A New Lower Bound to the Traveling Tournament Problem

Sebastián Urrutia, Celso C. Ribeiro, and Rafael A. Melo

***Abstract*— Optimization in sports is a field of increasing interest. The traveling tournament problem abstracts certain characteristics of sports scheduling problems. We propose a new method for determining a lower bound to this problem. The new bound improves upon the previously best known lower bound. Numerical results on benchmark instances showed reductions as large as 38.6% in the gaps between lower and upper bounds.**

I. Introduction

Optimization in sports is a field of increasing interest. Some applications have been reviewed by Ribeiro and Urrutia [1]. Combinatorial optimization techniques have been applied e.g. to game scheduling [2], playoff elimination/qualification [3] and referee assignment [4]. We also refer to [5], [6] for recent surveys on the sports scheduling literature.

The traveling tournament problem proposed by Easton et al. [7] abstracts certain characteristics of sports scheduling problems and has been tackled by several authors, see e.g. [8], [9], [10]. We propose in this paper a new method for determining a better lower bound for the traveling tournament problem than those previously known.

The paper is organized as follows. The traveling tournament problem and its mirrored version are introduced in Section 2. The independent lower bound proposed in [7] is described in Section 3 and we show that it can be tightened. An algorithm to improve the independent lower bound is proposed in Section 4. Numerical results illustrating the quality of the new lower bound are presented in Section 5. The last section closes the paper with some concluding remarks.

II. Traveling tournament problem

All teams face each other twice in a double round robin tournament, once at home and the other away. We say that a tournament is compact if every team plays exactly one game every round. We assume that every team is at its home city in the beginning of the tournament. A team returns to its home city whenever it plays its last game away. Furthermore, whenever a team plays two consecutive games away, it goes from the home city of the first opponent directly to that of the second, without making a stop at its home city. We count a trip every time a team travels from one city to another.

The *Traveling Tournament Problem* (TTP) is defined as follows [7]. Given an even number n of teams and an $n \times n$ distance matrix $D = \{d_{ij}\}_{i=1,\ldots,n}^{j=1,\ldots,n}$ representing the distances between the home cities of each pair of teams, the problem consists in building the schedule of games for a compact double round robin tournament, such that each team plays at most three consecutive home games and at most three consecutive away games, no repeaters (two games between the same pair of teams in consecutive rounds) are allowed, and the sum of the distances traveled by each team is minimized. Schedules for a *Mirrored Traveling Tournament Problem* (mTTP) satisfy an additional constraint: the games played in round $k = 1, \ldots, n-1$ are exactly the same played in round $k + (n-1)$, but with reversed venues. Repeaters do not occur in mirrored schedules.

NL instances of the TTP were defined in [7] for subsets of teams playing in the National League of the Major League Baseball. CIRCLE instances [7] are those in which the teams correspond to vertices of a circle graph with unit distances between neighbor vertices. The distance between every pair of teams is equal to the shortest path between them in the graph. Later, Urrutia and Ribeiro [11] defined the CONSTANT instances, in which the distance between every pair of teams is equal to one. Only small NL and CIRCLE instances with $n \leq 8$ teams can be exactly solved to date [9]. CONSTANT instances with up to 16 teams for the TTP and with up to 18 teams for the mTTP were already solved exactly [11], [12], [13].

The TTP has been tackled by integer programming [7], [9], constraint programming [7], [14], and local search metaheuristics [8], [10]. Benchmark instances and their best known feasible solutions and lower bounds are available in [15]. The best known solutions have been updated several times since 2001 and this trend seems to be strengthening. As an example, the best known solution for instance NL12 was updated 12 times. Contrarily, the lower bounds were never updated for the large majority of the instances. Finding improved lower bounds for the traveling tournament problem to reduce the gaps between lower and upper bounds is one of the challenges to be faced in the development of exact algorithms for this problem.

III. Independent lower bound

A very simple lower bound to the TTP can be obtained by determining the minimum distance every team has to travel to visit all others, independently of any other constraints [7].

