i and belongs to a given shipment $s_i \in S$. All items belonging to the same shipment have to be sent via the same route, but not necessarily in the same container. Furthermore, there is a set T of container types such that a container of type $t \in T$ has capacity C_t. We denote by f_{tr} the long transportation cost of sending a container of type $t \in T$ along route $r \in R$ and by c_{ir} the short transportation cost of delivering item $i \in I$ along route $r \in R$. The goal is to load all items into containers so that the total sum of short and long transportation costs is minimized.

One way to tackle this problem is to define B as a large enough set of available containers. Each container $j \in B$ has a type $t_j \in T$ and will follow a route $r_j \in R$. The size V_j of container $j \in B$ is defined by its type $t_j \in T$. The long transportation cost of sending a container $j \in B$ along route $r_j \in R$ is $p_j = f_{t_j,r_j}$, while the short transportation cost of sending item $i \in I$ from the unloading point of a container $j \in B$ to its final destination is $r_{ij} = c_{i,r_j}$. We define the following decision variables:

$$y_j = \begin{cases} 1, & \text{if container } j \in B \text{ is used,} \\ 0, & \text{otherwise;} \end{cases}$$

$$x_{ij} = \begin{cases} 1, & \text{if item } i \in I \text{ is loaded into container } j \in B, \\ 0, & \text{otherwise; and} \end{cases}$$

$$z_{sr} = \begin{cases} 1, & \text{if shipment } s \in S \text{ is sent along route } r \in R, \\ 0, & \text{otherwise.} \end{cases}$$

A standard formulation for problem FCCP is given by model STD-FCCP:

$$z_{STD} = \min \sum_{j \in B} p_j y_j + \sum_{i \in I} \sum_{j \in B} r_{ij} x_{ij}$$

$$\sum_{j \in B} x_{ij} = 1, \quad \forall i \in I \tag{1}$$

$$x_{ij} \leq y_j, \quad \forall i \in I, \forall j \in B \tag{2}$$

$$x_{ij} \leq z_{sr}, \quad \forall i \in I : s_i = s, \forall j \in B : r_j = r \tag{3}$$

$$\sum_{r \in R} z_{sr} = 1, \quad \forall s \in S \tag{4}$$

$$\sum_{i \in I} v_i x_{ij} \leq V_j y_j, \quad \forall j \in B \tag{5}$$

$$y_j, x_{ij}, z_{sr} \in \{0, 1\}, \quad \forall j \in B, \forall i \in I, \forall s \in S, \forall r \in R. \tag{6}$$

The objective function minimizes the total sum of the long transportation costs of the containers plus the short transportation costs of the items after each container is unloaded. Constraints (1) enforce that each item is loaded into exactly one container. Constraints (2) imply that an item can only be allocated to an used container. Constraints (3) say that if an item is loaded into some container, then the shipment to which this item belongs is sent along the route used by this container. Constraints (4) determine that each shipment follows a single route. Constraints (5) limit the volume loaded into each container. Constraints (6) are the integrality constraints. We notice that the difference between this integer programming formulation and that presented in [11] is the introduction of the y variables in constraints (5), which contribute to obtaining improved, stronger linear relaxation bounds.

We remark that model STD-FCCP has a considerable amount of symmetric solutions. This observation comes from the fact that the contents of different containers with the same capacity and route can be swapped without changing the objective value.

3 The shipment containerization problem

Although items belonging to the same shipment do not necessarily have to be loaded into the same container, in some situations it can be advantageous to do so. Analyzing the instances available in the literature for the FCCP, we could observe that the total size of the shipments is usually not large, specially for the instances with small items. Therefore, the probability of complete shipments (i.e., all the items belonging to the same shipment) being loaded together in a single container in a good solution is possibly high. In consequence, in the remaining of this paper, we assume that each shipment is entirely loaded into some container. Of course, we naturally assume that any shipment can be loaded at least in the largest available container.

This problem is therefore a restriction of problem FCCP, in which each shipment is considered as a single, indivisible unit. Instead of items and shipments, we now have shipments that are formed by one single "super-item" resulting from aggregating the original items. Therefore, the resolution of the aggregated problem may be seen as a heuristic to the original problem FCCP, since any optimal solution for this aggregated problem is feasible (but not necessarily optimal) for the original problem FCCP.

New definitions and notation are imposed by the aggregation of items into shipments. The size of a shipment $s \in S$ is now defined by $v'_s = \sum_{i \in I: s_i = s} v_i$, while the short transportation cost of shipment $s \in S$ loaded into container $j \in B$ is $r'_{sj} = \sum_{i \in I: s_i = s} r_{ij}$. As noticed before, we assume thereafter that each shipment fits into at least one single container, i.e., $v'_s \leq \max_{j \in B} V_j$. We define new decision variables:

$$w_{sj} = \begin{cases} 1, & \text{if shipment } s \in S \text{ is loaded into container } j \in B, \\ 0, & \text{otherwise;} \end{cases}$$

The other variables y_j and z_{sr} have the same definitions as for model STD-FCCP. The new formulation STD-AGG for the aggregated problem follows:

$$z_{AGG} = \min \sum_{j \in B} p_j y_j + \sum_{s \in S} \sum_{j \in B} r'_{sj} w_{sj}$$

$$\sum_{j \in B} w_{sj} = 1, \quad \forall s \in S \tag{7}$$

$$w_{sj} \leq y_j, \quad \forall s \in S, \forall j \in B \tag{8}$$

$$w_{sj} \leq z_{sr}, \quad \forall s \in S, \forall j \in B : r_j = r \tag{9}$$

$$\sum_{r \in R} z_{sr} = 1, \quad \forall s \in S \tag{4}$$

$$\sum_{s \in S} v'_s w_{sj} \leq V_j y_j, \quad \forall j \in B \tag{10}$$

$$y_j, w_{sj}, z_{sr} \in \{0, 1\}, \quad \forall j \in B, \forall s \in S, \forall r \in R. \tag{11}$$

4 Reformulations by representatives

In this section, we propose two reformulations by representatives of model STD-AGG, whose main goal is to tackle more efficiently the symmetries that are inherent to the problem.

The key idea of this approach is to identify each container by the index of the shipment whose index is minimum among all shipments loaded into this container. As an illustration, suppose that k different shipments $s_{l_1}, s_{l_2}, \ldots, s_{l_k}$ are loaded in the same container, with $s_{l_j} \in S$ for $j = 1, 2, \ldots, k$ and $l_1 < l_2 < \ldots < l_k$. Therefore, this specific container will be represented by l_1 and, as a consequence, all shipments s_{l_1}, s_{l_2}, $\dots, s_{l_k}$ are considered to belong to the container associated to l_1.

We first observe that, under the aggregation assumption, the short transportation cost of delivering a shipment $s \in S$ along route $r \in R$ is given by $c'_{sr} = \sum_{i \in I: s_i = s} c_{ir}$.

For any shipments $s, h \in S$ with $s \geq h$, let α^t_{shr} be a new binary variable defined as:

$$\alpha^t_{shr} = \begin{cases} 1, & \text{if shipment } s \in S \text{ is sent along route } r \in R \text{ by a container} \\ & \quad \text{of type } t \in T \text{ represented by the lowest indexed shipment } h \in S, \\ 0, & \text{otherwise.} \end{cases}$$

With these new definitions, problem STD-AGG has a new formulation REP1:

$$z_{REP1} = \min \sum_{t \in T} \sum_{s \in S} \sum_{r \in R} f_{tr} \alpha^t_{ssr} + \sum_{t \in T} \sum_{s \in S} \sum_{h \in S: h \leq s} \sum_{r \in R} c'_{sr} \alpha^t_{shr}$$

$$\sum_{t \in T} \sum_{h \in S: h \leq s} \sum_{r \in R} \alpha^t_{shr} = 1, \quad \forall s \in S \tag{12}$$

$$\alpha^t_{shr} \leq \alpha^t_{hhr}, \quad \forall t \in T, \forall s, h \in S : s > h, \forall r \in R \tag{13}$$

$$\sum_{s \in S} v'_s \alpha^t_{shr} \leq \alpha^t_{hhr} C_t, \quad \forall t \in T, \forall h \in S, \forall r \in R \tag{14}$$

$$\alpha^t_{shr} \in \{0, 1\}, \quad \forall s, h \in S : s \geq h, \forall t \in T, \forall r \in R. \tag{15}$$

The objective function minimizes the total sum of the long and short transportation costs. Constraints (12) state that each shipment is loaded into exactly one container. Constraints (13) imply that a shipment can only be loaded into a container if that container is used. Constraints (14) limit the amount loaded into a container. Constraints (15) are the integrality constraints on the variables.

