POWER CONTROL IN AD HOC WIRELESS NETWORK FAULT-TOLERANT DESIGN

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Abstract

We consider the problem of power assignment to the nodes of an ad hoc wireless network, so as that the total power consumption is minimized and the resulting network is biconnected, i.e., there are at least two node-disjoint paths between any pair of nodes. A biconnected communication graph is necessary to ensure fault tolerance, since ad hoc networks are used in critical application domains where failures are likely to occur. We present a mixed integer programming formulation for the problem, whose optimal solutions can be computed by a commercial solver for moderately-sized networks. Four problem variants are discussed. We also propose a heuristic for solving large problem instances, based on a greedy randomized algorithm that builds feasible solutions and on a local search strategy to improve them. Computational experiments are presented and discussed.

Key words: *Wireless networks, ad hoc networks, topology control, biconnectivity, fault tolerance, energy consumption optimization, power control, mixed integer programming, GRASP.*

1. Introduction

An *ad hoc network* consists of a collection of transceivers, in which a packet may have to traverse multiple consecutive wireless links to reach its final destination. They have become an increasingly common and important object of study, due to their applications in battlefield communication, disaster relief communication, and sensor networks, among others.

Ad hoc networks can be represented by a set V of transceivers (nodes), numbered $0, 1, \ldots, |V| - 1$, together with their locations or the distances between them. A transmission power p_u is associated with each node $u \in V$. For each ordered pair (u, v) of transceivers, with $u, v \in V$, we are given a non-negative arc weight $e(u, v)$ such that a signal transmitted by the transceiver u can be received at node v if and only if the transmission power of u is at least equal to $e(u, v)$, i.e. if $p_u \geq e(u, v)$.

Wireless networks face a variety of constraints that do not appear in wired networks. Nodes in a wireless network are typically battery-powered, and it is expensive and sometimes even infeasible to recharge the device. We focus on radio power consumption, since radios tend to be the major source of power dissipation in wireless networks [1]. Instead of transmitting with maximum power, the proposed algorithms adjust the transmission power of each node.

There are also fault-tolerance requirements, due to their evolving critical application domains and to the large number of failures that may result from mobility, fading, or obstructions [2]. A connected graph is usually assumed as the minimum connectivity requirement by the algorithms running in different layers of the network, such as routing protocols [3]. However, if there is only one path between a pair of nodes, failure of a single node (or link) between them will result in a disconnected graph. Therefore, topologies with multiple, alternative disjoint paths between any pair of nodes are often required [4].

The *transmission graph* $G = (V, E)$, where $E = \{(u, v) : u \in V, v \in V, p_u \geq e(u, v)\}$ is said to be 2-node connected if, for any two nodes $u, v \in V$, there exist two node-disjoint paths connecting u to v. Since a 2-node connected graph is also 2-edge connected, but the converse is not necessarily true, we say that a graph is biconnected if it is 2-node connected.

Given the node set V and non-negative arc weights $e(u, v)$ for any $u, v \in V$, the *biconnected minimum power consumption problem* consists of finding an optimal assignment of transmission powers $p : V \rightarrow R+$ to every node $u \in V$, such that the total power consumption $\sum_{u \in V} p_u$ is minimized and the resulting transmission graph $G = (V, E)$ is biconnected. This problem was proved to be NP-hard by Calinescu and Wan [5].

Four variants of the biconnected minimum power consumption problem are discussed in this paper. The system model is described in detail in the next section. Previous work is reviewed in Section 3. A mixed integer programming formulation to exactly solve problems in moderately-sized networks is given in Section 4. A GRASP heuristic to approximately solve large problems instances is proposed in Section 5. Computational results are reported and discussed in Section 6. Concluding remarks are made in the last section.

2. System Model

We are given a set V of transceivers, with $|V| = n$, each of them equipped with an omnidirectional antenna which is responsible for sending and receiving signals. An ad hoc network is established by assigning a transmission power p_u to each transceiver $u \in V$.
Each node can (possibly dynamically) adjust its transmitting power, based on the distance to the receiving nodes and on the background noise. In the most common power attenuation model [6], the signal power falls with $1/d^{\varepsilon}$, where d is the distance from the transmitter and ε is the loss exponent (typical values of ε are between 2 and 4). Under this model, the power requirement at node u for supporting the transmission through a link from u to v is given by

