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Invited Review

Scheduling in sports: An annotated bibliography

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ABSTRACT

Sports have worldwide appeal. Professional sport leagues involve significant investments in players. Events such as the Olympics Games, the Football World Cup and the major golf and tennis tournaments generate huge worldwide television audiences and many sports are multi-million dollar industries. A key aspect of sporting events is the ability to generate schedules that optimize logistic issues and that are seen as fair to all those who have an interest. This is not just restricted to generating the fixtures, but also to other areas such as assigning officials to the games in the competitions. This paper provides an annotated bibliography for *sports scheduling* articles. This area can be traced back over 40 years. It is noticeable that the number of papers has risen in recent years, demonstrating that scientific interest is increasing in this area.
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Contents

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1. Introduction

Sports have become a big business. In a globalized economy, many countries and cities battle for the rights to organize major events such as the Olympic Games and the Football World Cup, which often bring thousands of jobs, urban regeneration and economic opportunities to their hosts. Tournaments are followed by millions of people across the world, eager for information on their teams progress in each competition. Fans check newspapers, radio, television and, more recently, the Internet in their quest for information.

Professional sport leagues involve millions of fans and significant investments in players, broadcast rights, merchandizing, and advertising. Multiple agents, such as the organizers, media, players, fans, security forces, and airlines, play an important role in the leagues and tournaments. Professional sports leagues are therefore part of a major economic activity and face challenging optimization problems, such as revenue maximization and logistic optimization. On the other side, amateur leagues usually do not have access to the same amounts of money, but the number of tournaments and competitors can be very large, also requiring coordination and logistical efforts.

Sports scheduling and management has been attracting the attention of an increasing number of researchers in multidisciplinary areas such as operations research, scheduling theory, constraint programming (CP), graph theory, combinatorial optimization, and applied mathematics. Different optimization techniques have been applied to solve problems arising from sports scheduling and management. The difficulty of the problems in the field lead to the use of a number of exact and approximate approaches, including integer programming (IP), constraint programming, metaheuristics, and hybrid methods.

Teams and leagues want to optimize their investments by playing a good schedule which seeks to meet various criteria. Good fixtures are important in order to maximize revenues, ensure the attractiveness of the games, and to keep the interest of both the media and the fans. Good schedules can have significant financial implications and interfere (for the bad and the good) in the performance of every team participating in a tournament. Finding the best schedule of games is a difficult task with multiple decision makers, constraints, and objectives (involving logistics, organizational, economical, and fairness issues).

The general problem of scheduling the games of a tournament is certainly the most studied area of sports scheduling. It consists in determining the date and the venue in which each game will be played. Applications to real-life problems in the scheduling of tournaments of sports such as football, baseball, basketball, cricket, hockey, and others are common in the literature. However, there are also other relevant scheduling problems in sports. One of them is that of assigning referees to games, which also involve multiple objectives.

Whilst the above motivations, by themselves, entirely justify the study of scheduling problems in sports, we also remark that they are very interesting and, arguably, more interesting and motivating than other scheduling problems. Most people can relate to sporting events. Even if they do not participate or watch sport, either live or on TV, everybody has some relationship with sport. It might be the fact that one can hardly avoid the coverage that is given to events such as the Olympic Games, the Football World Cup or the Super Bowl. Or it could be that parents need to organize their personal schedules around their children's sporting endeavors. Sporting events (and therefore their schedules) are part of everybody's life and one cannot avoid them. As a single illustrative example of the role played by amateur sport leagues, in the MOSA (Monmouth & Ocean Counties Soccer Association) league, New Jersey, children of ages 8–18 make up six divisions per age and gender group with six teams per division, making a total of 396 games every Sunday. These games are officiated by hundreds of certified referees and attended by thousands of parents, relatives, and friends, who travel to support the players and to enjoy the games.

Due to our daily and continuous involvement with sports, most sports scheduling problems are very easy to explain and understood by researchers, developers, and practitioners. Sports applications tend to motivate students and can be successfully used in Operations Research courses and classrooms (see [10]).

Sports scheduling covers an extremely wide range of problems and there are many possible ways to survey the area or to view the different classes of problems. For example, we could be interested in the underlying theoretical fundamentals (such as graph theory, factorizations, or latin squares) of the problems, or in the solution of real world applications in different sports and leagues. Or we could look at different types of problems that are faced by different competitions (such as scheduling fixtures and officials, or maximizing gate receipts). In fact, the way we view sports scheduling depends on the motivation for the individual researchers and practitioners involved.

This annotated bibliography has two main goals. Firstly, to present a comprehensive list of sports scheduling papers. Although we understand that it is almost impossible to provide a complete coverage of all the work in the area, we hope this bibliography will be as broad as possible and will serve as a starting point for those requiring information about this area, or for those simply wishing to bring their knowledge up to date. This would be beneficial, not only for researchers, but also for potential users responsible for real-life applications. We would be glad to hear from the readers with suggestions and additional references to those presented here, with a view that we can include any omissions in any subsequent revision. The second aim is to present the methodologies that have been utilized in solving these types of problems, so that other researchers and practitioners are able to make an informed choice in case of need.

The rest of this paper is organized as follows. Section 2 presents the main definitions, principles, and fundamentals. Section 3 gives an account of the use of different approaches in the solution of scheduling problems in sports: decomposition strategies; integer programming; constraint programming; heuristic search, metaheuristics and their hybrids. Finally, Section 4 surveys applications in different sports disciplines like football, baseball, basketball, cricket, and hockey. This section closes with references on further problems such as referee assignment. Finally, the last section summarizes some conclusions.

Some remarks on the style and organization of this annotated bibliography should be made:

- We refer to *football* as the sport regulated by FIFA (Fédération Internationale de Football Association), at which countries such as Argentina, Brazil, England, and Germany excel and have won 11 out of the 18 preceding world cups (it is not by chance that these are the countries of the coauthors of this bibliography). We also use *soccer* as a synonym, to maintain consistency with the citations. We use *American football* to refer to the sport played in the United States.
- Some papers are relevant for different sections of this annotated bibliography. Papers in this situation are usually discussed in one of the relevant sections and merely cited elsewhere. A few papers are discussed twice under different points of views.
- Due to space limitations, we mainly limit this annotated bibliography to final versions of journal papers or full papers that appeared in conference proceedings. Except for some relevant exceptions, we do not cite unrefereed papers, abstracts, and theses.
- We have limited ourselves to scheduling problems in sports. There are other optimization problems in sports that do not fall in this

category and, consequently, are not reviewed here. Some of them are briefly cited in Section 4.8.

We conclude this section by presenting a short list of some seminal, introductory, relevant, or survey papers in the area. In particular, readers new to this area might find these useful starting points.

[1] Anderson I. Combinatorial designs and tournaments. In: Oxford lecture series in mathematics and its applications, 1997.

This book covers many of the basics that are of interest to sports scheduling researchers. This includes latin squares, room squares, scheduling leagues, balanced tournament designs, and whist tournaments.

[2] Dinitz JH, Fronček, D, Lamken ER, Wallis WD. Scheduling a tournament. In: Colbourn CJ, Dinitz JH, editors. Handbook of combinatorial designs, 2nd ed. Boca Raton: CRC Press; 2007. p. 591–606.

Provides an excellent introduction to some of the fundamental issues in tournament scheduling, including round-robin tournaments, 1-factorizations and balanced tournament designs. The chapter also considers specific sports such as softball, golf, whist and bridge, as well as other issues such as tournaments for trios, spouse avoiding doubles tournaments, and elimination tournaments.

[3] Drexl A, Knust S. Sports league scheduling: graph- and resource-based models. Omega 2007;35:465–71.

Surveys graph models for sports league scheduling and introduces a model based on resource-constrained project scheduling.

[4] Easton K, Nemhauser GL, Trick MA. The travelling tournament problem: description and benchmarks. In: Walsh T, editor. Principles and practice of constraint programming. Lecture notes in computer science, vol. 2239. Berlin: Springer; 2001. p. 580–5.

This is the seminal paper introducing the traveling tournament problem (TTP) (see also Section 2.8), which was motivated by the problem of scheduling the Major League Baseball (MLB). Its formulation captures the fundamental difficulties of minimizing the travel distance for a sports league.

[5] Easton K, Nemhauser GL, Trick MA. Sports scheduling. In: Leung JT, editor. Handbook of scheduling. Boca Raton: CRC Press; 2004. p. 52.1–19.

Provides a survey of sports scheduling literature from the 1970s to 2003. During this period much of the literature was concerned with single and double round-robin tournaments, but other aspects (such as balanced tournament designs and the bipartite tournament problem) are also featured.

[6] Knust S. Classification of literature on sports scheduling, 2005. Available online at: ⟨http://www.inf.uos.de/knust/sportssched/sportlit_class/⟩, last visited on September 29, 2008.

This website provides an up to date account of sports scheduling literature.

[7] Machol RE, Ladany SP, Morrison DG. Management science in sports, studies in the management sciences, vol. 4. Amsterdam: North-Holland; 1976.

An early account of applications of quantitative methods to problems in sports.

[8] Rasmussen RV, Trick MA. Round robin scheduling—a survey. European Journal of Operational Research 2008;188:617–36.

Presents a comprehensive survey of the literature on round-robin tournaments and helps to unify the terminology. Contributions presented during the last 30 years are outlined, with the papers being divided into two categories according with their focus: break minimization and distance minimization. Directions for future research in the area are discussed.

[9] Rokosz FM. Procedures for structuring and scheduling sports tournaments: elimination, consolation, placement, and round-robin design, 3rd ed. Charles C Thomas; 2000.

This book, whilst not referencing other work, provides a basic introduction to many types of tournaments. The types of tournaments discussed are single elimination, double elimination, round robin, ladder, novelty elimination, and novelty placement. The book provides a set of homework questions/answers which might make it suitable for an introductory course on sports scheduling.