The next formulation aims at reducing the excessive number of variables, but still keeping a symmetry breaking structure. First, the type index is removed from each variable α^t_{shr}, giving rise to a new variable α'_{shr}:

$$\alpha'_{shr} = \begin{cases} 1, & \text{if shipment } s \in S \text{ is sent along route } r \in R \text{ by a container} \\ & \quad \text{represented by the lowest indexed shipment } h \in S, \\ 0, & \text{otherwise.} \end{cases}$$

In addition, a new binary variable is defined:

$$\beta_{tsr} = \begin{cases} 1, & \text{if the container represented by shipment } s \in S \text{ is sent along} \\ & \quad \text{route } r \in R \text{ and is one of type } t \in T, \\ 0, & \text{otherwise;} \end{cases}$$

A new asymmetric reformulation by representatives is given by model REP2:

$$z_{REP2} = \min \sum_{t\in T}\sum_{s\in S}\sum_{r\in R} f_{tr}\beta_{tsr} + \sum_{s\in S}\sum_{h\in S:h\leq s}\sum_{r\in R} c'_{sr}\alpha'_{shr}$$

$$\sum_{h\in S:h\leq s}\sum_{r\in R}\alpha'_{shr} = 1, \quad \forall s \in S \tag{16}$$

$$\alpha'_{shr} \leq \alpha'_{hhr}, \quad \forall s,h \in S : s > h, \forall r \in R \tag{17}$$

$$\alpha'_{hhr} \leq \sum_{t\in T}\beta_{thr}, \quad \forall h \in S, \forall r \in R \tag{18}$$

$$\sum_{s\in S} v'_s\alpha'_{shr} \leq \sum_{t\in T}\beta_{thr}C_t, \quad \forall h \in S, \forall r \in R \tag{19}$$

$$\alpha'_{shr}, \beta_{tsr} \in \{0,1\}, \quad \forall s,h \in S : s \geq h, \forall t \in T, \forall r \in R. \tag{20}$$

The objective function minimizes the total sum of the long and short transportation costs. Constraints (16) state that each shipment is loaded into exactly one container. Constraints (17) imply that a shipment can only be loaded into a container if that container is used. Constraints (18) associate a container type to the container used for loading a given shipment. Constraints (19) limit the amount loaded into a container. Constraints (20) are the integrality constraints on the variables.

We observe that a similar symmetry breaking approach could also be applied to the original formulation FCCP. However, as it will be seen later, the large number of possible items makes this approach intractable.

Proposition 1. *Let $\underline{z}_{REP1}$ and $\underline{z}_{REP2}$ be the linear relaxation bounds of formulations REP1 and REP2, respectively. Then, $\underline{z}_{REP1} \geq \underline{z}_{REP2}$.*

Proof. The proof is concise and consists in demonstrating that REP2 is a relaxation of REP1. To do so, we derive a relaxation of REP1 that is equivalent to REP2. Consider a relaxation rREP1 of REP1 obtained by keeping constraints (12) unchanged and summing up constraints (13) and (14) over all $t \in T$ to obtain constraints (21) and (22), respectively:

$$z_{rREP1} = \min \sum_{t\in T}\sum_{s\in S}\sum_{r\in R} f_{tr}\alpha^t_{ssr} + \sum_{t\in T}\sum_{s\in S}\sum_{h\in S:h\leq s}\sum_{r\in R} c'_{sr}\alpha^t_{shr}$$

$$\sum_{t\in T}\sum_{h\in S:h\leq s}\sum_{r\in R}\alpha^t_{shr} = 1, \quad \forall s \in S \tag{12}$$

$$\sum_{t\in T}\alpha^t_{shr} \leq \sum_{t\in T}\alpha^t_{hhr}, \quad \forall s,h \in S : s \geq h, \forall r \in R \tag{21}$$

$$\sum_{t\in T}\sum_{s\in S} v'_s\alpha^t_{shr} \leq \sum_{t\in T}\alpha^t_{hhr}C_t, \quad \forall h \in S, \forall r \in R \tag{22}$$

$$\alpha^t_{shr} \in \{0,1\}, \quad \forall s,h \in S : s \geq h, \forall t \in T, \forall r \in R. \tag{15}$$

Now, let $\alpha'_{shr} = \sum_{t\in T}\alpha^t_{shr}$ and $\beta_{thr} = \alpha^t_{hhr}$. By direct substitution of these variables in the above formulation rREP1, we obtain:

$$z_{rREP1} = \min \sum_{t\in T}\sum_{s\in S}\sum_{r\in R} f_{tr}\beta_{tsr} + \sum_{s\in S}\sum_{h\in S:h\leq s}\sum_{sr\in R} c'_{sr}\alpha'_{shr}$$

$$\sum_{h\in S:h\leq s}\sum_{r\in R}\alpha'_{shr} = 1, \quad \forall s\in S \tag{16}$$

$$\alpha'_{shr} \leq \alpha'_{hhr}, \quad \forall s,h\in S: s\geq h, \forall r\in R \tag{17}$$

$$\alpha'_{hhr} \leq \sum_{t\in T}\beta_{thr}, \quad \forall h\in S, \forall r\in R \tag{18}$$

$$\sum_{s\in S} v'_s\alpha'_{shr} \leq \sum_{t\in T}\beta_{thr}C_t, \quad \forall h\in S, \forall r\in R \tag{19}$$

$$\alpha'_{shr}, \beta_{tsr} \in \{0,1\}, \quad \forall s,h\in S: s\geq h, \forall t\in T, \forall r\in R. \tag{20}$$

Observe that the integrality constraints (20) on the binary variables α'_{shr} and β_{tsr} follow from constraints (12). Finally, we notice that formulation rREP1 is indeed equivalent to REP2. □

5 Computational experiments

All experiments were performed on a machine running under Debian GNU/Linux, kernel 2.6.24-1-amd64 with an Intel Core 2 Quad 2.40GHz processor, with 8 Gb of RAM memory and using FICO Xpress 7.6.0.

We notice that the shipment containerization problem is the one considered in the computational experiments, and not the original freight consolidation and containerization problem, observing that any feasible solution to the former can be easily transformed into a feasible solution to the latter by simply disaggregating the shipments. Therefore, the concepts of optimality and gaps in the computational experiments always refer to the shipment containerization problem.

We have considered the same set of instances used by Qin et al. [11], in which a more detailed description of these test problems can be found. The instances are identified by groups $|S|$-$|R|$-γ-type defined by four parameters: $|S|$ denotes the number of shipments, $|R|$ denotes the number of routes, γ is a parameter that relates the long and short transportation costs (larger values of gamma mean larger delivery costs), and a type that is 1 for small items and 2 for large items. These parameters assumed the following values in the computational experiments: $|S| \in \{20, 50, 80\}$, $|R| \in \{5, 10\}$, and $\gamma \in \{0.08, 0.16, 0.32\}$. There are ten instances for each possible combination of the four parameters, with a total of 180 instances with small items and 180 instances with large items. The reader is referred to [11] for more details about these instances and how they have been generated.

The computational experiments are organized as follows. In Section 5.1 we present the results obtained by applying the different formulations to instances with small items. In Section 5.2 we show how these results can be further improved by giving priorities to specific variables in the formulations. In Section 5.3 we show how to restrict some variables in the standard formulation, in order to obtain good solutions in small running times; and then we use this restricted problem together with the symmetry breaking formulations with the goal of improving the obtained solutions and possibly solving some additional instances to optimality (considering the shipment problem). The presented computational results show that our approach outperforms the heuristics available in the literature reaching all the best known solutions and encountering new best solutions that were previously unknown for several instances. Finally, computational results for instances with large items are shown in Section 5.4 A total time limit of 1800 seconds has been imposed for every execution of the MIP solver along the computational experiments.

5.1 Formulations

Table 1 presents the results obtained by the standard aggregated formulation STD-AGG for small items. For each instance group, we give the number of instances for which STD-AGG obtained a solution that is better than or equal to that reported by Qin et al. [11] (#≤), the number of instances for which STD-AGG obtained a solution strictly better than that reported in [11] (#<), the geometric mean of the execution times over the ten instances (gm(time)), the number of aggregated problems solved to optimality (#agg-opt), and the geometric mean of the gaps between the value of the best feasible solution found and the best lower bound at the end of the execution (over the instances that have not been solved to optimality within the time limit) (gm(agg-gap). The table shows that the standard aggregated formulation found a solution as good as the best known for only 106 out of the 180 test instances. In total only 85 instances could be solved to optimality.