$$p_u \geq d_{uv}^{\varepsilon} . q_v, \tag{1}$$

where d_{uv} is the Euclidean distance between the transmitter u and the receiver v, and q_v is the receiver's power threshold for signal detection, which is usually normalized to one.
We first define the *symmetric input* version of the biconnected minimum power consumption problem. In this case, the power requirement (also referred to as the weight of arc (u, v)) for supporting a transmission between nodes u and v separated by a distance d_{uv} becomes $e(u, v) = e(v, u) = d_{uv}^{\varepsilon}$. Although the symmetric version is widely accepted as reasonable, inequality (1) holds only for free-space environments with non-obstructed lines of sight. It does not consider the possible occurrence of reflections, scattering, and diffraction caused e.g. by buildings and terrains. In practice, power requirement values for two nodes u and v may be asymmetric because of many reasons. For example, asymmetric arc weights can be used to model batteries with different power levels [7] and heterogeneous nodes [8]. Also, the ambient noise levels of the regions containing the two nodes may be different [9]. Therefore, we also study the more general *asymmetric input* version of the problem. Under this model, there may be pairs of transceivers $u, v \in V$ such that $e(u, v) \neq e(v, u)$.
Communication from node u to node v is enabled whenever $p_u \geq e(u, v)$. The transmission graph associated with a power assignment p_u to each transceiver $u \in V$ is defined as the direct graph $G = (V, E)$, where $E = \{(u, v) : u \in V, v \in V, p_u \geq e(u, v)\}$.
Two different graph topology structures may be used to enforce biconnectedness. In a *unidirectional topology*, all arcs established by the power settings in the transmission graph $G = (V, E)$ are considered to enforce the connectivity constraints. In a *bidirectional topology*, the edge $[u, v]$ is used as a communication link to enforce biconnectedness not only if v is within the transmission range of u, but if u is also within the transmission range of v. In this case, the arc set considered to enforce the connectivity constraints in the transmission graph is restrained to $B = \{(u, v) : u \in V, v \in V, p_u \geq e(u, v), p_v \geq e(v, u)\} \subseteq E$.

3. Previous Work

Four versions of the biconnected minimum power consumption problem are considered:

- symmetric input with unidirectional topology,
- symmetric input with bidirectional topology,
- asymmetric input with unidirectional topology, and
- asymmetric input with bidirectional topology.

The symmetric version of the minimum power consumption problem establishing a unidirectional connected transmission graph was proved to be NP-hard by Chen and Huang [10], who presented a 2-approximation algorithm based on minimum spanning trees. Kirousis et al. [11] proved that the problem is NP-hard in the three-dimensional Euclidean space, and described a 2-approximation algorithm. Clementi et al. [12] gave a reduction proving that the same problem is also NP-hard in the two-dimensional Euclidean space. Calinescu and Wan [5] discussed algorithms for the symmetric input with unidirectional topology version of the biconnected minimum power consumption problem and established its NP-hardness. They also described a 4-approximation algorithm for the problem.
Although implementing wireless unidirectional links is technically feasible [13], and imposing the requirement of symmetry incurs in a considerable additional cost, the advantage of using unidirectional links is questionable. There is a potential for packet loss and error in realistic networks, and thus acknowledgments and retransmissions are required [10]. Therefore, to improve the network performance, link bidirectionality is implicitly assumed in many routing protocols [14]. Marina and Das [15] showed that the overhead needed to handle unidirectional links in routing protocols outweighs the benefits they can provide, and that better performance can be achieved by simply avoiding them.
The minimum power consumption problem with a bidirectional connected transmission graph and symmetric inputs was proposed in [16], where its decision version was proved to be NP-complete. Cheng et al. [17] showed the importance of the problem in the case of sensor networks, proved its NP-completeness, and proposed two approximate algorithms. The 2-approximation algorithm in [11] solves the symmetric version of the minimum power consumption problem with bidirectional connectivity. These approximation factors have been improved by Althaus et al.

[18] to $5/3+\epsilon$, who also gave an exact branch-and-cut algorithm based on a new integer programming formulation. Another exact algorithm was presented in [19]. Lloyd et al. [20] studied the symmetric input with bidirectional topology version of the biconnected minimum power consumption problem. They gave an algorithm with an approximation ratio of at most $2(2-2/n)(2+1/n)$.

While the symmetric version of the minimum power consumption problem has received significant attention in recent years, only a few approximation algorithms have been proposed for the case with asymmetric power requirements. Krumke et al. [9] considered the asymmetric version of the unidirectional connected minimum power consumption problem. They showed that an $\Omega(\log n)$-approximation algorithm cannot exist unless $P = NP$ and presented an $O(\log n)$-approximation algorithm. Independently, Calinescu et al. [7] achieved a similar approximation bound by an algorithm which incrementally constructs a tree. Caragiannis et al. [21] also obtained an $O(\log n)$-approximation algorithm.