[10] Trick MA. Using sports scheduling to teach integer programming. INFORMS Transactions on Education 2004;5:10–7. Available online at: ⟨http://archive.ite.journal.informs.org/Vol5No1/Trick/⟩.

Examples of sports scheduling models are given, showing how they can be used to illustrate key concepts in an MBA level integer programming course. The models have been selected so that they can be solved within the standard version of Excel's solver (from Frontline Systems, included in the standard Excel distribution). The models are accessible to every student without the need for additional software.

2. Fundamentals, problems and definitions

Scheduling problems in sports leagues may be divided into two main classes: temporally constrained and temporally relaxed problems. In the time-constrained case, the planning horizon consists of the minimum number of periods (so-called rounds) required to schedule all the games and, hence, each team has to play exactly one game in each round. Tournaments following this pattern are said to be compact. On the other hand, in the time-relaxed case the number of periods is generally larger than the minimum number of rounds needed for scheduling all games. In this situation not every team necessarily plays in each round and thus teams may have some periods without a game.

The basic temporally constrained problem for scheduling a sports league may be formulated as follows. The league consists of an even number n of different teams indexed by $i \in \{1,\dots,n\}$, where each team has to play against each other team exactly $\ell \geq 1$ times. The number of rounds available to schedule these $\binom{n}{2}\ell = n(n-1)\ell/2$ games is equal to $(n-1)\ell$, where each team has to play exactly one game in each round. Thus, one has to determine which teams $i,j \in \{1,\dots,n\}$ play against each other in each round $t=1,\dots,(n-1)\ell$ and, for each of these pairings, whether it is played in the home stadium of team i (home game for team i) or in the home stadium of team j (away game for team i).

In most cases we have $\ell = 1$ (single round-robin tournament, SRRT) or $\ell = 2$ (double round-robin tournament, DRRT). For double round-robin tournaments the season is often partitioned into two half series, where each pairing has to occur exactly once in each half (with different home rights). The second half series is usually not scheduled independently from the first. In a so-called mirrored schedule, the second series is planned complementarily to the first, i.e. the pairings of round $t=1,\dots,n-1$ in the second half are exactly the same as in round t of the first half, but with exchanged home rights. Another possibility is the so-called "English" system, where the pairings in the first round of the second half are the same as in the last round of the first half and rounds $2,\dots,n-1$ in the second half equal rounds $1,\dots,n-2$ of the first half.

If the league consists of an odd number of teams, in each round one team has a bye, i.e. does not play. This situation may be reduced to the previous case with an even number of teams by adding a dummy team $n+1$. Then, in each round the team playing against $n+1$ has a bye.

Usually, a schedule for a sports league is described by a so-called opponent schedule and a so-called home–away pattern (HAP). An opponent schedule may be represented by an $n\times(n-1)$-matrix where the entry $opp_{it} \in \{1,\dots,n\}\backslash\{i\}$ specifies the opponent of team i in round t. If this matrix is enlarged by an additional column containing the teams $1,\dots,n$, its structure is a latin square (each column and each row is a permutation of $1,\dots,n$ such that in any column or row no number occurs twice). Additionally, the latin squares fulfill the symmetry condition that $opp_{it} = j$ if and only if $opp_{jt} = i$ (i.e. if team i plays against team j in round t, also team j plays against team i in that round).

A home–away pattern is defined as $n \times (n-1)$-matrix $H = (h_{it})$, where h_{it} equals “H” (resp. “A”) when team i has a home (resp. away) game in round t. If two consecutive entries $h_{i,t-1}$ and h_{it} in a row i are equal for some $t = 2, \ldots, n-1$, then team i has a so-called break in round t (i.e. the alternating sequence of home and away games is broken).

2.1. Graph coloring and factorizations

Traditional models for constructing sports league schedules are based on graphs. The complete graph K_n on n nodes is used to represent single round-robin tournaments. Its nodes $i = 1, \ldots, n$ represent the teams, while the edge $i-j$ represents the game between teams $i, j \in \{1, \ldots, n\}$. An edge coloring with $n-1$ colors, i.e. a partitioning of the edge set into 1-factors $F_1, \ldots, F_{n-1}$ (each consisting of $n/2$ non-adjacent edges), corresponds to the games scheduled in the rounds $t = 1, \ldots, n-1$. If additionally home and away games have to be distinguished, the edges are directed into arcs (i, j) for a game in the home stadium of team j or (j, i) for a game in the stadium of team i.

[11] de Werra D. Geography, games, and graphs. Discrete Applied Mathematics 1980;2:327–37.

Graph theoretical models are used to deal with geographical constraints for teams located in different cities. For the situation that the number of teams is a multiple of four and the teams are partitioned into two subsets of equal, size schedules with $n-2$ breaks are constructed where each round is either “internal” (i.e. each team plays against a team belonging to the same subset) or “external” (i.e. each team plays against a team belonging to the other subset).

[12] de Werra D. Scheduling in sports. In: Hansen P, editor. Studies on graphs and discrete programming. Amsterdam: North-Holland; 1981. p. 381–95.

Based on the complete graph model, it is shown that schedules for n teams with even n have at least $n-2$ breaks. Corresponding schedules which achieve this lower bound value are constructed by the so-called canonical 1-factorization. Furthermore, for odd n schedules without any breaks are constructed.

[13] de Werra D. Minimizing irregularities in sports schedules using graph theory. Discrete Applied Mathematics 1982;4:217–26.

Graph theoretical properties are stated for schedules with $n-2$ breaks. Furthermore, schedules are constructed in which the number of rounds with breaks is minimal.

[14] de Werra D. On the multiplication of divisions: the use of graphs for sports scheduling. Networks 1985;15:125–36.

A generalization of the situation in de Werra [11] with more than two subsets (the so-called divisions) is studied.

[15] de Werra D. Some models of graphs for scheduling sports competitions. Discrete Applied Mathematics 1988;21:47–65.

Graph theoretical models are introduced for scheduling sports leagues with a minimum number of breaks.

[16] de Werra D, Descombes J, Masson P. A constrained sports scheduling problem. Discrete Applied Mathematics 1990;26:41–9.

Schedules are constructed by graph theoretical methods for two Australian basketball leagues in which constraints on breaks and teams sharing the same stadium are regarded.

[17] Frončeк D, Meszka M. Round robin tournaments with one bye and no breaks in home–away patterns are unique. In: Kendall G, Burke E, Petrovic S, Gendreau M, editors. Multidisciplinary scheduling: theory and applications. Berlin: Springer; 2003. p. 331–40.

It is shown that for an odd number of teams a schedule with one bye per team and no breaks is unique (up to a permutation of teams). Furthermore, the same results holds if for an even number of teams one more round is used (i.e. there are n rounds for a tournament with n teams).

[18] Griggs T, Rosa A. A tour of European soccer schedules, or testing the popularity of GK_{2n}. Bulletin of the Institute of Combinatorics and its Applications 1996;18:65–8.

It is investigated according to which 1-factorizations 25 European soccer leagues are scheduled.

[19] Knust S, von Thaden M. Balanced home–away assignments. Discrete Optimization 2006;3:354–65.

A balanced home–away assignment is a collection of modes determining the home and away team for each game such that the difference between the number of home games and the number of away games is at most one for each team. By graph theoretical considerations it is shown how balanced home–away assignments can be constructed, how they can be changed according to a connected neighborhood structure, and how mode preassignments can be handled with network flow techniques.

[20] Mendelsohn E, Rosa A. One-factorizations of the complete graph—a survey. Journal of Graph Theory 1985;9:43–65.

Results on the existence and enumeration of different 1-factorizations of the complete graph are surveyed. Especially, applications to round-robin tournaments are discussed.

[21] Rosa A, Wallis W. Premature sets of 1-factors or how not to schedule round robin tournaments. Discrete Applied Mathematics 1982;4:217–26.

Greedy approaches for scheduling a round-robin tournament round by round are studied. It is shown that such a procedure does not always lead to feasible complete schedules.

[22] Schreuder J. Constructing timetables for sport competitions. Mathematical Programming Study 1980;13:58–67.

Based on graph theoretical results and the canonical 1-factorization, an algorithm is presented which constructs schedules with $n-2$ breaks.

[23] Schreuder J. Combinatorial aspects of construction of competition Dutch professional football leagues. Discrete Applied Mathematics 1992;35:301–12.

Schedules are determined for the Dutch soccer league by first constructing a schedule with $n-2$ breaks and assigning the teams afterwards.

[24] van Weert A, Schreuder J. Construction of basic match schedules for sports competitions by using graph theory. In: Burke E, Carter M, editors. The 2nd international conference on the practice and theory of automated timetabling. Lecture notes in computer science, vol. 1408. Berlin: Springer; 1998. p. 201–10.

For a Dutch volleyball competition where each team has to play two games in each round, schedules are constructed using 2-factors in the complete graph.

2.2. Model development

Most of the sports scheduling problems discussed in this paper present some form of objective function, together with a set of constraints. Some of these models are presented formally (e.g. as an integer program formulation) and some are simply descriptive.

It is not possible to provide standard objective functions and sets of constraints for what may first appear to be very similar problems; at least for real world problems. A good example is provided by the referee assignment. Yavuz et al. [156] (see Section 4.7) assigns referees in the context of professional football leagues. The objective function is a summation of five soft constraint violations which capture the following aspects: (1) a fair dispersion of referees to games throughout the season, (2) the number of times a referee is assigned to a fixture throughout the season, (3) the number of times a referee is assigned a team when it is playing at home, (4) the higher skilled referees should be assigned more games than lesser qualified referees, and (5) the same referee does not officiate both games between

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the same pair of teams. In Duarte and Ribeiro [145] (see Section 4.7) the problem is essentially the same, however, the objective functions (and constraints) are quite different. The objective is to allocate each referee to an agreed number of games (which might be different for each referee), penalizing both over and under assignments, and to provide the referees with a good spread of games throughout each round (actually defined as minimizing the sum of the idle times between consecutive games assigned to the same referee). It is apparent that for seemingly similar problems the objective functions and constraints can have similarities (e.g. give a fair spread of games), but they are essentially tailored for the problem at hand and their formulations are presented in different ways.