Table 1: Results using the standard formulation STD-AGG for the instances with small items.

	STD-AGG				
Instance group	#≤	#<	gm(time)	#agg-opt	gm(agg-gap)
20-5-0.08-1	10	0	22.0	10	-
20-5-0.16-1	10	0	4.9	10	-
20-5-0.32-1	10	0	0.8	10	-
20-10-0.08-1	10	0	58.4	10	-
20-10-0.16-1	10	0	6.8	10	-
20-10-0.32-1	10	0	1.9	10	-
50-5-0.08-1	6	0	1800.0	0	1.5
50-5-0.16-1	7	0	587.0	7	1.6
50-5-0.32-1	7	1	350.4	7	3.2
50-10-0.08-1	2	1	1800.0	0	2.6
50-10-0.16-1	7	1	1263.4	5	1.2
50-10-0.32-1	8	1	504.9	6	0.7
80-5-0.08-1	1	1	1800.0	0	4.7
80-5-0.16-1	0	0	1800.0	0	4.5
80-5-0.32-1	3	3	1800.0	0	2.8
80-10-0.08-1	0	0	1800.0	0	10.1
80-10-0.16-1	4	3	1800.0	0	4.4
80-10-0.32-1	1	0	1800.0	0	4.2
Total (out of 180)	106	11		85	

Table 2 depicts the same information for the results obtained with the two symmetry breaking formulations REP1 and REP2.

The results in Tables 1 and 2 show that formulation REP2 performed much better than REP1. In particular, REP2 obtained solutions that are at least as good as the best known solutions in the literature for 178 out of 180 test instances, while REP1 found only 138. In addition, REP2 improved the best known solutions in the literature for 68 instances, while REP1 did the same for only 35 instances. It is worthy noting that 113 out of the 180 instances could be solved to optimality using REP2.

Table 2: Results using the symmetry breaking formulations REP1 and REP2 for the instances with small items.

	REP1					REP2				
Instance group	#≤	#<	gm(time)	#agg-opt	gm(agg-gap)	#≤	#<	gm(time)	#agg-opt	gm(agg-gap)
20-5-0.08-1	10	0	6.9	10	-	10	0	4.1	10	-
20-5-0.16-1	10	0	1.8	10	-	10	0	0.6	10	-
20-5-0.32-1	10	0	0.4	10	-	10	0	0.1	10	-
20-10-0.08-1	10	0	20.3	10	-	10	0	10.4	10	-
20-10-0.16-1	10	0	3.5	10	-	10	0	2.1	10	-
20-10-0.32-1	10	0	0.8	10	-	10	0	0.4	10	-
50-5-0.08-1	9	2	229.1	7	3.7	10	3	379.1	6	1.3
50-5-0.16-1	9	1	28.7	9	6.5	10	1	20.6	9	5.8
50-5-0.32-1	10	4	85.9	9	1.4	10	4	26.1	9	1.7
50-10-0.08-1	7	3	422.3	5	2.1	10	5	506.5	4	1.0
50-10-0.16-1	10	2	30.0	10	-	10	2	31.3	10	-
50-10-0.32-1	10	2	23.3	10	-	10	2	20.0	10	-
80-5-0.08-1	2	2	1800.0	0	3.8	10	10	1800.0	0	2.4
80-5-0.16-1	1	1	1800.0	0	2.7	9	6	1800.0	0	2.0
80-5-0.32-1	5	4	1800.0	0	1.1	9	7	1569.3	1	0.8
80-10-0.08-1	2	2	1800.0	0	6.0	10	10	1800.0	0	3.3
80-10-0.16-1	8	8	1203.7	1	2.5	10	10	1256.8	1	1.4
80-10-0.32-1	5	4	1243.7	1	1.0	10	8	899.2	3	0.8
Total (out of 180)	138	35		112		178	68		113	

5.2 Changing the branching priorities

Changing the default settings of a commercial solver is not always a good idea. However, in some situations in which the problem structure is well known, some default settings can be improved. In the case of the problem under study, potentially important variables for the enumeration process can be identified and used to guide the solver by giving priorities to them. Basically, we propose to give branching priority to the variables that determine the containers to be used. In the case of formulation STD-AGG, this strategy amounts to give priority to the y variables, while in formulation REP1 higher priorities will be given to the α^t_{hhr} variables and in formulation REP2 priority will be given to the α_{hhr} variables. Numerical results are shown in Table 3 for the standard formulation and in Table 4 for the two formulations by representatives. The same statistics presented in Tables 1 and 2 are provided.

Considerable improvements have been shown by the use of branching priorities in the standard formulation STD-AGG: solutions at least as good as the best known in the literature have been obtained for 158 out of 180 test instances, which represents 52 in addition to the 106 previously found with the default settings. In addition, a much larger number of 138 instances have been solved to optimality with the improved branching priorities, while only 85 have been optimally solved with the default settings.

However, the branching priorities have not been as much effective in the case of the formulations by representatives. Formulation REP2 with the new priorities found solutions at least as good as the best known in 176 out of the 180 instances (in opposition to 178 when the original settings have been used). But on the other hand an optimal solution was encountered in 120 instances, an increase of 7 instances.

5.3 Obtaining good feasible solutions quickly with a restricted problem

We have observed that the optimal solutions for some instances of the aggregated problem make use of few containers. Therefore, it seems logical to make attempts to find feasible solutions with a small number of containers.

Table 3: Results using the standard formulation STD-AGG and branching priorities for the instances with small items.

	STD-AGG				
Instance group	#≤	#<	gm(time)	#aggopt	gm(agggap)
20-5-0.08-1	10	0	13.4	10	-
20-5-0.16-1	10	0	4.0	10	-
20-5-0.32-1	10	0	0.8	10	-
20-10-0.08-1	10	0	34.6	10	-
20-10-0.16-1	10	0	6.4	10	-
20-10-0.32-1	10	0	1.8	10	-
50-5-0.08-1	10	3	201.5	10	-
50-5-0.16-1	10	3	155.1	10	-
50-5-0.32-1	10	4	95.0	10	-
50-10-0.08-1	9	5	944.1	8	0.5
50-10-0.16-1	10	2	445.5	10	-
50-10-0.32-1	10	2	292.1	9	0.3
80-5-0.08-1	8	8	1340.2	5	0.8
80-5-0.16-1	8	5	1079.9	7	2.3
80-5-0.32-1	9	7	758.8	9	2.5
80-10-0.08-1	4	4	1800.0	0	5.8
80-10-0.16-1	4	4	1800.0	0	2.9
80-10-0.32-1	6	4	1800.0	0	1.3
Total (out of 180)	158	51		138	

Table 4: Results using the symmetry breaking formulations by representatives and branching priorities for the instances with small items.

	REP1					REP2				
Instance group	#≤	#<	gm(time)	#aggopt	gm(agggap)	#≤	#<	gm(time)	# aggopt	gm(agggap)
20-5-0.08-1	10	0	4.8	10	-	10	0	13.4	10	-
20-5-0.16-1	10	0	1.5	10	-	10	0	0.5	10	-
20-5-0.32-1	10	0	0.4	10	-	10	0	0.1	10	-
20-10-0.08-1	10	0	12.4	10	-	10	0	6.3	10	-
20-10-0.16-1	10	0	3.0	10	-	10	0	1.6	10	-
20-10-0.32-1	10	0	0.8	10	-	10	0	0.4	10	-
50-5-0.08-1	10	3	99.9	8	4.4	10	3	167.1	7	1.1
50-5-0.16-1	9	1	21.6	9	6.4	10	2	13.9	9	3.9
50-5-0.32-1	10	4	39.1	9	0.7	10	4	15.0	9	1.3
50-10-0.08-1	9	4	167.6	7	3.6	10	6	319.1	6	1.4
50-10-0.16-1	10	2	26.1	10	-	10	2	22.6	10	-
50-10-0.32-1	10	2	18.3	10	-	10	2	11.8	10	-
80-5-0.08-1	5	5	1800.0	0	3.0	9	9	1800.0	0	2.3
80-5-0.16-1	2	2	1800.0	0	2.3	9	6	1800.0	0	1.3
80-5-0.32-1	7	6	1293.0	2	1.6	9	7	1288.6	4	1.2
80-10-0.08-1	3	3	1800.0	0	5.4	10	10	1800.0	0	2.9
80-10-0.16-1	6	6	1184.2	1	2.6	10	10	1253.5	1	0.9
80-10-0.32-1	6	5	1112.7	4	1.3	9	7	679.8	4	0.5
Total # (out of 180)	147	43		119		176	68		120	

The task of restricting the number of containers cannot be easily achieved with the formulations by representatives. This, because the containers are identified by shipment indices and the use of specific containers would imply that the shipments associated with the used containers would never be put together, thus limiting the different possible combinations of shipments. Therefore, we used the aggregated formulation STD-AGG as a heuristic by limiting the number $|B|$ of containers. We call this problem with a limited number of containers a restricted problem and use it as an initial heuristic.