For the asymmetric version of the bidirectional connected minimum power consumption problem, Althaus et al. [18] obtained an inapproximability result within a factor of $O(\log n)$. Caragiannis et al. [21] developed an $O(1.35 \ln n)$-approximation algorithm for the same problem. An $O(\ln n)$-approximation algorithm has been independently obtained in [7] by different techniques.

In the following, we present an integer programming formulation for tackling the four variants of the biconnected minimum power consumption problem, together with computational results obtained with a commercial integer programming solver. For the more interesting case in practice, corresponding to the asymmetric input and bidirectional topology variant, we propose a GRASP heuristic to approximately solve large, real-size problem instances.

4. Integer Programming Formulation

We propose a mixed integer programming multicommodity flow model for the bidirectional biconnected minimum power consumption problem. Let C denote a set of $\lceil |V|/2 \rceil$ commodities. For each commodity $c \in C$, let $o(c)$ be its origin and $d(c)$ its destination. For any node $i \in V$ and any commodity $c \in C$, let $D_c(i) = -2$ if $i = o(c)$, $D_c(i) = +2$ if $i = d(c)$, $D_c(i) = 0$ otherwise. The discrete variable f_{ij}^c and the continuous variable p_i represent, respectively, the flow of commodity c through arc (i,j) and the power assignment to node i. The binary variable f_{ij}^c is equal to one if arc (i,j) is used by commodity c for communication from node i to j, zero otherwise.

Let $P_i = [p_i^1, \ldots, p_i^{\phi(i)}]$ be a list of increasing power levels that can be assigned to node $i \in V$, where p_i^1 is the minimum power p_i such that transmissions from node i reach at least one node in $V \setminus \{i\}$ and $p_i^{\ell+1} > p_i^\ell$ for any $\ell = 1, \ldots, \phi(i)-1$. Also, let $p_i^0 = 0$. For any $\ell = 1, \ldots, \phi(i)$, let T_i^ℓ be the set of new nodes reachable from node i if the power level assigned to node i increases from $p_i^{\ell-1}$ to p_i^ℓ, as illustrated in Figure 1. The binary variable x_i^ℓ takes the value one if there is a node $j \in T_i^\ell$ such that (i,j) is used for communication from i to j, zero otherwise. Since the transmission graph G is required to be biconnected, each node must be able to communicate with at least two other nodes. Therefore, we denote by $p_i^{\bar{\ell}(i)}$ the minimum power level such that transmissions from node i reach at least two nodes in $V \setminus \{i\}$.

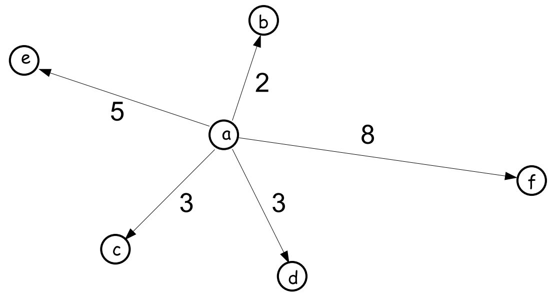

Fig. 1: Example with $P_a = [2,3,5,8]$ and $T_a^1 = \{b\}$, $T_a^2 = \{c,d\}$, $T_a^3 = \{e\}$, $T_a^4 = \{f\}$.

The mixed integer program defined by the objective function (2) and constraints (3)-(9) below is a valid formulation for the asymmetric input with unidirectional topology version of the biconnected minimum power consumption problem:

$$\min \sum_{i\in V} \sum_{\ell=1}^{\phi(i)} (p_i^\ell - p_i^{\ell-1}) \cdot x_i^\ell \qquad (2)$$

$$\sum_{j\in V} f_{ji}^c - \sum_{l\in V} f_{il}^c = D_c(i), \qquad \forall c \in C, \forall i \in V \qquad (3)$$

$$\sum_{j\in V} f_{ij}^c \le 1, \qquad \forall c \in C, \forall i \in V : i \ne o(c), i \ne d(c) \qquad (4)$$

$$x_i^\ell \ge f_{ij}^c, \qquad \forall i \in V, \forall c \in C, \forall j \in T_i^\ell, \ell = 1, \ldots, \phi(i) \qquad (5)$$

$$x_i^{\ell+1} \le x_i^\ell, \qquad \forall i \in V, \ell = 1, \ldots, \phi(i)-1 \qquad (6)$$

$$x_i^\ell = 1, \qquad \forall i \in V, \ell = 1, \ldots, \bar{\ell}(i) \qquad (7)$$

$$f_{ij}^c \in \{0,1\}, \qquad \forall i,j \in V, \forall c \in C \qquad (8)$$

$$x_i^\ell \in \{0,1\}, \qquad \forall i \in V, \ell = 1, \ldots, \phi(i). \qquad (9)$$