Some problems do not tackle a real world application, but have been introduced for the benefit of researchers to enable them to use a common model. Perhaps the most useful of these is the traveling tournament problem, which aims to minimize the total distance traveled (see Section 2.8). It provides a simplified objective of a real world problem, together with a set of benchmark instances. The TTP also provides researchers with the ability to compare their methodologies with those that have been previously reported. Kendall [120] (see Section 4.1) handles the same objective, but the problems are distinctly different. While the TTP assumes that teams go on road trips and the distance traveled is minimized across the entire season, Kendall [120] aims to minimize travel distances for English football teams for just two days of the season. As such, the objective is slightly different and the constraints which must be respected are different for each problem. For example, the constraints in the TTP are limited to playing three consecutive home or away matches and not playing the same team in consecutive rounds, while many more constraints have to be respected in a real world problem.

It is apparent that sports scheduling is as much about developing an appropriate model of the problem, as it is about the solution methodology that is employed. Researchers are able to draw upon previous work to provide inspiration, and may be able to use existing work for some parts of the problem but, to solve a real world problem, model development is as much of an issue as the choice of solution methodology.

2.3. Breaks optimization

The first objective in sports league scheduling (and perhaps the most studied subject in this area) is the minimization of breaks. A lot of league organizers restrict themselves to schedules having a minimum number of breaks (i.e. $n-2$ for a league with an even number n of teams, and 0 for an odd number n). De Werra was the first who did a lot of pioneering work concerning breaks, cf. de Werra [11] in Section 2.1, de Werra [12] in Section 2.1, de Werra [13] in Section 2.1, and de Werra [15] in Section 2.1.

In connection with “first-schedule, then break”-approaches (cf. Section 3.1) especially the problem of finding a good home–away pattern for a given opponent schedule was studied (see Brouwer et al. [76] in Section 3.1, Elf et al. [77] in Section 3.1, Miyashiro and Matsui [78] in Section 3.1, Miyashiro and Matsui [79] in Section 3.1, and Post and Woeginger [80] in Section 3.1).

Other papers dealing with breaks are Rasmussen and Trick [8] in Section 1, Regin [94] in Section 3.3, and Suzuka et al. [87] in Section 3.2.

Finally, Urrutia and Ribeiro [74] in Section 2.8, showed that sometimes also a large number of breaks is advantageous since then travel distances are minimized in the traveling tournament problem.

2.4. Carry-over effects

For a given single round-robin schedule with n teams, it is said that team i gives a carry-over effect to team j if some other team plays consecutively against teams i and j, i.e. some team plays against i in round t and against j in round $t+1$ for some $t \in \{1,\ldots,n-1\}$, where the rounds are considered cyclically (i.e. round n corresponds to round 1). If team i is a very strong or very weak team and several teams play consecutively against teams i and j, team j may be, respectively, handicapped or favored compared with other teams. In an “ideal” schedule with respect to carry-over effects, no teams i,j,x,y should exist such that teams x and y both play against team j immediately after playing against team i.

The carry-over effects $n \times n$-matrix $C=(c_{ij})$ has been introduced in order to measure the balance with respect to this criterion. Each entry c_{ij} of this matrix counts the number of carry-over effects given by teams i to j. It can be seen that $c_{ii}=0$ and $\sum_{j=1}^{n} c_{ij} = \sum_{i=1}^{n} c_{ij} = n-1$ for all rows and columns $i,j=1,\ldots,n$. The quality of a schedule with respect to carry-over effects is measured by the carry-over effects value $coe = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^2$. An ideal or balanced schedule is one in which the lower bound value $n(n-1)$ is achieved, i.e. all non-diagonal elements of the corresponding matrix C are equal to 1.

More generally, it is said that team i gives a carry-over effect to team j at level $\ell \geq 1$ if some other team plays against team i in round t and against team j in round $t+\ell$ (the carry-over effect introduced above corresponds to $\ell=1$). A schedule is called completely balanced if it is balanced for all levels $\ell = 1,\ldots,n-2$.

[25] Anderson I. Balancing carry-over effects in tournaments. In: Holroyd F, Quinn K, Rowley C, Webb B, editors. Combinatorial designs and their applications. Chapman & Hall/CRC research notes in mathematics; 1999. p. 1–16.

A construction scheme for schedules with small carry-over effects values is proposed. The construction is based on the algebraic concept of starters in the group $\mathbb{Z}_{n-1}$. For $n \leq 24$ this construction produced schedules with the smallest carry-over effect values known at the time of writing. For $n=20$ and 22 even balanced schedules are found, which implies that Russell's conjecture in Russell [32] is false. Finally, bipartite tournaments are studied, where two sets with n teams each are given and each team in one set has to play against each team of the other set. It is shown that for each even n schedules exist which are balanced for both sets.

[26] Anderson I, Bailey R. Completeness properties of conjugates of latin squares based on groups, and an application to bipartite tournaments. Bulletin of the Institute of Combinatorics and its Applications 1997;21:95–9.

A bipartite tournament with two groups of n teams (players) and n rounds is considered where every team has to play against each of the n teams in the other group once. Based on a construction of special latin squares, it is shown that for all even n a schedule can be constructed which is balanced with respect to carry-over effects for both groups.

[27] Anderson I, Ferguson C. Training schedules balanced for carryover effects. Bulletin of the Institute of Combinatorics and its Applications 2004;40:5–12.

Training schedules for $n(n-1)$ athletes which have to be balanced with respect to carry-over values in different ways are constructed. Each athlete has to perform n tasks twice in two consecutive permutations such that for each athlete no carry-over effect of the tasks occurs twice, in any two consecutive periods each of the $n(n-1)$ carry-over effects occurs exactly once, and each athlete performs different tasks in the two permutations at the same position. Corresponding schedules are constructed when n is an odd prime or $n \in \{9,15,25\}$.

[28] Beintema M, Bonn J, Fitzgerald R, Yucas J. Orderings of finite fields and balanced tournaments. Ars Combinatoria 1998;49:41–8.

It is shown that completely balanced schedules exist for all n which are a power of 2 by modifying Russell's construction method in Russell [32] based on Galois fields.

[29] Guedes A, Ribeiro CC. A hybrid heuristic for minimizing weighted carry-over effects in round robin tournaments. In: Pro-

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ceedings of the 4th multidisciplinary international conference on scheduling theory and applications, Dublin, 2009.

A new, weighted variant of the minimum carry-over effects value problem is introduced and discussed. The problem is formulated by integer programming and a heuristic based on the hybridization of the iterated local search (ILS) metaheuristic with a multistart strategy is proposed for its approximate solution. Numerical results are presented.

[30] Keedwell A. Construction, properties and application of finite neofields. Commentationes Mathematicae Universitatis Carolinae 2000;41:283–97.

Based on the algebraic structure of the so-called finite neofields, it is shown that a sufficient condition for the existence of a balanced schedule is that a cyclic neofield with additional properties exist. These neofields are known to exist for all n which are a power of 2 or $n=20$ and 22.

[31] Miyashiro R, Matsui T. Minimizing the carry-over effects value in a round robin tournament. In: Burke EK, Rudová H, editors. Proceedings of the 6th international conference on the practice and theory of automated timetabling, Brno, 2006. p. 460–3.

A simple heuristic is proposed to quickly generate schedules with small carry-over effect values. The heuristic makes use of the circle method for constructing canonical 1-factorizations associated with schedules with high carry-over effect values. Permutations of the rounds of the schedules generated by the circle method give better solutions with small carry-over effect values in less than 1 s.

[32] Russell K. Balancing carry-over effects in round robin tournaments. Biometrika 1980;67:127–31.

The concepts of carry-over effects and balanced schedules are introduced. It is shown that balanced schedules exist for all $n=2^m$ by giving a construction scheme based on the Galois fields $GF(2^m)$. It is conjectured that balanced schedules exist if and only if n is a power of 2. Later on in Anderson [25] it was proved that this conjecture is false by providing balanced schedules for $n=20$ and 22. A heuristic is proposed for the case $n=p^m+1$ where p is a prime, which constructs schedules with small (but not necessarily minimal) carry-over effects values.

2.5. Group-changing schedules

For sports leagues in which the teams are partitioned into $g\geq 2$ different strength groups, the objective is to avoid that a team plays against extremely weak or extremely strong teams in consecutive rounds. Two concepts have been introduced assuring a certain degree of fairness with respect to groups. A schedule for a SRRT is called group-changing if no team plays against teams of the same group in two consecutive games. It is called group-balanced if no team plays more than once against teams of the same group within g consecutive games.

The second property implies the first one, since each group-balanced schedule (where each team plays against each group exactly once within $g\geq 2$ consecutive matches) is also group-changing.

[33] Briskorn D. Combinatorial properties of strength groups in round robin tournaments. European Journal of Operational Research 2009;192:744–54.

The concepts of group-changing and group-balanced schedules are introduced. For several cases (even number of teams, $g=2$ or $g\geq 4$ groups) corresponding schedules are constructed or it is shown that they cannot exist.

[34] Briskorn D, Knust S. Constructing fair sports leagues schedules with regard to strength groups. Discrete Applied Mathematics 2008, in press.