The number of usable containers of each type $t \in T$ will be limited by an ad hoc parameter θ_t for each route $r \in R$. Therefore, a maximum of $|R| \sum_{t \in T} \theta_t$ containers will be used. Branching priority is given to the y variables.

We have used $\theta_1 = 1$ for the small containers and $\theta_2 = \theta_3 = 5$. For each instance, first the solver was run for the restricted problem with a time limit of 900 seconds. Next, the solver was run using the asymmetric formulation REP2 with the solution obtained in the previous step as its initial solution and a time limit of 1800 seconds. To illustrate the interaction between the restricted STD-AGG and REP2, Figure 1 shows an example in which four shipments are loaded as follows: shipments 1 and 2 are assigned to a container of type 1 to be shipped through route 5, while shipments 3 and 4 are allocated to a container of type 2 to be shipped through route 7. Considering that these two containers are indexed by 6 and 8, in STD-AGG this solution is associated to the nonzero variables $y_6 = y_8 = w_{16} = w_{26} = w_{38} = w_{48} = z_{15} = z_{25} = z_{37} = z_{47} = 1$. This solution for STD-AGG would imply in the following initial solution for REP2: one container represented by shipment 1 of type 1 with shipments 1 and 2 (nonzero variables $\alpha'_{115} = \alpha'_{215} = \beta_{115} = 1$), and another container represented by shipment 3 of type 2 with shipments 3 and 4 (nonzero variables $\alpha'_{337} = \alpha'_{437} = \beta_{237} = 1$).

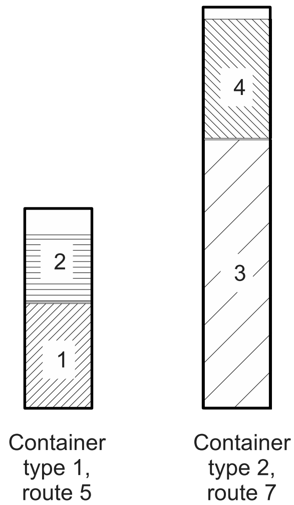

Figure 1: Example of shipments loaded into two different containers

The results of the experiments are shown in Table 5. These results show that this simple heuristic (using a restricted problem) performed extremely well. A solution at least as good as the best known in the literature was obtained for all test instances. A strictly better solution was found for 73 out of 180 instances, i.e., for 40.6% of the test problems in Table 5. The fourth column of this table shows that the restricted problem was solved quite fast in most cases: the largest geometric mean of the running times amounted at most to 195.6 seconds,

Table 5: Results using the restricted problem heuristic followed (or not) by using the solver over the formulation by representatives REP2 for the instances with small items.

	Initial Heuristic			Initial Heuristic + REP2				
Instance group	#≤	#<	gm(time)	#≤	#<	gm(time)	#aggopt	gm(agggap)
20-5-0.08-1	10	0	2.2	10	0	7.7	10	-
20-5-0.16-1	10	0	1.1	10	0	1.7	10	-
20-5-0.32-1	10	0	0.4	10	0	0.3	10	-
20-10-0.08-1	10	0	5.6	10	0	15.1	10	-
20-10-0.16-1	10	0	1.9	10	0	4.2	10	-
20-10-0.32-1	10	0	0.6	10	0	0.8	10	-
50-5-0.08-1	10	3	11.0	10	3	302.2	6	1.3
50-5-0.16-1	10	3	10.3	10	3	28.2	9	5.3
50-5-0.32-1	10	4	4.0	10	4	19.1	9	1.6
50-10-0.08-1	10	6	29.9	10	6	488.9	5	1.3
50-10-0.16-1	10	2	17.9	10	2	29.2	10	-
50-10-0.32-1	10	2	8.9	10	2	25.2	10	-
80-5-0.08-1	10	10	27.1	10	10	1800.0	0	2.4
80-5-0.16-1	10	7	22.1	10	7	1800.0	0	1.9
80-5-0.32-1	10	8	16.0	10	8	1667.7	1	0.5
80-10-0.08-1	10	10	195.6	10	10	1800.0	0	2.8
80-10-0.16-1	10	10	74.0	10	10	1081.3	3	1.5
80-10-0.32-1	10	8	62.7	10	8	770.9	5	0.4
Total (out of 180)	180	73		180	73		118	

observed for the instance group 80-10-0.08-1. Applying the solver to formulation REP2 with an initial feasible solution lead to 118 aggregated instances solved to optimality. Finally, we observe that the existence of good solutions already at the beginning of the execution of the solver using the formulation by representatives REP2 did not improve the overall performance. This observation comes from the fact that a few solutions that were proven to be optimal without the use of the initial solution could not be solved to optimality when a heuristic initial solution was provided: as shown in the last rows of Tables 4 and 5, 120 proven optimal solutions have been found without the use of the initial solution, while 118 have been found with its use.

5.4 Instances with larger items

Although the approach proposed in this work was not targeted to instances with large items, we show that it could still outperform the approach of Qin et al. [11] for most of the instances in terms of best solution found.

We present in this section computational results for such instances with the approach that performed best in the previous sections, namely a heuristic using a limited number of bins followed by the solver using the reformulation by representatives REP2. The results are summarized in Table 6.

The table shows that the simple heuristic using a limited number of bins found out solutions as good as the best known in the literature for 158 out of 180 instances, i.e., for 87.8% of them. Solutions strictly better than the best known have been found for 107 out of 180 instances, i.e., for 59.4% of them. Considering these solutions as starting point when using the solver with reformulation REP2 increased the number of instances for which a solution as good as the best known solution was found from 158 to 160, i.e., 88.9% of the total. This means that in only 11.1% of the instances our approach could not find the best available feasible solution. In addition, with the exception of group 20-10-0.08-2, which had only two unsolved instances with an average gap of 3.5%, the geometric mean of the remaining gap

Table 6: Results using the heuristic and the formulation by representatives for instances with large items.

	Initial heuristic			Initial heuristic + REP2				
Instance group	#≤	#<	gm(time)	#≤	#<	gm(time)	# aggopt	gm(agggap)
20-5-0.08-2	10	6	5.8	10	6	95.5	8	2.3
20-5-0.16-2	10	0	3.2	10	0	5.8	10	-
20-5-0.32-2	10	0	2.3	10	0	4.4	10	-
20-10-0.08-2	10	7	18.5	10	7	166.3	8	3.5
20-10-0.16-2	10	3	10.8	10	3	16.8	10	-
20-10-0.32-2	8	0	4.3	9	0	6.0	10	-
50-5-0.08-2	10	8	140.0	10	8	1800.0	0	1.0
50-5-0.16-2	8	7	299.5	9	7	1800.0	0	1.1
50-5-0.32-2	9	5	160.6	9	5	1800.0	0	1.0
50-10-0.08-2	9	9	900.0	9	9	1800.0	0	2.2
50-10-0.16-2	9	9	742.6	9	9	1800.0	0	1.3
50-10-0.32-2	8	6	317.4	8	6	1788.3	1	0.4
80-5-0.08-2	8	8	900.0	8	8	1800.0	0	1.8
80-5-0.16-2	8	8	868.8	8	8	1800.0	0	1.2
80-5-0.32-2	4	4	900.0	4	4	1800.0	0	1.2
80-10-0.08-2	10	10	900.0	10	10	1800.0	0	2.3
80-10-0.16-2	9	9	900.0	9	9	1800.0	0	2.1
80-10-0.32-2	8	8	900.0	8	8	1800.0	0	1.6
Total (out of 180)	158	107		160	107		57	

was always less than 2.3%. Therefore, we found solutions that are very close to the optimal for the shipment containerization problem.

6 Final remarks

We studied the freight consolidation and containerization problem and proposed a solution approach which consists in treating a simplified shipment-based problem, rather than the original item-based formulation, with a reduced number of variables. We showed that this heuristic approach obtained very good solutions using a MIP solver.