Constraints (3) are the flow conservation equations. Inequalities (4) ensure node-disjointness. Inequalities (5) state that x_i^ℓ must be set to one if there is a node $j \in T_i^\ell$ such that arc (i, j) is used for communication from node i to j by commodity c. Constraints (6) enforce $x_i^{\ell+1}$ to be equal to zero if the previous increment level was not used, i.e. if $x_i^\ell = 0$. Constraints (7) set to one the power increments that are necessary to reach at least the two closest nodes to each node. Constraints (8) and (9) express the integrality requirements. Whenever a bidirectional topology is sought, it suffices to replace constraints (5) by

$$x_i^\ell \geq f_{ij}^c + f_{ji}^c, \qquad \forall i \in V, \forall c \in C, \\ \forall j \in T_i^\ell, \ell = 1, \ldots, \phi(i) \quad (10)$$

to ensure the existence of one arc in each direction.

5. GRASP Heuristic

A greedy randomized adaptive search procedure (GRASP) [22] is a multistart process. Each of its iterations consists of two phases: a construction phase, in which a feasible solution is built, and a local search phase, in which a local optimum in the neighborhood of the current solution is sought. The best overall solution is returned. in the neighborhood. In the remainder of this section, we customize a GRASP heuristic for the asymmetric input with bidirectional topology version of the biconnected minimum power consumption problem.

5.1. Construction Phase

The first stage of the construction phase builds a bidirectional connected graph one node at a time. Given an undirected input graph $D = (V, A)$, the algorithm sets $p_u = 0$ for all $u \in V$, and initializes a working graph $H = (V', E)$ with $V' = \{r\}$ and $E = \emptyset$, where $r \in V$ is any randomly selected initial node. The greedy function that guides the construction is based on the wireless multicast advantage property [23]: if p_u is the current power assignment to node u and there is a node v such that $e(u, v) > p_u$, then the power required to set up communication from u to v is $e(u, v) - p_u$. Therefore, the greedy function is $g(u, v) = \max\{0, e(u, v) - p_u\} + \max\{0, e(v, u) - p_v\}$, for any $u, v \in V$. If $g(u, v) = 0$, the bidirectional communication between u and v is already set up.
For every node $u \notin V'$, let $g(u) = \min_{v \in V'}\{g(u, v)\}$ be the minimum power increment to connect it to a node in V'. Let $\underline{g} = \min_{u \in V \setminus V'}\{g(u)\}$ and $\overline{g} = \max_{u \in V \setminus V'}\{g(u)\}$ be, respectively, the minimum and maximum power increments over all candidate nodes (those not in the current solution). The restricted candidate list RCL is formed by all nodes $u \in V \setminus V'$ such that $g(u) \leq \underline{g} + \alpha(\overline{g} - \underline{g})$, with $0 \leq \alpha \leq 1$. A node u is randomly selected from RCL and inserted into V'. The power assignments of node $u \in V \setminus V'$ and node $v \in V'$ such that $g(u) = g(u, v)$ are increased by $\max\{0, e(u, v) - p_u\}$ and $\max\{0, e(v, u) - p_v\}$, respectively, and the bidirectional edge $[u, v]$ is inserted into E. This stage finishes when $V' = V$, ensuring that a connected graph $H = (V, E)$ is obtained.
The next stage produces a biconnected graph $G = (V, B)$ with $E \subseteq B$. Its edge set starts with $B = E$. A node is an articulation point of a graph if it belongs to more than one of its biconnected components. Tarjan's algorithm [24] is used to compute the biconnected components and articulation points of the current solution. For every node $u \in V$ that is not an articulation point, let $g'(u) = \min_{v \in V}\{g(u, v) : u \neq v$, node v is not an articulation point and does not belong to the same component as $u\}$ be the minimum power increment to connect it to a node in a different biconnected component which is not an articulation point. Let $\underline{g}' = \min_{u \in V}\{g'(u) : u \text{ is not an articulation point}\}$ and $\overline{g}' = \max_{u \in V}\{g'(u) : u \text{ is not an articulation point}\}$ be, respectively, the minimum and maximum power increments over all nodes which are not articulation points. The restricted candidate list RCL' contains all nodes $u \in V$ which are not articulation points and such that $g'(u) \leq \underline{g}' + \alpha(\overline{g}' - \underline{g}')$, with $0 \leq \alpha \leq 1$. A node u is randomly selected from RCL', with $g'(u) = g(u, v)$ for some node v which is not an articulation point. The power assignments of nodes u and v are increased by $\max\{0, e(u, v) - p_u\}$ and $\max\{0, e(v, u) - p_v\}$, respectively, the bidirectional edge $[u, v]$ is inserted into B, and a new iteration resumes. Since linking two biconnected components by an edge reduces their number at least by one. the algorithm stops when a biconnected graph is built.