Group-balanced schedules based on graph models are constructed for an odd number of teams. Group-changing schedules are constructed for an even number of teams and $g=3$ groups. Additionally, generalized problems with arbitrary group sizes are studied.

2.6. Minimizing costs

In a very general model, it is assumed that costs (or benefits) depending on the rounds are given for every game. For all teams $i\neq j$ and rounds t, a cost c_{ijt} is incurred for scheduling the game between teams i and j as a home game for team i in round t. The objective is to find a feasible single or double round-robin tournament schedule satisfying several constraints with minimum total costs (or maximum total benefits).

[35] Briskorn D, Drexl A. A branch-and-price algorithm for scheduling sport leagues. Journal of the Operational Research Society 2009;60:84–93.

A branch-and-price algorithm is presented in order to find a feasible schedule for a round-robin tournament with minimum number of breaks and minimum total costs. Computational results are presented for leagues with up to 12 teams.

[36] Briskorn D, Drexl A. A branching scheme for finding cost-minimal round robin tournaments. European Journal of Operational Research 2009;197:68–76.

For the same problem as in Briskorn and Drexl [35] a branch-and-bound algorithm is developed which uses an integer programming formulation and a branching scheme based on home–away pattern sets. Computational results are presented for leagues with up to 10 teams.

[37] Knust S, Lücking D. Minimizing costs in round robin tournaments with place constraints. Computers & Operations Research 2009;36:2937–43.

The objective is to find a feasible schedule for a round-robin tournament with minimum number of breaks and minimum total costs, where additionally place constraints are taken into account. A “first-break, then schedule”-approach is presented which uses an enumerative procedure to generate home–away patterns and integer programming for finding corresponding schedules. Computational results are presented for leagues with up to 14 teams.

2.7. Problems with venues and balanced tournament designs

Special requirements regarding the venues (courts, fields, stadiums, locations) have to be taken into account for some sports scheduling problems. In the so-called balanced tournament problem, $2n$ teams (or players) have to play a single round-robin tournament in $2n-1$ rounds using n different courts in each round. In order to balance the effects of the different courts, it is required that no team plays more than twice on any court (i.e. each team plays twice on $n-1$ courts and once on the remaining court). Such a schedule is also called a balanced tournament design BTD(n) of order n. It may be represented by a matrix where the columns correspond to the rounds and the rows to the courts. In some versions of the problem instead of courts the assigned time slots in the rounds (starting times for the games) have to be balanced. This problem is also known as prob026 of the CSPLib in the constraint programming community. In the BTD*(n) variant of the balanced tournament design problem, each of the $2n$ teams plays $2n$ games (implying that each team plays against one opponent twice) and each team plays exactly twice on any court.

A factored balanced tournament design FBTD(n) is a BTD(n) with the property that each team plays exactly once on each court in the first n rounds. A partitioned balanced tournament design PBTD(n) is a FBTD(n) where each team additionally plays exactly once on each court in the last n rounds.

Although it has been shown that a BTD(n) or even an FBTD(n) exists for every integer $n\neq 2$ (cf. Schellenberg et al. [60], Lamken and

Vanstone [54]), the proofs do not lead to a concrete constructive method for building such schedules. Therefore, besides some construction schemes for specific values of n, several procedures have been developed for the general case using tabu search, constraint programming, or integer programming.

[38] Blest D, Fitzgerald D. Scheduling sports competitions with a given distribution of times. Discrete Applied Mathematics 1988;22:9–19.

Balanced tournament designs are constructed for an odd number of teams and for an even number of teams which is not a multiple of four.

[39] Corriveau J. Enumeration of balanced tournament designs. Ars Combinatoria 1988;25:93–105.

It is shown that 47 non-isomorphic balanced tournament designs BTD(4) exist.

[40] de Werra D, Ekim T, Raess C. Construction of sports schedules with multiple venues. Discrete Applied Mathematics 2006;154;47–58.

The variant BTD* of balanced tournament designs is considered, where each of the $2n$ teams plays $2n$ games. An inductive construction based on a graph model is given for all $2n$ which are a power of two.

[41] Dinitz J, Dinitz M. Enumeration of balanced tournament designs on 10 points. Journal of Combinatorial Mathematics and Combinatorial Computing 2005;52:51–64.

It is shown that 30 220 557 non-isomorphic balanced tournament designs BTD(5) exist.

[42] Finizio N. Tournament designs balanced with respect to several bias categories. Bulletin of the Institute of Combinatorics and its Applications 1993;9:69–95.

A generalization of balanced tournaments is studied in which not only the courts are regarded, but several other bias categories should be balanced (e.g. day of the week, time of the day, number of games per week). Existence results and specific schedules are presented.

[43] Franek F, Frončck D, Rosa A. Imbalance in tournament designs. Australasian Journal of Combinatorics 2001;23:237–51.

Two measures of imbalance are introduced for schedules using a given number of courts: the team imbalance and the court imbalance, which are both defined by counting how often each team plays on any court. Bounds are given for the general case after determining all possible imbalances for up to eight teams. Tournaments with large imbalances are constructed.

[44] Geinoz A, Ekim T, de Werra, D. Construction of balanced sports schedules using partitions into subleagues. Operations Research Letters 2008;36:279–82.

Balanced tournaments BTD(n) are constructed for specific values of n using a partition of the league into subleagues.

[45] Gelling E, Odeh R. On 1-factorizations of the complete graph and the relationship to round robin schedules. Congressus Numerantium 1974;9:213–21.

Introduces the concept of balanced tournament designs. The non-isomorphic 1-factorizations of the complete graphs K_8 (six equivalence classes) and K_{10} (396 equivalence classes) are analyzed with respect to different properties. As an application, a BTD(4) for eight teams is constructed.

[46] Hamiez J-P, Hao J-K. Solving the sports league scheduling problem with tabu search. In: Nareyek A, editor. Local search for planning and scheduling. Lecture notes in artificial intelligence, vol. 2148. Berlin: Springer; 2001. p. 24–36.

The balanced tournament problem (prob026 of CSPLib) is solved with a tabu search algorithm based on a swap neighborhood. Computational results are presented for instances with up to 40 teams.

[47] Hamiez J-P, Hao J-K. A linear-time algorithm to solve the sports league scheduling problem (prob026 of csplib). Discrete Applied Mathematics 2004;143:252–65.

The balanced tournament problem (prob026 of CSPLib) for $2n$ teams is considered for the special situation where $(2n-1) \bmod 3 \neq 0$, which was previously studied in Haselgrove and Leech [49]. It is shown that this case can be solved by a linear-time repair-based algorithm. Starting with a conflicting schedule with specific properties iteratively conflicts are removed by exchanging games.

[48] Hamiez J-P, Hao J-K. Using solution properties within an enumerative search to solve a sports league scheduling problem. Discrete Applied Mathematics 2008;156:1683–93.

The balanced tournament problem (prob026 of CSPLib) is solved by an enumerative search algorithm where problem-specific properties are exploited in order to reduce the search space. Computational experiments have shown that instances with up to 50 teams can be solved.

[49] Haselgrove J, Leech J. A tournament design problem. American Mathematical Monthly 1977;84:198–201.

A construction method for BTD(n) is given for all integers n with $(2n-1) \bmod 3 \neq 0$.

[50] Ikebe Y, Tamura A. On the existence of sports schedules with multiple venues. Discrete Applied Mathematics 2008; 156:1694–710.

Gives theoretical methods for creating sports schedules for the BTD* variant of balanced tournament designs where there are multiple venues for the games, and the number of times each team uses each venue should be balanced. A construction for leagues having $2^p \geq 8$ teams was given by de Werra et al. [40]. It is shown that feasible schedules exist when the league has an arbitrary even number of teams greater than or equal to 8.

[51] Knust S. Scheduling sports tournaments on a single court minimizing waiting times. Operations Research Letters 2008;36:471–6.

A single round tournament for an odd number of teams is considered where each team plays exactly two games per round and all games in a round are scheduled consecutively on a single court. Graph theoretical results are used to construct schedules minimizing the number of long waiting times and the total waiting time simultaneously.

[52] Lamken E. A few more partitioned balanced tournament designs. Ars Combinatoria 1996;43:121–34.

It is shown that a PBTD(n) exists for all integers $n \geq 5$ with the possible exceptions for $n \in \{9, 11, 15\}$.

[53] Lamken E. Balanced tournament designs. In: Colbourn CJ, Dinitz JH, editors. Handbook of combinatorial designs. Boca Raton: CRC Press; 2006. p. 333–6.

Surveys definitions and existence results for balanced tournament designs.

[54] Lamken E, Vanstone S. The existence of factored balanced tournament designs. Ars Combinatoria 1985;19:157–60.

It is shown that a FBTD(n) exists for all integers $n \neq 2$.

[55] Lamken E, Vanstone S. Balanced tournament designs and related topics. Discrete Mathematics 1989;77:159–76.

Surveys known results on balanced tournament designs and discusses several extensions and generalizations.

[56] Lim A, Rodrigues B, Zhang X. Scheduling sports competitions at multiple venues—revisited. European Journal of Operational Research 2006;175;171–86.

The problems studied in Urban and Russell [61] are solved heuristically using beam search and simulated annealing. Feasible solutions could be found for up to $2n = 16$ teams.

[57] Mendelsohn E, Rodney P. The existence of court balanced tournament designs. Discrete Mathematics 1994;133:207–16.

For a given number of courts, a court balanced tournament design (CBTD) is a single round-robin tournament where every team plays the same number of games on each court. Necessary and sufficient conditions for the existence of a CBTD are provided.

[58] Rodney P. The existence of interval-balanced tournament designs. Journal of Combinatorial Mathematics and Combinatorial Computing 1995;19:161–70.