The symmetry breaking formulations made it possible to achieve better solutions than those obtained using a standard formulation. The use of an alternative asymmetric formulation with fewer variables lead to better results than a larger formulation, even though the latter is stronger in terms of its linear relaxation.

Information about the structure of good solutions was used to guide the solver and helped it to obtain much better results. The solutions obtained by the symmetry breaking formulations allowed us to identify characteristics of potentially good solutions. As a consequence, an even simpler approach was devised taking these characteristics into account. This approach found solutions that are as good as or better than the best available in the literature almost all instances in a few minutes of running time (all 180 instances with small items and 160 out of the 180 instances with large items).

Finding improved solutions for instances with large items remains a challenge. Better bounds using a column generation approach seems to be a promising approach, as far as this might help to close the gap for unsolved instances.

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Appendix

In this Appendix we present the objective values of the solutions found for all test instances. For each instance in Tables 7-18, PBest denotes the previously known best solution value, while the other columns denote the best solution value found by each strategy.

Table 7: Best solution values using the formulations by representatives and the heuristic for the small items instances with 20 shipments.

			Standard solver settings			Branching priorities			
	Instance	PBest	b(REP1)	b(REP2)	b(HEUR)	b(REP1)	b(REP2)	b(HEUR)	b(HEUR+REP2)
data_O20A08R5U1	0.in	9190.1	9190.1	9190.1	9190.1	9190.1	9190.1	9190.1	9190.1
	1.in	7843.8	7843.8	7843.8	7843.8	7843.8	7843.8	7843.8	7843.8
	2.in	7659.5	7659.5	7659.5	7659.5	7659.5	7659.5	7659.5	7659.5
	3.in	8528.6	8528.6	8528.6	8528.6	8528.6	8528.6	8528.6	8528.6
	4.in	8384.1	8384.1	8384.1	8384.1	8384.1	8384.1	8384.1	8384.1
	5.in	8558.5	8558.5	8558.5	8558.5	8558.5	8558.5	8558.5	8558.5
	6.in	8514.1	8514.1	8514.1	8514.1	8514.1	8514.1	8514.1	8514.1
	7.in	9103.5	9103.5	9103.5	9103.5	9103.5	9103.5	9103.5	9103.5
	8.in	10190.5	10190.5	10190.5	10190.5	10190.5	10190.5	10190.5	10190.5
	9.in	8163.4	8163.4	8163.4	8163.4	8163.4	8163.4	8163.4	8163.4
data_O20A16R5U1	0.in	8764.3	8764.3	8764.3	8764.3	8764.3	8764.3	8764.3	8764.3
	1.in	12587.3	12587.3	12587.3	12587.3	12587.3	12587.3	12587.3	12587.3
	2.in	10857.4	10857.4	10857.4	10857.4	10857.4	10857.4	10857.4	10857.4
	3.in	10774.6	10774.6	10774.6	10774.6	10774.6	10774.6	10774.6	10774.6
	4.in	10684.1	10684.1	10684.1	10684.1	10684.1	10684.1	10684.1	10684.1
	5.in	10084.2	10084.2	10084.2	10084.2	10084.2	10084.2	10084.2	10084.2
	6.in	11126.0	11126.0	11126.0	11126.0	11126.0	11126.0	11126.0	11126.0
	7.in	11528.6	11528.6	11528.6	11528.6	11528.6	11528.6	11528.6	11528.6
	8.in	7830.1	7830.1	7830.1	7830.1	7830.1	7830.1	7830.1	7830.1
	9.in	11197.2	11197.2	11197.2	11197.2	11197.2	11197.2	11197.2	11197.2
data_O20A32R5U1	0.in	16136.9	16136.9	16136.9	16136.9	16136.9	16136.9	16136.9	16136.9
	1.in	12737.0	12737.0	12737.0	12737.0	12737.0	12737.0	12737.0	12737.0
	2.in	13371.0	13371.0	13371.0	13371.0	13371.0	13371.0	13371.0	13371.0
	3.in	16024.0	16024.0	16024.0	16024.0	16024.0	16024.0	16024.0	16024.0
	4.in	14447.8	14447.8	14447.8	14447.8	14447.8	14447.8	14447.8	14447.8
	5.in	17397.2	17397.2	17397.2	17397.2	17397.2	17397.2	17397.2	17397.2
	6.in	15631.7	15631.7	15631.7	15631.7	15631.7	15631.7	15631.7	15631.7
	7.in	13726.3	13726.3	13726.3	13726.3	13726.3	13726.3	13726.3	13726.3
	8.in	16164.1	16164.1	16164.1	16164.1	16164.1	16164.1	16164.1	16164.1
	9.in	14450.9	14450.9	14450.9	14450.9	14450.9	14450.9	14450.9	14450.9

Table 8: Best solution values using the formulations by representatives and the heuristic for the small items instances with 20 shipments.

	Instance	PBest	Standard solver settings b(REP1)	Standard solver settings b(REP2)	Standard solver settings b(HEUR)	Branching priorities b(REP1)	Branching priorities b(REP2)	Branching priorities b(HEUR)	Branching priorities b(HEUR+REP2)
data_O20A08R10U1	0.in	8132.1	8132.1	8132.1	8132.1	8132.1	8132.1	8132.1	8132.1
	1.in	7304.4	7304.4	7304.4	7304.4	7304.4	7304.4	7304.4	7304.4
	2.in	7927.1	7927.1	7927.1	7927.1	7927.1	7927.1	7927.1	7927.1
	3.in	8343.0	8343.0	8343.0	8343.0	8343.0	8343.0	8343.0	8343.0
	4.in	7533.5	7533.5	7533.5	7533.5	7533.5	7533.5	7533.5	7533.5
	5.in	7936.3	7936.3	7936.3	7936.3	7936.3	7936.3	7936.3	7936.3
	6.in	7255.8	7255.8	7255.8	7255.8	7255.8	7255.8	7255.8	7255.8
	7.in	7568.9	7568.9	7568.9	7568.9	7568.9	7568.9	7568.9	7568.9
	8.in	7861.0	7861.0	7861.0	7861.0	7861.0	7861.0	7861.0	7861.0
	9.in	7847.2	7847.2	7847.2	7847.2	7847.2	7847.2	7847.2	7847.2
data_O20A16R10U1	0.in	10748.8	10748.8	10748.8	10748.8	10748.8	10748.8	10748.8	10748.8
	1.in	9114.6	9114.6	9114.6	9114.6	9114.6	9114.6	9114.6	9114.6
	2.in	10519.8	10519.8	10519.8	10519.8	10519.8	10519.8	10519.8	10519.8
	3.in	10971.7	10971.7	10971.7	10971.7	10971.7	10971.7	10971.7	10971.7
	4.in	10613.5	10613.5	10613.5	10613.5	10613.5	10613.5	10613.5	10613.5
	5.in	8311.8	8311.8	8311.8	8311.8	8311.8	8311.8	8311.8	8311.8
	6.in	9339.8	9339.8	9339.8	9339.8	9339.8	9339.8	9339.8	9339.8
	7.in	9976.5	9976.5	9976.5	9976.5	9976.5	9976.5	9976.5	9976.5
	8.in	9985.5	9985.5	9985.5	9985.5	9985.5	9985.5	9985.5	9985.5
	9.in	10062.4	10062.4	10062.4	10062.4	10062.4	10062.4	10062.4	10062.4
data_O20A32R10U1	0.in	17389.5	17389.5	17389.5	17389.5	17389.5	17389.5	17389.5	17389.5
	1.in	15452.2	15452.2	15452.2	15452.2	15452.2	15452.2	15452.2	15452.2
	2.in	13331.9	13331.9	13331.9	13331.9	13331.9	13331.9	13331.9	13331.9
	3.in	15099.5	15099.5	15099.5	15099.5	15099.5	15099.5	15099.5	15099.5
	4.in	13992.1	13992.1	13992.1	13992.1	13992.1	13992.1	13992.1	13992.1
	5.in	15212.1	15212.1	15212.1	15212.1	15212.1	15212.1	15212.1	15212.1
	6.in	12951.9	12951.9	12951.9	12951.9	12951.9	12951.9	12951.9	12951.9
	7.in	15400.3	15400.3	15400.3	15400.3	15400.3	15400.3	15400.3	15400.3
	8.in	13880.5	13880.5	13880.5	13880.5	13880.5	13880.5	13880.5	13880.5
	9.in	15480.0	15480.0	15480.0	15480.0	15480.0	15480.0	15480.0	15480.0

Table 9: Best solution values using the formulations by representatives and the heuristic for the small items instances with 50 shipments.