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This can be done by solving the capacitated vehicle routing problem CVRP (1) to (4) formulated below for each team $t = 1, \ldots, n$:

$$dist(t) = \min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij} \quad (1)$$

subject to:

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} (x_{ij} + x_{ji}) = 2 \quad \forall i = 1, \ldots, n, \quad i \neq t \quad (2)$$

$$\sum_{i \in S} \sum_{j \notin S} (x_{ij} + x_{ji}) \geq 2\lceil |S|/3 \rceil \quad \forall S \subset \{1, \ldots, n\}, \quad t \notin S \quad (3)$$

$$x_{ij} \in \{0, 1\} \quad \forall i = 1, \ldots, n, \quad \forall j = 1, \ldots, n. \quad (4)$$

In this formulation, $x_{ij} = 1$ if team t goes directly from the home city of i to the home city of j, $x_{ij} = 0$ otherwise. The clients for capacitated vehicle routing problem correspond to the teams to be visited by team t. The demand of each client (i.e., each team to be visited) is equal to one, while the capacity of the vehicle (i.e., team t) is equal to three, since no team can play more than three consecutive away games. The minimum value $dist(t)$ of the objective function in equation (1) gives the minimum distance traveled by team t. Constraints (2) express that team t must enter and leave the home city of each of its opponents. Finally, constraints (3) enforce that the number of teams visited in a single tour cannot exceed three. Since there are exponentially many constraints of this type, they should be entered into the model gradually when they are violated. The independent lower bound is then given by the sum of all individual team bounds, i.e. $ILB = \sum_{t=1}^{n} dist(t)$.

We show that ILB is not a tight bound by considering the CONSTANT instances. Since in CONSTANT instances the distance between every pair of teams is equal to one, the objective function value of every feasible solution is equal to the number of trips that the teams perform in this solution. The minimization of the traveled distance for the CONSTANT instances gives the minimum number of trips to be performed by all teams in a feasible solution. Furthermore, the number of trips performed by all teams in a feasible solution of the TTP on any kind of instance cannot be smaller than the optimal value of the corresponding CONSTANT instance [11]. We illustrate below by an example that in the computation of the independent lower bound teams may perform less trips than this minimum, showing that ILB is not tight.

We consider the four team instance in Figures 1 and 2, in which the circles represent teams and the squared circles correspond to the team visiting all other teams in each diagram. Solving CVRP for each team yields four trips: each team is able to visit all other teams in a single tour, since the number of opponents is not greater than three. Adding up the number of trips of all teams gives a total of 16 trips, as showed in Figure 1.

However, Rasmussen and Trick [13] have shown that the optimal value of the CONSTANT instance with four teams is 17. Therefore, it is possible to enforce that one of the four teams should perform an extra trip without eliminating any feasible solution. In Figure 2 team d performs one additional trip, making the total number of trips equal to the lower bound 17.

Since ILB considers the minimum number of trips for each team without imposing constraints on the total number of trips of all teams. It may be smaller than the total minimum of trips in a feasible TTP solution. Therefore, the lower bound ILB may be improved.

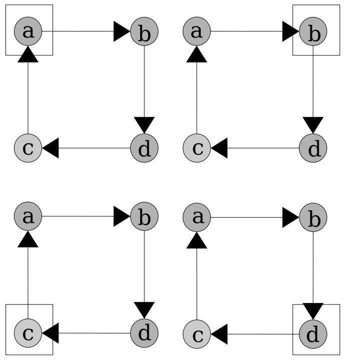

Fig. 1. Solution of CVRP for each team and the lower bound ILB.

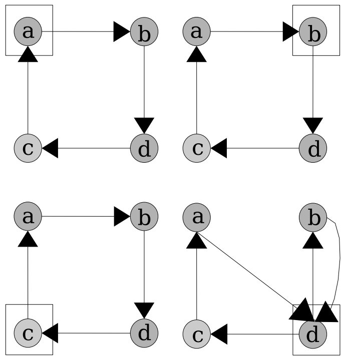

Fig. 2. Teams performing a total of 17 trips.

IV. Minimum number of trips lower bound

The minimum number of trips lower bound MNT is computed considering the difference between the value of the optimal solution (or the best known lower bound) of the related CONSTANT instance and the sum of the individual minimum number of trips. Algorithm 1 shows the pseudo-code of the algorithm to compute this lower bound.