For a given number c of courts, an interval-balanced tournament design (IBTD) is a single round-robin tournament where resting times for the teams between consecutive games are fairly distributed. Necessary and sufficient conditions for the existence of an IBTD are provided.

[59] Russell R, Urban T. A constraint programming approach to the multiple-venue, sport-scheduling problem. Computers & Operations Research 2006;33:1895–906.

The problems studied in Urban and Russell [61] are solved using constraint programming. Solutions are found for $2n \in \{12, 14, 16\}$. The constraint programming approach is compared against the integer goal programming approach of Urban and Russell. Optimal solutions are produced for 16 team problems, and near-optimal solutions for problems with up to 30 teams. By comparison, mathematical programming was only able to find optimal solutions for problem sizes up to ten teams. The success of the method relies on a formulation that enables significant domain reduction during constraint propagation. If this is not possible, then a mathematical programming approach might be more appropriate.

[60] Schellenberg P, van Rees G, Vanstone S. The existence of balanced tournament designs. Ars Combinatoria 1977;3:303–18.

It is shown that a BTD(n) exists for all integers $n \neq 2$ by giving a recursive construction scheme.

[61] Urban T, Russell R. Scheduling sports competitions on multiple venues. European Journal of Operational Research 2003;148:302–11.

A BTD*(n) has to be constructed in order to schedule intra-squad competitions on various drill stations. The problem is formulated as an integer goal programming model and solved for $2n \in \{8, 10\}$. Additionally, it is shown that no solution exists for $2n \in \{4, 6\}$. Finally, generalizations of the problem with less than n courts or home/away assignments are studied.

2.8. Traveling tournament problem

The traveling tournament problem was introduced by Easton et al. [67]. It is a challenging combinatorial optimization problem in sports scheduling that abstracts the most important aspects in creating timetables where team travel is an important issue. Given two integer numbers L and U, and an even number n of teams whose distances between their sites are known beforehand, the problem calls for the schedule of a double round-robin tournament minimizing the total distance traveled by the teams and respecting a set of constraints. Every team begins the tournament at home and must return to home after its last away game. Whenever a team plays two consecutive away games, it goes directly from the site of the first opponent to that of the second, without going back to home. No repeaters are allowed, i.e. no two teams can face each other in two consecutive rounds. Furthermore, every sequence of consecutive home games (or consecutive away games) played by any team is formed by at least L and at most U games. Research on the TTP was inspired by work done for the Major League Baseball in the United States. The complexity of the traveling tournament problem is still open to date.

The mirrored traveling tournament problem and the traveling tournament problem with predefined venues (TTPPV) are two variants of the traveling tournament problem, described in Ribeiro and Urrutia [73] and Melo et al. [71], respectively. The mirrored traveling tournament problem has the additional constraint that games played in round t are exactly the same played in round $t + (n - 1)$ for $t = 1, \ldots, n - 1$, but with reversed venues. Such tournaments are a common tournament structure in Latin America.

The traveling tournament problem with predefined venues is a single round-robin variant of the TTP, in which the venue of each game to be played is known beforehand.

The TTP and its variants have been tackled by different exact and approximate solution methods. Three sets of test instances available from the literature are widely used for benchmark studies: circle instances CIRCn, National League instances NLn, and constant instances (for which the minimization of the travel distance is equivalent to the maximization of the number of breaks). The two first sets were created by Easton et al. [67] and the third by Urrutia and Ribeiro [74]. Instances derived from the National Football League and from the Brazilian national football championship are also available. Input data, together with the best lower and upper bounds considering $L = 1$ and $U = 3$ for all these instances can be found at http://mat.tepper.cmu.edu/TOURN/. The largest instances regularly solved to optimality to date consider only eight teams.

[62] Araújo A, Boeres MC, Rebello VE, Ribeiro CC, Urrutia S. Exploring grid implementations of parallel cooperative metaheuristics: a case study for the mirrored traveling tournament problem. In: Doerner K, Gendreau M, Greistorfer P, Gutjahr W, Hartl R, editors. Metaheuristics: progress in complex systems optimization. Berlin: Springer; 2007. p. 297–322.

Different strategies for the parallelization of greedy randomized adaptive search procedures (GRASP) with ILS hybrid heuristic proposed by Ribeiro and Urrutia [73] for the mirrored TTP are presented, with the objective of harnessing the benefits of grid computing, which offers significantly more computing power than traditional clusters. Computational experiments on a dedicated environment illustrate the effectiveness and the scalability of the proposed strategies. The parallel strategy implementing cooperation through a pool of elite solutions scales better and is able to find solutions that cannot be found by the others. Pioneering computational experiments on a shared computational grid formed by 82 machines distributed over four clusters in three cities illustrate the potential of the application of computational grids in the fields of metaheuristics and combinatorial optimization.

[63] Benoist T, Laburthe F, Rottembourg B. Lagrange relaxation and constraint programming collaborative schemes for travelling tournament problems. In: Proceedings of the third international workshop on integration of AI and OR techniques in constraint programming for combinatorial optimization problems, Ashford, 2001. p. 15–26.

The TTP is solved by a hybrid algorithm combining Lagrangian relaxation and constraint programming in a hierarchical architecture. The main component is a constraint programming model capturing the entire problem. This model can be either solved directly or together with a global constraint to improve the bounds during the search. This global constraint is obtained from the Lagrangian relaxation, which is solved by subgradient optimization or modified gradient techniques. The Lagrange multipliers are adjusted by solving one subproblem for each team. Each subproblem corresponds to an instance of the traveling salesman problem, consisting in scheduling the games for the corresponding team such that its travel distance is minimized.

[64] Cheung KKH. Solving mirrored traveling tournament problem benchmark instances with eight teams. Discrete Optimization 2008;5:138–43.

A two-phase method based on generating timetables from 1-factorizations and finding optimal home/away assignments solves the mirrored traveling tournament problem benchmark instances NL8 and CIRC8.

[65] Cheung KKH. A Benders approach for computing lower bounds for the mirrored traveling tournament problem. Discrete Optimization 2009;6:189–96.

A Benders approach for computing lower bounds for the mirrored traveling tournament problem is proposed.

[66] Costa FN, Urrutia S, Ribeiro CC. An ILS heuristic for the traveling tournament problem with fixed venues. In: Burke EK, Gendreau M, editors. Proceedings of the 7th international conference on the practice and theory of automated timetabling, Montréal, 2008.

An ILS heuristic for solving real-size instances of the TTPPV is proposed, making use of two types of local search moves and two types of perturbations. Initial solutions are derived from canonical 1-factorizations of the tournament graph or of its subgraphs. Computational results showed that the ILS heuristic performs much better than heuristics based on integer programming formulations and improves the best-known solutions for benchmark instances.

[67] Easton K, Nemhauser G, Trick MA. The travelling tournament problem: description and benchmarks. In: Walsh T, editor. Principles and practice of constraint programming. Lecture notes in computer science, vol. 2239. Berlin: Springer; 2001. p. 580–5.

This paper is the seminal work on the traveling tournament problem, in which the latter was formulated for the first time. Two classes of benchmark test instances are described and tackled by constraint programming and integer programming: circle instances and National League instances. Computational experiments have shown that test problems with four teams are easily solvable in a few seconds of computation time, while instances with six teams can be exactly solved in several hours. No instance with eight teams could be solved to proven optimality.

[68] Easton K, Nemhauser GL, Trick MA. Solving the travelling tournament problem: a combined integer programming and constraint programming approach. In: Burke E, de Causmaecker P, editors. The 4th international conference on the practice and theory of automated timetabling. Lecture notes in computer science, vol. 2740. Berlin: Springer; 2003. p. 100–9.

Solution algorithms for the TTP are proposed. At first the so-called "independent lower bound" is described which is obtained by considering a TSP for each team separately. Afterwards, a parallel implementation of a branch-and-price algorithm that uses integer programming to solve the master problem and constraint programming to solve the pricing problem is presented. The master problem is solved by column generation, with the columns corresponding to tours for the teams. Columns are generated only if a node is about to be cut off. Constraint programming is used as a primal heuristic to find feasible solutions. The first provably solution for a TTP instance with eight teams is presented, although the no-repeaters constraint has been discarded.

[69] Fujiwara N, Imahori S, Matsui T, Miyashiro R. Constructive algorithms for the constant distance traveling tournament problem. In: Burke EK, Rudová, H, editors. Practice and theory of automated timetabling VI. Lecture notes in computer science, vol. 3867. Berlin: Springer; 2007. p. 135–46.

A lower bound is proposed to the optimal value of constant distance instances of the traveling tournament problem, together with two constructive algorithms that produce feasible solutions whose objective values are close to the proposed lower bound.

[70] Irnich S. A new branch-and-price algorithm for the traveling tournament problem. Technical Report, OR-01-2009, RWTH Aachen; 2009.

Based on a new compact IP-formulation the traveling tournament problem is solved with branch and price. Contrary to Easton et al. [68] who solved the tour-generation subproblem with constraint programming, in this work the network structure is explicitly utilized. The column-generation subproblem is reduced to a shortest-path problem, which is efficiently solved. Computational results are presented for the National League and circle instances. On the one hand, the benchmark instance NL8 is solved to optimality for the first time, additionally, some best known lower bound values are improved.

[71] Melo RA, Urrutia S, Ribeiro CC. The traveling tournament problem with predefined venues. Journal of Scheduling 2009, in press.

Three integer programming formulations for the TTPPV are proposed and compared. Simple enumeration strategies to generate feasible solutions to real-size instances in a reasonable amount of time are also proposed. Numerical results comparing the three formulations are presented. They show that the formulation with the largest number of decision variables produces better lower bounds and smaller enumeration trees. They also show that two original enumeration strategies outperformed an improvement heuristic embedded within the commercial solver CPLEX.