	Instance	PBest	Standard solver settings			Branching priorities			
			b(REP1)	b(REP2)	b(HEUR)	b(REP1)	b(REP2)	b(HEUR)	b(HEUR+REP2)
data_O50A08R5U1	0.in	13908.1	13787.8	13771.8	13740.1	13771.8	13740.1	13740.1	13740.1
	1.in	15504.7	15504.7	15504.7	15504.7	15504.7	15504.7	15504.7	15504.7
	2.in	13840.3	13840.3	13840.3	13840.3	13840.3	13840.3	13840.3	13840.3
	3.in	17332.5	17332.3	17332.3	17332.3	17332.3	17332.3	17332.3	17332.3
	4.in	16761.1	16761.1	16761.1	16761.1	16761.1	16761.1	16761.1	16761.1
	5.in	13750.7	13750.7	13750.7	13750.7	13750.7	13750.7	13750.7	13750.7
	6.in	15639.2	15639.2	15639.2	15639.2	15639.2	15639.2	15639.2	15639.2
	7.in	18101.6	18138.2	17955.4	17903.4	17943.9	17903.4	17903.4	17903.4
	8.in	16180.6	16180.6	16180.6	16180.6	16180.6	16180.6	16180.6	16180.6
	9.in	17811.3	17811.3	17811.3	17811.3	17811.3	17811.3	17811.3	17811.3
data_O50A16R5U1	0.in	20062.3	20062.3	20062.3	20062.3	20062.3	20062.3	20062.3	20062.3
	1.in	21005.7	21005.7	21005.7	21005.6	21005.7	21005.6	21005.6	21005.7
	2.in	23238.2	23246.2	23238.2	23178.4	23246.2	23238.2	23178.4	23178.4
	3.in	19915.1	19915.1	19915.1	19915.1	19915.1	19915.1	19915.1	19915.1
	4.in	22272.4	22272.4	22272.4	22272.4	22272.4	22272.4	22272.4	22272.4
	5.in	19801.4	19759.0	19759.0	19759.0	19759.0	19759.0	19759.0	19759.0
	6.in	21474.7	21474.7	21474.7	21474.7	21474.7	21474.7	21474.7	21474.7
	7.in	20091.8	20091.8	20091.8	20091.8	20091.8	20091.8	20091.8	20091.8
	8.in	20613.4	20613.4	20613.4	20613.4	20613.4	20613.4	20613.4	20613.4
	9.in	19688.0	19688.0	19688.0	19688.0	19688.0	19688.0	19688.0	19688.0
data_O50A32R5U1	0.in	30711.3	30711.3	30711.3	30711.3	30711.3	30711.3	30711.3	30711.3
	1.in	32385.9	32385.9	32385.9	32385.9	32385.9	32385.9	32385.9	32385.9
	2.in	28972.8	28972.8	28972.8	28972.8	28972.8	28972.8	28972.8	28972.8
	3.in	27231.5	27231.5	27231.5	27231.5	27231.5	27231.5	27231.5	27231.5
	4.in	44403.0	44296.4	44297.3	44296.4	44296.4	44296.4	44296.4	44296.4
	5.in	32766.5	32610.7	32610.7	32610.7	32610.7	32610.7	32610.7	32610.7
	6.in	36783.0	36781.7	36781.7	36781.7	36781.7	36781.7	36781.7	36781.7
	7.in	28739.5	28739.5	28739.5	28739.5	28739.5	28739.5	28739.5	28739.5
	8.in	37757.3	37722.3	37722.3	37722.3	37722.3	37722.3	37722.3	37722.3
	9.in	29950.9	29950.9	29950.9	29950.9	29950.9	29950.9	29950.9	29950.9

Table 10: Best solution values using the formulations by representatives and the heuristic for the small items instances with 50 shipments.

	Instance	PBest	Standard solver settings			Branching priorities			
			b(REP1)	b(REP2)	b(HEUR)	b(REP1)	b(REP2)	b(HEUR)	b(HEUR+REP2)
data_O50A08R10U1	0.in	15466.9	15466.9	15466.9	15466.9	15466.9	15466.9	15466.9	15466.9
	1.in	13615.4	13615.4	13615.4	13615.4	13615.4	13615.4	13615.4	13615.4
	2.in	13564.7	13564.7	13564.7	13564.7	13564.7	13564.7	13564.7	13564.7
	3.in	16536.4	16734.5	16521.6	16521.6	16701.7	16521.6	16521.6	16521.6
	4.in	14975.9	14993.8	14886.0	14886.0	14886.0	14920.3	14886.0	14886.0
	5.in	15666.6	15617.4	15617.4	15617.4	15617.4	15617.4	15617.4	15617.4
	6.in	14346.1	14346.1	14346.1	14346.1	14346.1	14346.1	14346.1	14346.1
	7.in	15243.2	14671.4	14671.4	14671.4	14671.4	14671.4	14671.4	14671.4
	8.in	15424.1	15375.5	15375.5	15375.5	15375.5	15375.5	15375.5	15375.5
	9.in	15279.1	15312.7	15279.1	15278.7	15279.1	15278.7	15278.7	15278.7
data_O50A16R10U1	0.in	20451.6	20451.6	20451.6	20451.6	20451.6	20451.6	20451.6	20451.6
	1.in	21536.4	21517.4	21517.4	21517.4	21517.4	21517.4	21517.4	21517.4
	2.in	20592.4	20592.4	20592.4	20592.4	20592.4	20592.4	20592.4	20592.4
	3.in	19034.2	19034.2	19034.2	19034.2	19034.2	19034.2	19034.2	19034.2
	4.in	19862.5	19862.5	19862.5	19862.5	19862.5	19862.5	19862.5	19862.5
	5.in	20953.7	20953.7	20953.7	20953.7	20953.7	20953.7	20953.7	20953.7
	6.in	18700.7	18694.3	18694.3	18694.3	18694.3	18694.3	18694.3	18694.3
	7.in	17390.5	17390.5	17390.5	17390.5	17390.5	17390.5	17390.5	17390.5
	8.in	19578.9	19578.9	19578.9	19578.9	19578.9	19578.9	19578.9	19578.9
	9.in	19137.3	19137.3	19137.3	19137.3	19137.3	19137.3	19137.3	19137.3
data_O50A32R10U1	0.in	29945.5	29945.5	29945.5	29945.5	29945.5	29945.5	29945.5	29945.5
	1.in	30798.6	30728.6	30728.6	30728.6	30728.6	30728.6	30728.6	30728.6
	2.in	27509.9	27509.9	27509.9	27509.9	27509.9	27509.9	27509.9	27509.9
	3.in	31287.7	31287.7	31287.7	31287.7	31287.7	31287.7	31287.7	31287.7
	4.in	29572.8	29572.8	29572.8	29572.8	29572.8	29572.8	29572.8	29572.8
	5.in	27682.5	27682.5	27682.5	27682.5	27682.5	27682.5	27682.5	27682.5
	6.in	30091.0	29614.5	29614.5	29614.5	29614.5	29614.5	29614.5	29614.5
	7.in	29314.1	29314.1	29314.1	29314.1	29314.1	29314.1	29314.1	29314.1
	8.in	28097.2	28097.2	28097.2	28097.2	28097.2	28097.2	28097.2	28097.2
	9.in	30895.1	30895.1	30895.1	30895.1	30895.1	30895.1	30895.1	30895.1

Table 11: Best solution values using the formulations by representatives and the heuristic for the small items instances with 80 shipments.