5.2. Local Search Phase

$P_i = [p_i^1, \ldots, p_i^{\phi(i)}]$ was defined as a list of increasing power levels that can be assigned to node $i \in V$ in Section 4. For a given power assignment p_i to each node $i \in V$, let $(s_i^1, \ldots, s_i^{\phi(i)})$ be a vector with components $s_i^\ell \in \{0, 1, 2\}$, for $\ell = 1, \ldots, \phi(i)$:

- $s_i^\ell = 0$ if $p_i^\ell > p_i$ (node i operates with a power assignment smaller than p_i^ℓ);
- $s_i^\ell = 2$ if $p_i^\ell \leq p_i$ and there exist a node $j \in T_i^\ell$ and a level $k = 1, \ldots, \phi(j)$ such that $p_j \geq p_j^k$ and $i \in T_j^k$ (power level p_i^ℓ supports a bidirectional edge with node j); and
- $s_i^\ell = 1$ otherwise (power level p_i^ℓ is used, but only a unidirectional arc from i to j is established).

Local search and the definition of the neighborhoods make use of two basic operations for decreasing and increasing the power assignments. Applied to a node $i \in V$, the first operation decreases its current power assignment $p_i = p_i^\ell$ (with $\ell \geq 2$) to $p_i = p_i^{\ell'}$, where

Table 1: Exact optimal solutions.

Instance		Asymmetric						Symmetric					
		unidirectional			bidirectional			unidirectional			bidirectional		
	$\|V\|$	solved	time (s)	gap (%)	solved	time (s)	gap (%)	solved	time (s)	gap (%)	solved	time (s)	gap (%)
Euclidean	10	15	0.89	11.06	15	0.47	7.51	15	0.78	10.90	15	0.48	7.25
	15	15	16.20	13.75	15	7.55	10.34	15	16.03	14.23	15	7.24	10.14
	20	15	177.59	13.40	15	66.61	8.10	15	179.02	12.80	15	47.26	8.27
	25	15	1563.94	11.96	15	298.53	7.71	15	1600.28	12.15	15	509.83	7.70
	30	5	2837.09	7.47	12	1351.98	4.56	6	4875.97	11.51	12	1373.72	4.20
	50	–	–	–	–	–	–	–	–	–	–	–	–
Random	10	15	0.07	1.19	15	0.48	5.98	15	0.11	1.56	15	0.15	0.82
	15	15	0.16	0.00	15	6.99	10.83	15	0.74	0.40	15	0.23	0.22
	20	15	0.87	0.01	15	117.36	10.87	15	6.78	0.29	15	2.69	0.28
	25	15	2.36	0.01	15	872.44	13.48	15	20.43	0.32	15	10.95	0.12
	30	15	5.69	0.02	1	5559.86	13.55	15	102.12	0.22	15	73.71	0.24
	50	15	126.89	0.02	0	–	–	12	2827.35	0.07	11	562.42	0.06