The first step in the evaluation of the new lower bound MNT for a given instance consists in computing in lines 1 to 5 the minimum number of trips a single team must perform. A team must travel $k+1$ times to visit k teams in a single road trip and to return to its home city, with $k \leq 3$ for the TTP. Every team must visit the other $n-1$ teams. Therefore, the minimum number of trips performed by each team is $mnt = 4(n-1)/3$ if $(n-1) \mod 3 = 0$, or $mnt = 4\lfloor (n-1)/3 \rfloor + (n-1) \mod 3 + 1$ if $(n-1) \mod 3 > 0$.

Variable $ExtraTrips$ is set in line 6 as the difference between the optimal solution value OPT (or a lower bound,

Algorithm: ComputeLowerBoundMNT()

Input: TTP instance

Output: Lower bound MNT

1 **if** $(n-1) \mod 3 = 0$ **then**
2 | $mnt \leftarrow 4(n-1)/3$;
3 **else**
4 | $mnt \leftarrow 4\lfloor(n-1)/3\rfloor + (n-1) \mod 3 + 1$
5 **end**
6 $ExtraTrips \leftarrow OPT - mnt \cdot n$;
7 Compute the independent lower bound ILB;
8 **if** $ExtraTrips = 0$ **then**
9 | $MNT \leftarrow ILB$;
10 **else**
11 | $p \leftarrow \min\{ExtraTrips, 2(n-1) - mnt\}$;
12 | **for** $t = 1, \ldots, n$ **do**
13 | | Compute $dist(t)$ by solving CVRP for team t;
14 | | **for** $j = 1, \ldots, p$ **do**
15 | | | Compute $\overline{dist}(t,j)$ by solving CVRP for team t with the additional constraint $\sum_{i \neq j} x_{ij} \geq mnt + j$;
16 | | | $e_{tj} \leftarrow \overline{dist}(t,j) - dist(t)$;
17 | | **end**
18 | **end**
19 | Solve problem (5) to (8) to minimize the additional distance the teams must travel to reach the minimum total number of trips;
20 | $MNT \leftarrow ILB + \Delta$;
21 **end**

Algorithm 1: Algorithm for computing the minimum number of trips lower bound

in case the optimal is not known) of the corresponding CONSTANT instance and n times the minimum number of trips mnt a single team must perform. The independent lower bound is computed in line 7. If $ExtraTrips$ is equal to zero (line 8), then the minimum number of trips lower bound is equal to the independent lower bound. In this case, there are feasible solutions in which every team performs exactly the minimum number of trips considered in the computation of the independent lower bound.

Otherwise, the extra distance matrix $E = \{e_{tj}\}_{t=1,\ldots,n}^{j=1,\ldots,p}$ is computed. Matrix E has one row for each team (n) and as many columns (p) as the minimum between $ExtraTrips$ and the difference between the maximum ($2(n-1)$) and the minimum (mnt) number of trips every team may perform. Each element e_{tj} is equal to the additional distance team t must travel if it is forced to perform at least j trips more than the minimum number of trips it may perform. The number p of columns of matrix E is computed in line 11. The loop in lines 12 to 18 scans all teams $t = 1, \ldots, n$. The individual lower bound $dist(t)$ for each team is computed in line 13. The loop in lines 14 to 17 scans all columns. To compute the extra distance matrix for team t and j additional trips, we first solve in line 15 the capacitated vehicle routing problem (1) to (4) with the following additional constraint $\sum_{i \neq j} x_{ij} \geq mnt + j$, which states that the total number of trips must be greater than or equal to $mnt + j$. The value of e_{tj} is computed in line 16.

In the last step, we solve in line 19 problem (5) to (8) to minimize the additional distance the teams must travel to reach the minimum total number of trips:

$$\Delta = \min \sum_{t=1}^{n} \sum_{j=1}^{p} e_{tj} y_{tj} \tag{5}$$

subject to:

$$\sum_{j=1}^{p} y_{tj} \leq 1 \quad \forall t = 1, \ldots, n \tag{6}$$

$$\sum_{t=1}^{n} \sum_{j=1}^{p} j \cdot y_{tj} = ExtraTrips \tag{7}$$

$$y_{tj} \in \{0,1\} \quad \forall t = 1, \ldots, n, \quad \forall j = 1, \ldots, p. \tag{8}$$

In the above model, $y_{tj} = 1$ if team t performs exactly j extra trips, $y_{tj} = 0$ otherwise. Constraints (6) express that each team may perform just a given number of extra trips, which can be at most equal to p. Constraint (7) enforces that the total number of additional trips is equal to $ExtraTrips$. Finally, the minimum number of trips lower bound is computed in line 20.