	Instance	PBest	Standard solver settings			Branching priorities			
			b(REP1)	b(REP2)	b(HEUR)	b(REP1)	b(REP2)	b(HEUR)	b(HEUR+REP2)
data_O80A08R5U1	0.in	23239.7	23006.7	22917.7	22917.7	22947.5	22917.7	22917.7	22917.7
	1.in	24950.5	25051.0	24863.1	24863.1	24884.9	24863.1	24863.1	24863.1
	2.in	22238.7	22143.0	22143.0	22143.0	22148.4	22146.0	22143.0	22143.0
	3.in	22048.6	22889.6	21981.2	21970.1	22588.6	22102.2	21970.1	21970.1
	4.in	23994.8	24976.5	23958.5	23929.3	24517.2	23929.3	23929.3	23929.3
	5.in	25289.8	25949.5	25218.8	25177.6	25653.0	25223.3	25177.6	25177.6
	6.in	25810.6	25928.2	25506.1	25506.1	25733.0	25521.6	25506.1	25506.1
	7.in	21795.9	22202.0	21662.6	21565.2	21675.7	21565.2	21565.2	21565.2
	8.in	26610.8	26747.4	26582.8	26582.8	26651.7	26583.8	26582.8	26582.8
	9.in	23828.4	24264.5	23719.1	23614.3	23845.6	23744.4	23614.3	23613.2
data_O80A16R5U1	0.in	30931.6	31138.3	30931.6	30931.6	30940.0	30931.6	30931.6	30931.6
	1.in	31507.7	31802.1	31501.6	31499.6	31676.5	31506.6	31499.6	31499.6
	2.in	33263.9	33268.9	33262.4	33262.4	33402.0	33262.4	33262.4	33262.4
	3.in	31621.2	31872.5	31621.2	31621.2	31935.9	31621.2	31621.2	31621.2
	4.in	29794.3	30371.5	29748.4	29748.4	30162.7	29748.4	29748.4	29748.4
	5.in	46548.2	46807.1	46408.3	46361.5	46875.1	46371.0	46361.5	46361.5
	6.in	31408.0	31836.2	31381.9	31375.2	31660.1	31492.8	31375.2	31375.2
	7.in	31226.8	31265.1	31226.8	31226.8	31244.7	31226.8	31226.8	31226.8
	8.in	31349.3	31259.8	31114.4	31114.4	31247.3	31114.4	31114.4	31114.4
	9.in	39187.9	39364.7	39190.1	39147.2	39147.2	39147.2	39147.2	39147.2
data_O80A32R5U1	0.in	46910.8	46245.1	46245.1	46245.1	46280.3	46245.1	46245.1	46245.1
	1.in	47633.6	47620.1	47620.1	47620.1	47620.1	47620.1	47620.1	47620.1
	2.in	46898.6	46650.6	46650.6	46501.8	46730.1	46501.8	46501.8	46501.8
	3.in	46436.3	46506.0	46390.7	46373.7	46390.7	46373.7	46373.7	46373.7
	4.in	48984.9	48984.9	48984.9	48984.9	48984.9	48984.9	48984.9	48984.9
	5.in	40332.1	41020.7	40323.7	40312.3	40362.0	40366.0	40312.3	40312.3
	6.in	51115.6	51176.1	51185.1	51064.0	51064.0	51064.0	51064.0	51064.0
	7.in	48288.2	49704.6	48133.9	48133.9	48314.7	48133.9	48133.9	48133.9
	8.in	56830.6	56816.8	56816.8	56816.8	56816.8	56816.8	56816.8	56816.8
	9.in	50167.8	50270.9	50167.8	50167.8	50326.3	50167.8	50167.8	50167.8

Table 12: Best solution values using the formulations by representatives and the heuristic for the small items instances with 80 shipments.

	Instance	PBest	Standard solver settings			Branching priorities			
			b(REP1)	b(REP2)	b(HEUR)	b(REP1)	b(REP2)	b(HEUR)	b(HEUR+REP2)
data_O80A08R10U1	0.in	23172.4	22719.7	22584.7	22509.3	22826.0	22509.3	22509.3	22509.3
	1.in	21197.3	21820.6	21045.3	20942.9	21777.9	21055.8	20942.9	20942.9
	2.in	22258.6	22834.1	22021.8	21703.8	22475.1	21874.8	21703.8	21703.8
	3.in	25713.5	25221.6	24669.5	24635.8	25026.1	24638.8	24635.8	24635.8
	4.in	22444.5	22846.6	22271.3	21844.3	22604.6	22083.7	21844.3	21844.3
	5.in	22835.4	23220.7	22772.2	22446.6	22663.2	22589.5	22446.6	22446.6
	6.in	23142.4	23551.1	22622.9	22544.5	23551.1	22689.4	22544.5	22544.5
	7.in	20665.0	21255.1	20544.4	20530.4	21297.4	20530.4	20530.4	20530.4
	8.in	22572.6	23165.2	22253.1	22220.6	22877.1	22329.5	22220.6	22220.6
	9.in	24938.4	25533.1	24765.1	24747.6	25516.8	24844.0	24747.6	24747.6
data_O80A16R10U1	0.in	27836.1	28278.2	27794.0	27568.0	28224.9	27632.9	27568.0	27568.0
	1.in	30849.5	29545.8	29545.8	29545.8	29545.8	29545.8	29545.8	29545.8
	2.in	30724.8	30339.6	30289.3	30289.3	30296.4	30289.3	30289.3	30289.3
	3.in	33480.6	31858.4	31848.5	31834.2	31884.8	31859.2	31834.2	31834.2
	4.in	32453.7	31264.2	31015.8	31015.8	33427.2	31015.8	31015.8	31015.8
	5.in	30981.3	30679.7	30454.3	30454.3	30701.1	30473.0	30454.3	30454.3
	6.in	31294.4	31010.2	30843.6	30738.5	31186.9	30775.8	30738.5	30738.5
	7.in	30277.5	31290.3	30125.5	30125.5	30633.2	30203.0	30125.5	30125.5
	8.in	30533.1	30406.8	30370.0	30370.0	30610.3	30370.0	30370.0	30370.0
	9.in	31268.6	31035.1	30548.7	30545.1	31035.1	30545.1	30545.1	30545.1
data_O80A32R10U1	0.in	47313.0	47313.0	47313.0	47313.0	47313.0	47313.0	47313.0	47313.0
	1.in	43191.3	43223.6	43102.7	43053.1	43255.6	43058.7	43053.1	43053.1
	2.in	50835.5	49786.4	49786.4	49786.4	49786.4	49786.4	49786.4	49786.4
	3.in	46607.6	45488.2	45521.9	45253.1	45253.1	45365.6	45253.1	45253.1
	4.in	48756.4	47351.9	47351.9	47323.3	47323.3	47323.3	47323.3	47323.3
	5.in	42917.3	42968.3	42807.2	42807.2	42938.7	42807.2	42807.2	42807.2
	6.in	51543.9	52024.0	51480.3	51385.8	51629.0	51561.6	51385.8	51385.8
	7.in	37271.6	37279.8	37271.6	37271.6	37295.1	37271.6	37271.6	37271.6
	8.in	46107.6	48163.7	46019.8	46019.8	46087.2	46019.8	46019.8	46019.8
	9.in	45641.0	44463.8	44454.4	44454.3	44454.4	44454.4	44454.3	44454.3

Table 13: Best solution values using the formulations by representatives and the heuristic for the large items instances with 20 shipments.

	Instance	PBest	Branching priorities b(REP2)	b(HEUR+REP2)
data_O20A08R5U2	0.in	20406.1	20362.2	20362.2
	1.in	22809.2	22805.3	22805.3
	2.in	16081.0	16081.0	16081.0
	3.in	24626.5	24600.1	24600.1
	4.in	20433.2	20398.1	20398.1
	5.in	18610.1	18610.1	18610.1
	6.in	17751.5	17630.5	17630.5
	7.in	21588.0	21538.5	21538.5
	8.in	19822.8	19822.8	19822.8
	9.in	18454.4	18454.4	18454.4
data_O20A16R5U2	0.in	26528.2	26528.2	26528.2
	1.in	29058.9	29058.9	29058.9
	2.in	26337.8	26337.8	26337.8
	3.in	26900.0	26900.0	26900.0
	4.in	26972.4	26972.4	26972.4
	5.in	28753.0	28753.0	28753.0
	6.in	24135.9	24135.9	24135.9
	7.in	27957.8	27957.8	27957.8
	8.in	26647.7	26647.7	26647.7
	9.in	28336.6	28336.6	28336.6
data_O20A32R5U2	0.in	44999.4	44999.4	44999.4
	1.in	39514.1	39514.1	39514.1
	2.in	35285.0	35285.0	35285.0
	3.in	66526.1	66526.1	66526.1
	4.in	37743.7	37743.7	37743.7
	5.in	42341.9	42341.9	42341.9
	6.in	33069.1	33069.1	33069.1
	7.in	42269.1	42269.1	42269.1
	8.in	47704.2	47704.2	47704.2
	9.in	31442.6	31442.6	31442.6

Table 14: Best solution values using the formulations by representatives and the heuristic for the large items instances with 20 shipments.