ℓ' is the highest level which supports a bidirectional edge: $1 \le \ell' < \ell$, $s_i^{\ell'} = 2$, and $s_i^{\ell''} = 1$ for all $\ell'' = \ell'+1, \ldots, \ell-1$. It removes the links between nodes i and j for all $j \in T_i^{\ell'+1} \cup \cdots \cup T_i^{\ell-1} \cup T_i^{\ell}$ and the total power assignment is decreased by $p_i^{\ell} - p_i^{\ell'}$. Applied to a node $i \in V$, the second operation increases its current power $p_i = p_i^{\ell}$ (with $\ell \le \phi(i) - 1$) to $p_i = p_i^{\ell+1}$. If there exist a node $j \in T_i^{\ell+1}$ and a power level $k = 1, \ldots, \phi(j)$ such that $p_j \ge p_j^k$ and $i \in T_j^k$, then the objective function is increased by $p_i^{\ell+1} - p_i^{\ell}$. Otherwise, let $j \in T_i^{\ell+1}$ such that $p_j - p_j^{k(j)} = \min_{v \in T_i^{\ell+1}} \{p_v - p_v^{k(v)} : i \in T_v^{k(v)}$, for some $k(v) = 1, \ldots, \phi(v)\}$ and increase its current power p_j to $p_j^{k(j)}$. In this case, the objective function is increased by $(p_i^{\ell+1} - p_i^{\ell}) + (p_j^{k(j)} - p_j)$. In both cases, the bidirectional edge $[i,j]$ is inserted into solution.

The local search phase explores the neighborhood of the current solution, attempting to reduce the total power consumption. A move starts by decreasing the power assignment of one node, followed by as many power increases as needed to restablish biconnectivity. The first improving move is accepted and the search moves to the new neighbor. The procedure continues until no further improving moves exist.

The number of increase operations investigated may be reduced to speedup the local search. Whenever biconnectivity is destroyed by a power decrease, the biconnected components are computed and two acceleration schemes are implemented: (1) the *reduced scheme* restricts the increases to pair of nodes belonging to the same biconnected components of the pair of nodes affected by the previous decrease; and (2) the *extended scheme* considers increases involving any pair of nodes from different biconnected components. The local search procedure first makes use of the reduced scheme until no further improving moves can be found, followed by the extended scheme.

6. Computational Results

Computational experiments have been carried out on two classes of randomly generated asymmetric test problems with 10 to 800 nodes. For each problem size and type, 15 test instances have been generated:

- Euclidean instances: the nodes are uniformly distributed in the unit square grid. The weight of the arc between nodes $u, v \in V$ is $e(u,v) = F \cdot d_{u,v}^{\varepsilon}$, where $d_{u,v}$ is the Euclidean distance between nodes u and v, the loss exponent ε is set at 2, and $F \in [0.8, 1.2]$ is a random perturbation generated from a uniform distribution.

- Random instances: the weight $e(u,v)$ of the arc between nodes $u, v \in V$ is randomly generated in $(0, 1]$.

An Intel Core 2 Quad machine with a 2.40 GHz clock and 8 Gbytes of RAM memory running under GNU/Linux 2.6.24 was used in all experiments. CPLEX 11.0 was used to solve the integer programming formulation.

For each problem type and each size $|V| = 10, 15, 20, 25, 30, 50$, Table 1 shows the number of instances solved to optimality by CPLEX in less than three hours, the average running time in seconds over the instances exactly solved, and the average relative duality gap in percent between the linear relaxation value and the optimal value. Since CPLEX did not solve all instances in three hours, the numbers in Table 1 are average results over all instances solved to optimality. Cells in blank correspond to experiments not performed, due do the hardness of exactly solving the corresponding problems.

These results show that the minimum power consumption problem is hard to solve. The duality gaps are not small for the random instances of the asymmetric input with bidirectional topology variant and for the Euclidean instances, which makes it very difficult to the solver to find exact optimal solutions within the imposed time limits. The other variants of the random instances are easier to solve, because the optimal solutions of their linear relaxations in the root of the search tree are very close to their optimal integer solutions.

Since the computation times increase very fast with $|V|$, CPLEX could not solve to optimality in three hours of computations even moderately-sized networks with 30 nodes. The difficulty faced by a commercial solver to handle large instances supports the need for efficient heuristics, capable of finding good approximate solutions in reasonable computation times.

In the next, we focus our analysis into the asymmetric input with bidirectional variant, since it is the more interesting case in practice as discussed is Section 3. Parameter α was set by using the reactive strategy described in [25], with the probability distribution being updated after every 100 iterations. We limited the size of the candidate lists at $|V|^{\frac{1}{2}}$.

We first notice that the GRASP heuristic found the optimal solutions for all problems with up to 25 nodes. It obtained the optimal solutions for all Euclidean instances in less than one second, but the random instances were harder and took approximately 20 seconds on average.

For the instances with 200 and 400 nodes, Table 2 displays the average objective values over five runs for one instance of each type as the running time limit increases from five to 3125 seconds. The GRASP heuristic continues to improve their solutions as the time limit increases, showing that it may be benefited if longer running times are allowed. These results are summarized in Figure 2, which further illustrates the continuous improvement in solution quality along the total computation time.