The integer program (5) to (8) is relatively small for real-life instances and can be easily solved by standard integer programming solvers.

V. Numerical results

Numerical results illustrating the quality of the new minimum number of trips lower bound for the traveling tournament problem are presented in Table I. Results for the mirrored traveling tournament problem are presented in Table II. For each instance, we report the value of the best known feasible solution (BKS), the best lower bound to date (BLB), the independent lower bound (ILB), and the new minimum number of trips lower bound (MNT).

The new MNT lower bounds that improved the corresponding independent lower bound (ILB) are underlined. MNT lower bounds that improved the best previously known lower bound are outlined in boldface. The MNT lower bound improved the independent lower bound for ten out of 20 traveling tournament problem instances. For the mirrored version of the problem, the new bound was better than ILB for 13 out of 20 instances. Overall, the new lower bound improved the previously best known lower bound (or was the first lower bound ever to be computed) for 19 out of the 40 instances considered.

VI. Conclusions

We introduced a new lower bound to the traveling tournament problem, improving the previously known lower bounds for many of the benchmark CIRC, NL, and NFL instances. The MNT lower bound is the first lower bound to be proposed to improve the independent lower bound, which is known since the TTP was formulated in the literature. We believe this is a first step in finding good bounds for the traveling

TABLE I

RESULTS FOR THE TTP

Instance	BKS	BLB	ILB	MNT
circ4	20	20	16	18
circ6	64	64	60	60
circ8	132	128	128	128
circ10	242	220	220	**228**
circ12	408	384	384	384
circ14	654	588	588	588
circ16	928	832	832	**846**
circ18	1304	1188	1188	1188
circ20	1760	-	^{a}1600	**1600**
NL4	8276	8276	8044	8160
NL6	23916	23916	22557	22594
NL8	39721	39479	38670	38670
NL10	59436	57500	56506	56928
NL12	111248	107483	107483	**107494**
NL14	188728	182797	182797	182797
NL16	263772	248852	248852	**249477**
NFL16	237428	-	^{a}223079	**223800**
NFL18	296638	-	^{a}272834	**272834**
NFL20	346324	-	^{a}316721	**316721**
NFL22	412812	-	^{a}378692	**378813**

^{a} never computed before this work.

TABLE II

RESULTS FOR THE MIRRORED TTP

Instance	BKS	BLB	ILB	MNT
circ4	20	20	16	18
circ6	72	72	60	60
circ8	140	128	128	128
circ10	272	240	220	240
circ12	432	384	384	384
circ14	696	590	588	590
circ16	968	832	832	**876**
circ18	1306	1188	1188	1188
circ20	1852	-	^{a}1600	**1600**
NL4	8276	8276	8044	8160
NL6	26588	26588	22557	24112
NL8	41928	39479	38670	38670
NL10	63832	58190	56506	**58277**
NL12	119608	107483	107483	**110519**
NL14	199363	182797	182797	**182996**
NL16	279077	248852	248852	**253957**
NFL16	251289	-	^{a}223079	**228251**
NFL18	299903	-	^{a}272834	**276395**
NFL20	359748	-	^{a}316721	**316721**
NFL22	418086	-	^{a}378692	**378813**

^{a} never computed before this work.

tournament problem, for which most of the previous work was concentrated in the search of feasible good solutions.

The optimal solutions for the CONSTANT instances [11] were used in the computation of the new lower bound for general distance instances, illustrating the contribution of these artificially created instances to the research on the traveling tournament problem.

Rasmussen and Trick [6] stressed the usefulness of efficient strategies for the computation of improved lower bounds to the TTP, reducing the current gap between lower and upper bounds. The new lower bound proposed in this paper is a contribution in this direction, since it was able to reduce the gap between lower and upper bounds for several benchmark instances and for as much as 38.6%, e.g. for the circ10 instance.

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