	Instance	PBest	Branching priorities b(REP2)	b(HEUR+REP2)
data_O20A08R10U2	0.in	14809.3	14666.0	14666.0
	1.in	20032.9	20023.2	20023.2
	2.in	17208.5	17208.5	17208.5
	3.in	17286.9	17171.3	17119.7
	4.in	18579.6	18579.6	18579.6
	5.in	17555.3	17555.3	17555.3
	6.in	17644.3	17590.8	17590.8
	7.in	19095.1	18963.0	18963.0
	8.in	18710.0	18700.9	18700.9
	9.in	17906.6	17891.6	17891.6
data_O20A16R10U2	0.in	27089.4	26983.8	26983.8
	1.in	25999.2	25999.2	25999.2
	2.in	20786.6	20786.6	20786.6
	3.in	26558.3	26558.3	26558.3
	4.in	21557.6	21557.6	21557.6
	5.in	25346.5	25346.5	25346.5
	6.in	24179.6	24065.4	24065.4
	7.in	27083.8	27083.8	27083.8
	8.in	20852.7	20852.7	20852.7
	9.in	27840.1	27787.8	27787.8
data_O20A32R10U2	0.in	37630.8	37648.7	37648.7
	1.in	38286.4	38286.4	38286.4
	2.in	28281.4	28281.4	28281.4
	3.in	30571.6	30571.6	30571.6
	4.in	36984.9	36984.9	36984.9
	5.in	34375.9	34376.0	34375.9
	6.in	41133.1	41133.1	41133.1
	7.in	32642.6	32642.6	32642.6
	8.in	35004.5	35004.5	35004.5
	9.in	32235.3	32235.3	32235.3

Table 15: Best solution values using the formulations by representatives and the heuristic for the large items instances with 50 shipments.

	Instance	PBest	Branching priorities b(REP2)	b(HEUR+REP2)
data_O50A08R5U2	0.in	55330.2	55306.9	55306.9
	1.in	48588.1	48352.8	48352.8
	2.in	45869.0	45211.8	45211.8
	3.in	46123.2	45710.5	45710.5
	4.in	41674.9	41379.3	41379.3
	5.in	44907.1	44907.1	44907.1
	6.in	46346.1	46346.1	46346.1
	7.in	45637.8	44187.2	44187.2
	8.in	49423.6	49107.1	49107.1
	9.in	39887.1	39193.6	39193.6
data_O50A16R5U2	0.in	70971.2	70731.2	70731.2
	1.in	61319.9	61283.7	61283.7
	2.in	60375.9	60375.9	60375.9
	3.in	54368.5	54332.7	54332.7
	4.in	75357.8	75878.0	75627.7
	5.in	60051.7	59937.2	59937.2
	6.in	61421.9	61231.5	61231.5
	7.in	52707.6	52310.2	52310.2
	8.in	62476.4	62214.4	62214.4
	9.in	63184.9	63222.9	63184.9
data_O50A32R5U2	0.in	95124.0	95053.2	95053.2
	1.in	83807.2	83698.9	83698.9
	2.in	86934.5	86934.5	86934.5
	3.in	90705.5	90705.5	90705.5
	4.in	89389.3	89496.8	89496.8
	5.in	91797.7	91797.7	91797.7
	6.in	83741.4	83741.4	83741.4
	7.in	80117.7	80019.9	80019.9
	8.in	81899.6	81778.4	81778.4
	9.in	96449.3	95831.8	95831.8

Table 16: Best solution values using the formulations by representatives and the heuristic for the large items instances with 50 shipments.

	Instance	PBest	Branching priorities b(REP2)	b(HEUR+REP2)
data_O50A08R10U2	0.in	43892.3	42178.4	42073.8
	1.in	45444.6	44673.6	44496.3
	2.in	45814.5	42203.1	42203.1
	3.in	41740.3	39670.9	39574.9
	4.in	46843.4	45555.3	45553.7
	5.in	42355.7	41432.4	41394.8
	6.in	48086.9	48801.6	48801.6
	7.in	42578.4	41041.6	41041.6
	8.in	44170.9	43366.5	43366.5
	9.in	41397.8	39637.0	39588.0
data_O50A16R10U2	0.in	52130.1	50677.2	50677.2
	1.in	55846.4	54339.1	54316.2
	2.in	53117.1	53033.0	53033.0
	3.in	57437.4	57186.0	57156.2
	4.in	50365.4	49975.8	49975.8
	5.in	52943.6	53035.0	53035.0
	6.in	79065.9	77598.8	77598.8
	7.in	47984.4	47328.0	47328.0
	8.in	54845.6	54746.2	54663.2
	9.in	53498.6	53071.9	53071.9
data_O50A32R10U2	0.in	87027.2	87566.2	87566.2
	1.in	71918.4	71853.8	71853.8
	2.in	87699.2	87179.8	87171.0
	3.in	72869.5	72279.8	72279.8
	4.in	78854.7	78854.7	78854.7
	5.in	75639.3	75639.3	75639.3
	6.in	80946.4	81087.3	81080.8
	7.in	64403.1	64340.5	64340.5
	8.in	77038.2	76724.0	76724.0
	9.in	68574.5	68302.4	68302.4

Table 17: Best solution values using the formulations by representatives and the heuristic for the large items instances with 80 shipments.

	Instance	PBest	Branching priorities b(REP2)	b(HEUR+REP2)
data_O80A08R5U2	0.in	75076.6	74404.9	74404.9
	1.in	75679.5	75748.0	75748.0
	2.in	77026.4	76379.9	76379.9
	3.in	71701.3	70953.9	70953.9
	4.in	72639.4	69157.5	69157.5
	5.in	80591.4	80398.6	80398.6
	6.in	73420.8	72940.2	72940.2
	7.in	71604.8	68566.7	68566.7
	8.in	72055.4	73875.3	73875.3
	9.in	69299.1	68263.1	68263.1
data_O80A16R5U2	0.in	89963.8	89981.1	89981.1
	1.in	128280.8	128083.0	128083.0
	2.in	82215.9	81874.2	81874.2
	3.in	104838.6	103367.0	103367.0
	4.in	93761.0	93381.9	93381.9
	5.in	125481.6	125222.0	125222.0
	6.in	99560.9	99507.5	99507.5
	7.in	95947.3	95884.7	95884.7
	8.in	105456.5	105718.0	105718.0
	9.in	105542.7	104735.0	104735.0
data_O80A32R5U2	0.in	177291.4	177649.0	177581.0
	1.in	173297.2	173284.0	173284.0
	2.in	143387.1	144711.0	144711.0
	3.in	125602.4	125630.0	125630.0
	4.in	209939.6	209821.0	209820.0
	5.in	137207.0	137185.0	137185.0
	6.in	132075.8	132044.0	132044.0
	7.in	145864.9	145964.0	145964.0
	8.in	138082.0	138240.0	138240.0
	9.in	139324.3	139687.0	139687.0

Table 18: Best solution values using the formulations by representatives and the heuristic for the large items instances with 80 shipments.

	Instance	PBest	Branching priorities b(REP2)	b(HEUR+REP2)
data_O80A08R10U2	0.in	75578.8	73572.8	73572.8
	1.in	70272.9	67470.7	67458.3
	2.in	74408.0	72682.3	72612.7
	3.in	75956.9	71290.0	71290.0
	4.in	73691.7	67844.5	67844.5
	5.in	67323.7	65020.8	65020.8
	6.in	72515.7	68563.2	68563.2
	7.in	79486.2	74006.2	74006.2
	8.in	69057.5	66775.8	66613.1
	9.in	66037.6	59625.3	59625.3
data_O80A16R10U2	0.in	114857.5	112408.0	112408.0
	1.in	84183.3	82617.0	82617.0
	2.in	82071.1	80642.4	80584.0
	3.in	86937.1	87086.9	87086.9
	4.in	83796.1	83630.7	83630.7
	5.in	89442.7	88994.2	88994.2
	6.in	92209.6	91590.6	91590.6
	7.in	86450.7	84633.6	84633.6
	8.in	84441.4	84149.8	84149.8
	9.in	95797.6	92995.5	92995.5
data_O80A32R10U2	0.in	145559.3	144454.0	144164.0
	1.in	124454.3	124491.0	124456.0
	2.in	129904.1	129289.0	129289.0
	3.in	124318.3	123760.0	123760.0
	4.in	110210.4	109794.0	109794.0
	5.in	144003.6	144359.0	144359.0
	6.in	118741.1	117575.0	117575.0
	7.in	110523.7	109459.0	109459.0
	8.in	119762.0	118313.0	118132.0
	9.in	146277.4	145698.0	145332.0