In the final and more conclusive experiment, we compare the GRASP heuristic described in this work using a fixed amount of computation time (one hour) with the MST-aug heuristic of Calinescu [5]. Table 3 summarizes the average solution values over 15 instances of each size. For both algorithms, we give the number of arcs, the number of edges, the average node degree, the maximum node power assignment, and the total power consumption. The last column shows the reduction (i.e., the gain) observed in the total power consumption obtained by GRASP with respect to algorithm MST-aug.

The existing heuristic MST-aug does not take into account the structure of biconnected components and the gains offered by the wireless multicast advantage property. The proposed GRASP heuristic systematically finds better solutions in all aspects. In particular, GRASP outperformed MST-aug for the Euclidean instances with reductions in power consumption ranging from 37.59% to 40.87%. These improvements are even larger for random instances, ranging from 57.45% to 87.32%. In addition, the solutions produced by the GRASP heuristic are characterized by fewer unidirectional arcs and smaller power assignments, which are very useful to mitigate interference. The average node degree in the solutions produced by GRASP ranges from 2.45 to 2.69, being much smaller than those obtained with MST-aug. Since the degree of any node should be greater than or equal to two in a biconnected graph, we may conclude that these results are very close to the theoretical lower bounds.

7. Concluding Remarks

Ad hoc networks have become an increasingly important object of study, due to their many applications in situations where wired backbones are infeasible or economically inefficient. Nodes of ad hoc networks are typically equipped with limited power supply. Therefore, one of the primary goals of their design and operation consists of optimizing power consumption. Energy conservation often reduces the number of links, resulting in networks with smaller connectivity and more susceptible to system faults such as node failures and departures. In this paper, we considered the problem of power control and optimization in ad hoc networks to extend the functional lifetime of both individual units and the network, imposing fault-tolerant biconnectivity requirements.

We presented an integer programming formulation for the bidirectional topology version of the biconnected minimum power consumption problem. This formulation can be easily extended to account for problems with other connectivity requirements.

The formulation was applied to four variants of the problem, regarding the topologies of the input graph (symmetric or asymmetric) and of the solution (unidirectional or bidirectional). A commercial integer-programming solver was applied to the proposed formulation. Computational results were given for small- and medium-sized networks. We also showed that large instances cannot be solved to optimality by a state-of-the-art solver.

A GRASP heuristic was proposed to find good approximate solutions for real-size problems for the asymmetric input with bidirectional topology variant. Comparative experimental results for large networks with up to 800 nodes showed that GRASP is fast and finds effective solutions which significantly improve those obtained by a literature heuristic. Furthermore, we showed that the solutions obtained by the GRASP heuristic are very close to the optimal solutions, illustrating the effectiveness of the heuristic.