[72] Rasmussen RV, Trick MA. The timetable constrained distance minimization problem. In: Beck J, Smith B. editors. Integration of AI and OR techniques in constraint programming for combinatorial optimization problems. Lecture notes in computer science, vol. 3990. Berlin: Springer; 2006. p. 167–81.

The timetable constrained distance minimization problem is applicable for tournaments where the total travel distance must be minimized. The problem is defined and integer programming and constraint programming formulations are given. A hybrid approach combining integer programming and constraint programming is proposed for its solution. Also proposed is a branch-and-bound algorithm. The solution methods are tested and compared. The method showing the best performances is a two-phase hybrid IP/CP approach which generates all feasible patterns in a first phase using constraint programming and assigns teams to patterns in the second phase using integer programming.

[73] Ribeiro CC, Urrutia S. Heuristics for the mirrored traveling tournament problem. European Journal of Operational Research 2007;179:775–87.

A hybrid heuristic based on principles of the GRASP and ILS metaheuristics is proposed for the mirrored TTP. Four different neighborhood structures are used for local search, while an ejection chain mechanism is used for perturbations within the ILS phase. The results obtained by the hybrid heuristic were even better than those available at the time of writing for some instances of the less constrained TTP, with the execution times limited to 15 min. State-of-the-art algorithms at the time of writing usually reported up to several days of computation time.

[74] Urrutia, S, Ribeiro CC. Maximizing breaks and bounding solutions to the mirrored traveling tournament problem. Discrete Applied Mathematics 2006;154:1932–8.

A relation is established between two aspects of round-robin tournament scheduling problems: breaks and trips. The number of breaks plus twice the number of trips is proved to be equal to nR, where n is the number of teams and R is the number of rounds in a single or double round-robin tournament. A new class of constant instances of the TTP is proposed, in which the distance between any pair of venues is equal to one. In this case, the total distance traveled is equal to the number of trips. Lower bounds to the number of trips are derived for several tournament types. The lower bounds reached the primal bound found by the hybrid heuristic proposed by Ribeiro and Urrutia [73] for some instances. Instances of the mirrored TTP with constant distances and up to 16 teams were solved to optimality.

[75] Urrutia S, Ribeiro CC, Melo RA. A new lower bound to the traveling tournament problem. In: Proceedings of the IEEE symposium on computational intelligence in scheduling, Honolulu. IEEE; 2007. p. 15–8.

A new lower bound is proposed for the TTP. It improves the independent lower bound introduced in Easton et al. [68] by considering the difference between the sum of the minimum number of trips every team must perform and the known optimal solution values for the associated constant instances. Numerical results on benchmark instances showed reductions as large as 38.6% in the gaps between lower and upper bounds.

Additional algorithms for the TTP are reviewed in other sections, e.g. Anagnostopoulos et al. [108,109] in Section 3.4, Di Gaspero and Schaerf [111] in Section 3.4, Lim et al. [112] in Section 3.4, Rasmussen and Trick [86] in Section 3.2, Rasmussen and Trick [8] in Section 1, and Ribeiro and Urrutia [113] in Section 3.4.

3. Methodologies

Some solution approaches such as decomposition, integer programming, constraint programming, and metaheuristics are common to most optimization problems in sports scheduling.

3.1. Decomposition approaches

Sports scheduling problems are often decomposed into subproblems which are solved sequentially by exact or heuristic algorithms. Mainly, the following two approaches can be distinguished:

(1) "First-schedule, then-break": At first only the pairings are determined for each round (i.e. which teams play against each other in this round). Afterwards, a corresponding home–away pattern is calculated for these pairings (often with a minimum number of breaks).
(2) "First-break, then-schedule": At first a feasible home–away pattern (often with a minimum number of breaks) is determined. Afterwards, the pairings for the corresponding pattern are fixed. In this case, sometimes placeholders are used at first to determine the pairings and specific teams are assigned to the placeholders afterwards.

Both approaches have been studied in the literature for different problem settings. They are often used in combination with integer linear programming or constraint programming formulations.

In a "first-schedule, then-break" approach, the subproblem of the second stage is to find a home–away pattern (with a minimum number of breaks) corresponding to the pairings determined in the first stage. The complexity status of this problem is unknown, but it is conjectured to be NP-complete. On the other hand, it can be decided in polynomial time whether the given pairings can be scheduled with at most n breaks or not.

[76] Brouwer A, Post G, Woeginger G. Tight bounds for break minimization. Journal of Combinatorial Theory A 2008;115:1065–8.

For the break minimization problem it is shown that an infinite family of opponent schedules with n teams exist for which any home–away pattern has at least $n(n-2)/4$ breaks. This lower bound matches the upper bound derived in Post and Woeginger [80].

[77] Elf M, Jünger M, Rinaldi G. Minimizing breaks by maximizing cuts. Operations Research Letters 2003;36:343–9.

The break minimization problem for a given opponent schedule is transformed into a maximum cut problem in an undirected graph which is solved by a branch-and-cut algorithm.

[78] Miyashiro R, Matsui T. A polynomial-time algorithm to find an equitable home–away assignment. Operations Research Letters 2003;33:235–41.

It is proved that for a given opponent schedule with n teams it can be checked in polynomial time whether a home–away pattern with $n-2$ breaks exists or not by reducing the problem to the 2-satisfiability problem. The same idea can be applied in order to check whether a pattern with n breaks exists. Furthermore, it is proved that break maximization is equivalent to break minimization by showing that each home–away pattern with k breaks can be converted into a pattern with $n(n-2)-k$ breaks and vice versa. The idea of the transformation is to convert each break into a non-break and vice versa by keeping the home rights in the odd-numbered rounds $1,3,\ldots,n-1$ and flipping the home rights in the even-numbered rounds $2,4,\ldots,n-2$.

[79] Miyashiro R, Matsui T. Semidefinite programming based approaches to the break minimization problem. Computers & Operations Research 2006;33:1975–82.

The break minimization problem for a given opponent schedule is formulated as the MAX RES CUT and MAX 2SAT problems. These problems are heuristically solved using semidefinite programming.

[80] Post G, Woeginger G. Sports tournaments, home–away assignments, and the break minimization problem. Discrete Optimization 2006;3:165–73.

For the break minimization problem it is shown that for each opponent schedule with n teams a home–away pattern with at most $n(n-2)/4$ breaks exists. On the other hand, it is shown that opponent schedules exist which cannot be played with less than $n(n-1)/6$ breaks. Furthermore, it is proved that break minimization is NP-hard for a given partial opponent schedule with $r \geq 3$ rounds.

[81] Trick MA. A schedule-then-break approach to sports timetabling. In: Burke EK, Erben E, editors. The 3rd international conference on the practice and theory of automated timetabling. Lecture notes in computer science, vol. 2079. Berlin: Springer; 2001. p. 242–52.

A two-phase approach is presented for sports timetabling problems. In the first stage, a schedule is generated ignoring any home/away requirements using a constraint programming approach. The second phase (using an integer programming model), which proves to be more challenging than the first stage, makes the home/away assignments whilst minimizing bad structures such as consecutive home and away games. This paper has a useful discussion as to how the problem was formulated and it also presents the resultant OPL code, which other researchers might find useful.

In a "first-break, then-schedule" approach, the subproblem of the second stage consists in finding a feasible opponent schedule for a given home–away pattern. A HAP is called feasible if a corresponding opponent schedule exists which is compatible with this HAP. The pattern set feasibility problem is to determine whether a given HAP is feasible or not. The complexity status of this problem is still not known, but it is conjectured to be NP-complete.

[82] Briskorn D. Feasibility of home–away-pattern sets for round-robin tournaments. Operations Research Letters 2008;36:283–4.

The pattern set feasibility problem is studied and a necessary condition for feasibility based on a linear programming formulation is given. It is shown that this condition is strictly stronger than those provided in Miyashiro et al. [83].

[83] Miyashiro R, Iwasaki H, Matsui T. Characterizing feasible pattern sets with a minimum number of breaks. In: Burke E, de Causmaecker P, editors. The 4th international conference on the practice and theory of automated timetabling. Lecture notes in computer science, vol. 2740. Berlin: Springer; 2003. p. 78–99.

The pattern set feasibility problem is studied and necessary conditions for feasibility based on checking all possible subsets of teams are provided. For pattern sets with $n-2$ breaks these conditions can be checked in polynomial time. It is shown by computational experiments that in this case the conditions are also sufficient for problems with up to 26 teams.

Other algorithms based on a "first-break, then-schedule" approach can be found in the sections on integer and constraint programming, see e.g. Henz [128] in Section 4.2, Knust and Lücking [37] in Section 2.6, Nemhauser and Trick [129] in Section 4.2, Rasmussen [122] in Section 4.1, Rasmussen and Trick [72] in Section 2.8, Rasmussen and Trick [86] in Section 3.2, and Trick [81] in Section 3.1.

3.2. Integer programming

Integer programming is a useful tool to model and solve sports scheduling problems. Some round-robin tournament scheduling problems can be solved by directly applying an integer programming solver to the model. In most cases, decomposition schemes are used to tackle each stage of the problem by integer programming or other techniques such as constraint programming, complete enumeration, or heuristics.

If n denotes the number of teams and r the number of rounds, most models usually make use of the following variable definition:

$$x_{ijt} = \begin{cases} 1 & \text{if team } i \text{ plays at home against team } j \text{ in round } t, \\ 0 & \text{otherwise,} \end{cases}$$

for teams $i,j = 1,\dots,n$ (with $i \neq j$) and rounds $t = 1,\dots,r$. The constraints of a double round-robin tournament may be formulated as

$$\sum_{t=1}^{r} x_{ijt} = 1, \quad \forall 1 \le i,j \le n, \;\; i \neq j,$$

$$\sum_{j=1}^{n} x_{ijt} \le 1, \quad \forall 1 \le i \le n, \;\; 1 \le t \le r.$$

The first set of constraints imposes that every team must play against every other team at home. The second set establishes that no team plays more than once in each round. In the case of a compact schedule (where every team has to play in each round), the last inequality constraints turn into equalities.