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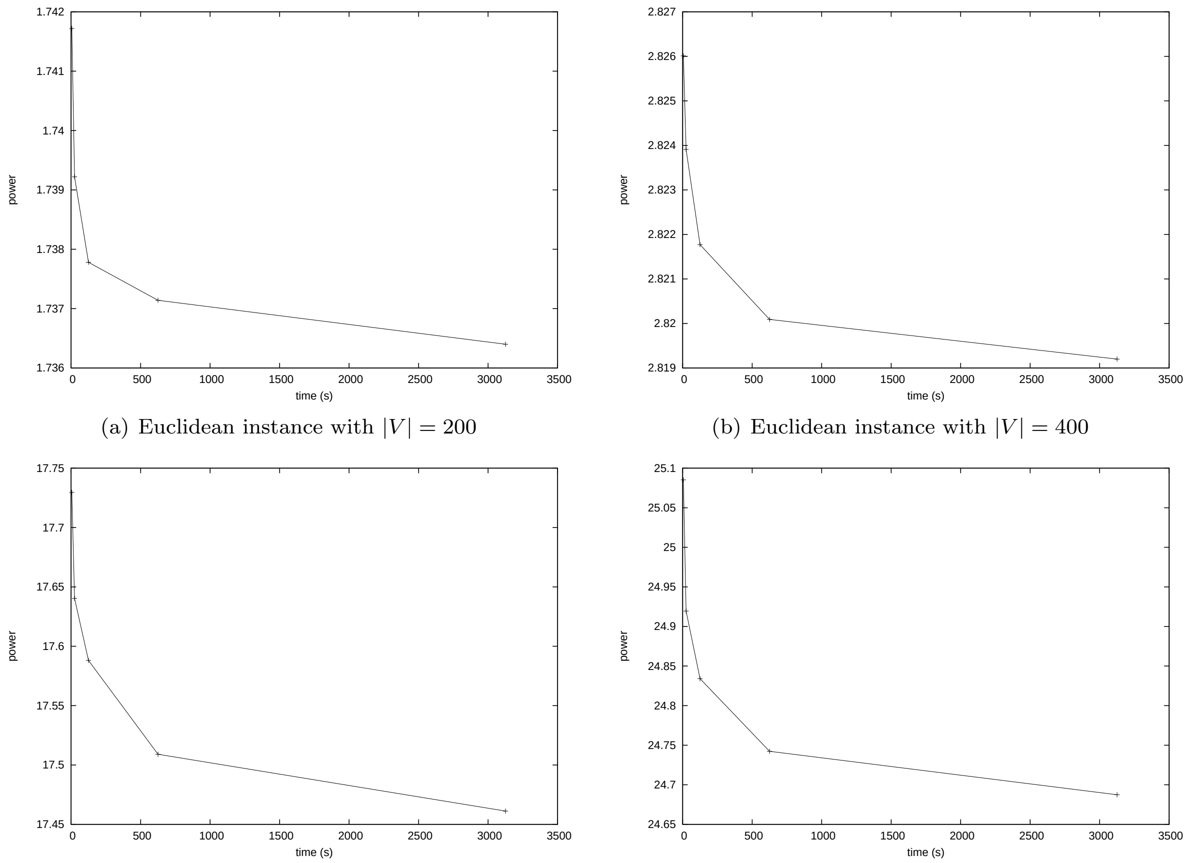
Table 2: Average total power consumption for instances with 200 and 400 nodes.

	$\lvert V\rvert = 200$					$\lvert V\rvert = 400$				
Instances	5 s	25 s	125 s	625 s	3125 s	5 s	25 s	125 s	625 s	3125 s
Euclidean	1.74172	1.73922	1.73778	1.73714	1.73640	2.82601	2.82391	2.82177	2.82009	2.81920
Random	17.72963	17.64030	17.58805	17.50900	17.46120	25.08528	24.91952	24.83406	24.74227	24.68739



(a) Euclidean instance with $|V| = 200$

(b) Euclidean instance with $|V| = 400$

(c) Random instance with $|V| = 200$

(d) Random instance with $|V| = 400$

Fig. 2: Improvement in the average total power consumption for instances with 200 and 400 nodes as a function of the running time.

Table 3: Comparative average results for MST-aug and GRASP on large problems.

		MST-aug					GRASP					
Instances	$\lvert V\rvert$	arcs (avg.)	edges (avg.)	degree (avg.)	power (max.)	total power	arcs (avg.)	edges (avg.)	degree (avg.)	power (max.)	total power	gain (%)
Euclidean	25	130.00	54.73	4.37	0.18784	2.18646	81.60	32.80	2.62	0.15052	1.35089	38.22
	50	275.73	111.60	4.46	0.11935	2.04944	168.33	65.66	2.62	0.07861	1.27900	37.59
	100	608.73	239.20	4.78	0.06809	2.15343	327.66	130.13	2.60	0.04066	1.32409	38.51
	200	1481.66	556.53	5.56	0.03857	2.84844	681.00	254.73	2.54	0.02249	1.76355	38.09
	400	4502.13	1524.40	7.62	0.03199	4.72278	1538.20	507.93	2.53	0.01394	2.82529	40.18
	800	14707.06	4530.40	11.32	0.02999	8.44331	3705.60	1033.06	2.58	0.01057	4.99252	40.87
Random	25	329.33	100.60	8.04	0.92757	12.84646	149.93	31.26	2.50	0.55168	5.46654	57.45
	50	1224.93	332.13	13.28	0.95827	24.20324	432.80	61.40	2.45	0.48981	8.35444	65.48
	100	4509.73	1083.60	21.67	0.97475	44.97218	1197.80	131.20	2.62	0.41067	11.86145	73.62
	200	16972.53	3748.93	37.48	0.99056	85.08285	3385.40	264.53	2.64	0.31773	17.14185	79.85
	400	63143.00	12738.66	63.69	0.99039	158.26410	9583.46	532.73	2.66	0.23502	25.00776	84.20
	800	233383.66	43080.53	107.70	0.99648	293.63736	27377.60	1077.20	2.69	0.18951	37.24512	87.32

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