Traveled distances cannot be directly modeled by variables x_{ijt}. If they are an issue in the problem, then new binary variables have to be used, e.g.

$$z_{kijt} = \begin{cases} 1 & \text{if team } k \text{ travels from the venue of team} \\ & i \text{ to that of } j \text{ in round } t, \\ 0 & \text{otherwise.} \end{cases}$$

When the length of the round trips and the length of the home stands are limited, the number of distinct round trips and home stands a given team may be involved in is polynomial with respect to the numbers of teams. This fact usually allows the use of more descriptive variables defining entire round trips or home stands, see e.g. Melo et al. [71] in Section 2.8, and Costa et al. [66] in Section 2.8. Exponentially many variables representing e.g. the entire sequence of games of a given team during the tournament are also used in some cases.

Integer programming methods applied to sport scheduling problems include branch and bound, branch and cut, Benders decomposition, and column generation. These methods are used to deal with real leagues scheduling problems as well as to tackle theoretical problems such as the traveling tournament problem or break minimization.

Hybridization with other methods is common in the literature. Integer programming methods are often used as subproblem solvers in several stage approaches involving, for example, constraint programming or heuristics techniques.

[84] Briskorn D. Sport leagues scheduling: models, combinatorial properties, and optimization algorithms. In: Lecture notes in economics and mathematical systems, vol. 603. Berlin: Springer; 2008.

This book presents the work carried out by the author in his Ph.D. thesis. Besides presenting basic and advanced formulations, the work also focusses on real world problems. Many integer programming formulations and proofs are provided. Some chapters can also be found in other publications, see e.g. Briskorn and Drexl [85] or the papers in Section 2.6.

[85] Briskorn D, Drexl A. IP models for round robin tournaments. Computers & Operations Research 2009;36:837–52.

Variations on basic round-robin tournaments are described. In addition, various real world constraints are discussed. These include matches which cannot be scheduled at a certain time (perhaps the stadium is in use for another event), limiting the number of matches in a given geographical region, and balancing the distribution of attractive matches. For each variation, the problem is formulated as an integer programming model and CPLEX is used to study the behavior of the various models.

[86] Rasmussen RV, Trick MA. A Benders approach for the constrained minimum break problem. European Journal of Operational Research 2007;177:198–213.

A hybrid algorithm combining constraint programming and integer programming is presented for finding a double round-robin tournament schedule with a minimum number of breaks and respecting place constraints. The algorithm uses constraint programming, integer programming, and Benders cuts in order to obtain feasible home–away patterns and constraint programming for finding the corresponding schedules. The algorithm is also applied to the traveling tournament problem with constant distances.

[87] Suzuka A, Miyashiro R, Yoshise A, Matsui T. The home–away assignment problems and break minimization/maximization problems in sports scheduling. Pacific Journal of Optimization 2007;3:113–33.

Presents a unified view of the home–away assignment problem (i.e. distance minimization) and the break minimization problem (i.e. the number of consecutive home/away games) in the context of round-robin tournaments. An integer programming formulation is given for the home–away assignment problem, together with some rounding algorithms. A technique is also presented to transform the home–away assignment problem to MIN RES CUT, to which Goemans and Williamson's algorithm for MAX RES CUT is applied. Computational results demonstrate that the approaches generate solutions with good approximation ratios, in fast computational times.

[88] Trick MA. Integer and constraint programming approaches for round-robin tournament scheduling. In: Burke E, de Causmaecker P, editors. The 4th international conference on the practice and theory of automated timetabling. Lecture notes in computer science, vol. 2740. Berlin: Springer; 2003. p. 63–77.

Shows that practical round-robin schedules can be modeled by both constraint programming and integer programming. Constraint programming is often faster, but it does depend on the constraints and the objective function. Several interesting research directions are suggested in the conclusions of the paper.

Additionally, integer programming was used for solving scheduling problems in football leagues in Bartsch et al. [115] in Section 4.1, Briskorn [82] in Section 3.1, Della Croce and Oliveri [116] in Section 4.1, Goossens and Spieksma [119] in Section 4.1, Noronha et al. [121] in Section 4.1, and Rasmussen [122] in Section 4.1. Furthermore, for the traveling tournament problem integer programming was applied in Easton et al. [68] in Section 2.8, Irnich [70] in Section 2.8, Melo et al. [71] in Section 2.8, and Rasmussen and Trick [72] in Section 2.8. It was also used to solve other applications in Farmer et al. [151] in Section 4.7, Nemhauser and Trick [129] in Section 4.2, Ribeiro and Urrutia [123] in Section 4.1, Trick [81] in Section 3.1, and Urban and Russell [61] in Section 2.7.

3.3. Constraint programming

In order to apply constraint programming techniques to an optimization problem, the problem must be modeled by inter-related sets of variables and constraints. Each variable has a finite domain of possible instantiation values. Constraint programming derives new constraints and fixes variable values by manipulating the original sets of constraints and variable domains. Furthermore, variables are fixed from logical conclusions. In defining a constraint satisfaction problem, typically (at least) the following questions have to be settled:

- Which variables are used to model the problem?
- Which values are those variables allowed to take?
- What are the constraints of the problem?
- What are the relationships between the variables so that only valid solutions are allowed once all the variables have been instantiated?
- How can decisions be propagated in order to derive new constraints?

There are many references describing constraint satisfaction methodologies. The reader is referred e.g. to Castillo et al. [89] and Marriott and Stuckey [93].

[89] Castillo L, Borrajo D, Salido M, Oddi A. Planning, scheduling and constraints satisfaction: from theory to practice. IOS Press; 2005.

Edited volume which might be useful for those interested in using constraint programming.

[90] Henz M. Constraint-based round robin tournament planning. In: de Schreye D, editor. Proceedings of the international conference on logic programming. Las Cruces, New Mexico: MIT Press; 1999. p. 545–57.

Proposes that the planning of a sports tournament can be split into three stages, which can all be solved using finite-domain constraint programming. The design of the Friar Tuck system (see also Henz [91]) is outlined, the former being a generic constraint-based round-robin planning tool. Results demonstrate the effectiveness of this approach.

[91] Henz M. Friar tuck—a constraint-based tournament-scheduling tool. IEEE Intelligent Systems 2000;15:5–7.

This is a short note on the Friar Tuck system (see also Henz [90]).

[92] Henz M, Müller T, Thiel S. Global constraints for round robin tournament scheduling. European Journal of Operational Research 2004;153:92–101.

Using the domain of round-robin tournaments, this paper analyzes arc-consistent propagation algorithms for the all-different and one-factor global constraints. The paper concludes that arc-consistent propagation for the all-different constraint is important for the tournament problems studied in the paper.

[93] Marriott K, Stuckey P. Programming with constraints. Cambridge: The MIT Press; 1998.

Textbook which might be of interest for those interested in using constraint programming.

[94] Regin J-C. Minimization of the number of breaks in sports scheduling problems using constraint programming. In: Freuder E, Wallace R, editors. Constraint programming large scale discrete optimization. DIMACS series in discrete mathematics and theoretical computer science, vol. 57, 2001. p. 115–30.

Presents constraint programming approaches for break minimization in single round-robin tournament problems. Solutions for the (polynomially solvable) problem of only finding a schedule with $n-2$ breaks for instances with $n=20$ teams are produced in 0.61 s and in less than a minute for 60 team instances. Afterwards, some experiments are done for the much more difficult situation when a timetable (opponent schedule) is given and breaks are to be minimized. Instances with up to 12 teams can be solved by introducing additional global constraints in the constraint programming formulation.

[95] Schaerf A. Scheduling sport tournaments using constraint logic programming. Constraints 1999;4:43–65.

Utilizing constraint logic programming, a two stage approach provides solutions to round-robin tournaments. The first stage produces a tournament pattern, with the second stage assigning teams to the pattern. The author shows that the problem being tackled is NP-complete. The paper also discusses the use of an interactive system in order to enable the user to participate in the generation of the schedule.

Additionally, constraint programming is used for the scheduling problems in Easton et al. [68] in Section 2.8, Henz [128] in Section 4.2, Ordonez and Knowles [154] in Section 4.7, Rasmussen and Trick [72] in Section 2.8, Rasmussen and Trick [86] in Section 3.2, Russell and Urban [59] in Section 2.7, Trick [81] in Section 3.1, and Trick [88] in Section 3.2.

3.4. Heuristic search and metaheuristics

Most optimization problems in scheduling are very hard in computational terms, in the sense that no polynomial-time algorithms are known for their solution. Although some scheduling problems in sports are amenable to be exactly solved by integer programming due to their small size or intrinsic structure, this is not the general case. Therefore, heuristics (or approximate algorithms) that find sub-optimal solutions in reasonable computation times are often used in practice. Research in heuristics started with the pioneering work of Hart et al. [96] on algorithm A*, which follows a scheme very similar to a branch-and-bound algorithm using a best-first strategy:

[96] Hart PE, Nilsson NJ, Raphael B. A formal basis for the heuristic determination of minimum cost paths. IEEE Transactions on Systems Science and Cybernetics 1968;4:100–7.

Algorithm A* is a graph-search based on heuristic information and lower bounds to speedup the search. It extends the less-informed breadth search strategy which corresponds to Dijkstra's shortest path algorithm when every edge has a unit weight. This is the seminal work describing algorithm A*, its principles and properties.

[97] Nilsson NJ. Problem-solving methods in artificial intelligence. New York: McGraw-Hill; 1971.

This book describes solution methods applied to search problems in artificial intelligence, such as theorem proving and two-person games. Special emphasis is given to algorithm A* and its basics.

[98] Nilsson NJ. Principles of artificial intelligence. Berlin: Springer; 1982.

This book is an extended and updated version of Nilsson [97].

Constructive heuristics build feasible solutions from scratch and are often based on quick greedy choices. The use of fast constructive algorithms to provide initial solutions to more sophisticated procedures often leads to impressive improvements in solution quality and computation times. Local search heuristics are based on the investigation of solution neighborhoods, successively replacing the current solution by a better one within its neighborhood and terminating when no better solution can be found. Metaheuristics are general high-level procedures that coordinate simple heuristics and rules to find good (often optimal) approximate solutions to computationally difficult combinatorial optimization problems. Among them, we find simulated annealing, tabu search, greedy randomized adaptive search procedures, genetic algorithms, iterated local search, variable neighborhood search (VNS), ant colonies, and others. They are based on distinct paradigms and offer different mechanisms to escape from locally optimal solutions, contrarily to greedy algorithms or to pure local search methods. The customization (or instantiation) of some metaheuristic to a given problem yields a heuristic to the latter. We provide below some references focusing on local search and the metaheuristics which are often applied to scheduling problems in sports:

[99] Aarts E, Korst J, Michiels W. Simulated annealing. In: Burke EK, Kendall E, editors. Search methodologies: introductory tutorials in optimization and decision support techniques. New York: Springer; 2005. p. 187–210.

[100] Aarts E, Korst J, Michiels W. Theoretical aspects of local search. Berlin: Springer; 2007.

[101] Dorigo M, Stützle T. The ant colony optimization metaheuristic: algorithms, applications, and advances. In: Glover F, Kochenberger G, editors. Handbook of metaheuristics. Boston: Kluwer Academic Publishers; 2003. p. 251–85.

[102] Glover F, Laguna M. Tabu search. Boston: Kluwer Academic Publishers; 1997.

[103] Goldberg DE. Genetic algorithms in search, optimization and machine learning. Reading: Addison-Wesley; 1989.

[104] Hansen P, Mladenović N. Variable neighborhood search. In: Glover F, Kochenberger G, editors. Handbook of metaheuristics. Boston: Kluwer Academic Publishers; 2003. p. 145–84.

[105] Hoos HH, Stützle T. Stochastic local search: foundations and applications. San Francisco: Morgan Kaufmann; 2005.

[106] Lourenço HR, Martin OC, Stützle T. Iterated local search. In: Glover F, Kochenberger G, editors. Handbook of metaheuristics. Boston: Kluwer Academic Publishers; 2003. p. 321–53.

[107] Resende MGC, Ribeiro CC. Greedy randomized adaptive search procedures. In: Glover F, Kochenberger G, editors. Handbook of metaheuristics. Boston: Kluwer Academic Publishers; 2003. p. 219–49.

Metaheuristics are among the most effective solution strategies for solving combinatorial optimization problems in practice. Hybridizations combining principles from different metaheuristics often produce the best results. In particular, metaheuristic implementations strongly benefit from good initial solutions such as those obtained by greedy randomized constructive heuristics and repair methods, as shown by numerical results reported e.g. in Ribeiro and Urrutia [77] and Duarte et al. [146], respectively. Metaheuristics and their hybrids have been applied to a broad variety of scheduling problems in sports. References with original algorithmic contributions, innovative components, or intricate hybridization strategies are cited, summarized, and discussed here. More straightforward implementations applied to specific problems are only reviewed in Section 4, e.g. Willis and Terrill [133] in Section 4.3, Wright [134] in Section 4.3, Wright [135] in Section 4.3, Wright [136] in Section 4.3, and Wright [131].

Additionally, heuristic methods were also used for the scheduling problems in Costa et al. [66] in Section 2.8, Della Croce et al. [141] in Section 4.1, Duarte et al. [146] in Section 4.7, Hamiez and Hao [46] in Section 2.7, and Lim et al. [112] in Section 2.7.

[108] Anagnostopoulos A, Michel L, Van Hentenryck P, Vergados Y. A simulated annealing approach to the traveling tournament problem. In: Proceedings of CPAIOR'03, 2003.

This is a preliminary version of Anagnostopoulo et al. [109]. This paper has been mentioned in order to highlight that some of the results presented in the 2006 journal paper were produced earlier in 2003.

[109] Anagnostopoulos A, Michel L, Van Hentenryck P, Vergados Y. A simulated annealing approach to the traveling tournament problem. Journal of Scheduling 2006;9:177–93.

A hybrid algorithm for the TTP is proposed, based on the simulated annealing metaheuristic and exploring both feasible and infeasible schedules. The heuristic buys some principles from other metaheuristics: it uses a large neighborhood with complex moves and includes advanced techniques such as strategic oscillation and reheats to balance the exploration of the feasible and infeasible regions and to escape local minima at very low temperatures. It matches the best-known solutions on the small instances and produces significant improvements over previous approaches on the larger instances. The algorithm is claimed to be robust, because the worst solution value it produced over 50 runs is always smaller than or equal to the best known solutions.

[110] Costa D. An evolutionary tabu search algorithm and the NHL scheduling problem. INFOR 1995;33:161–78.

The focus of this paper is to investigate the hybridization of genetic algorithms and tabu search in solving combinatorial optimization problems. The National Hockey League (NHL) is used as an example to illustrate the effectiveness of the approach.

[111] Di Gaspero L, Schaerf A. A composite-neighborhood tabu search approach to the traveling tournament problem. Journal of Heuristics 2007;13:189–207.

A family of tabu search solvers for the approximate solution of the TTP is proposed. They make use of complex combinations of many neighborhood structures. The different neighborhoods are thoroughly analyzed and experimentally compared. The solvers are evaluated on three sets of available benchmarks and their outcomes are compared with previous results presented in the literature.

[112] Lim A, Rodrigues B, Zhang X. A simulated annealing and hill-climbing algorithm for the traveling tournament problem. European Journal of Operational Research 2006;174:1459–78.

A hybridization of simulated annealing with hill-climbing is proposed for the TTP. The search space is divided into a timetable space and a team assignment space. The timetable space is explored by a simulated annealing algorithm, while the team assignment space is explored by a hill-climbing algorithm. A controller fixes team assignments and calls on the simulated annealing component to generate better timetables. The timetable with best schedules is passed on to the hill-climbing component, which searches for better team assignments. Team assignments that give best schedules for the given timetable are then passed back to the simulated annealing component. The process continues until there is no improvement for a specified fixed number of consecutive cycles or when a time limit is reached. The underlying idea is to look for better team assignments only for timetables with a higher chance of giving better schedules, and to search for better timetables only for team assignments that have a higher chance of giving better schedules.

[113] Ribeiro CC, Urrutia S. Heuristics for the mirrored traveling tournament problem. European Journal of Operational Research 2007;179:775–87.

A hybrid heuristic combining principles from the GRASP and ILS metaheuristics is proposed for the mirrored TTP. A three-step constructive heuristic is used to build good initial solutions. In the first step, the canonical 1-factorization is used for constructing a timetable with placeholders. Next, a greedy heuristic is used to assign teams to placeholders. The venues of the games are set round by round and local search is used to repair possible infeasibilities in the last step of the constructive heuristic. The hybrid heuristic makes use of four simple neighborhoods for local search and one ejection chain neighborhood for perturbations. The results obtained by the hybrid heuristic were even better than the best known at the time of writing for some instances of the less constrained TTP produced by Anagnostopoulos et al. [108], with execution times limited to 15 min. State-of-the-art algorithms at the time of writing usually reported up to several days of computation time. It is also shown that the constructive algorithm is very quick and produces good initial solutions that improve the quality of the best solution found by the hybrid heuristic.

[114] Van Hentenryck P, Vergados Y. Traveling tournament scheduling: a systematic evaluation of simulated annealing. In: Integration of AI and OR techniques in constraint programming for combinatorial optimization problems. Lecture notes in computer science, vol. 3990. Berlin: Springer; 2006. p. 228–43.

This paper presents extensions and enhancements of the algorithm in Anagnostopoulos et al. [108] for finding mirrored tournament schedules. The requirement of having a mirrored schedule is modeled as a soft constraint and its violation is penalized in the objective function. Variants of previously used neighborhoods that preserve the mirrored structure are presented. A new neighborhood is also described, consisting in the rearrangement of certain rounds to reverse a sequence of consecutive away games. Other refinements to the original algorithm are introduced. Numerical results showed that the new algorithm was able to improve some of the best known results at the time of writing.

4. Applications

Exact and approximate optimization methods have been widely applied to scheduling problems in sports. In this section we review some of these applications, classified by their corresponding fields or sports of application. Football and basketball are the sports with more applications.

4.1. Football

[115] Bartsch T, Drexl A, Kröger S. Scheduling the professional soccer leagues of Austria and Germany. Computers & Operations Research 2006;33:1907–37.

The creation of suitable schedules for national top soccer league in Europe has to address other constraints, besides numerous conflicting inner-league requirements and preferences. Additionally, the games of the European Cup matches (Champions League, UEFA Cup, National Cup Winners) have to be taken into account. This paper considers the case of Austria and Germany, that is the planning problem the Deutsche Fuball-Bund (DFB) and the Österreichische Fuball-Bund (ÖFB) are confronted with. For both leagues, models and algorithms are developed which yield reasonable schedules quickly. Heuristics and branch and bound are applied. The proposed approach generates schedules which have been accepted for play once by the DFB and six times by the ÖFB.

[116] Della Croce F, Oliveri D. Scheduling the Italian football league: an ILP-based approach. Computers & Operations Research 2006;33:1963–74.

Scheduling the Italian Major Football League (the so-called "Serie A") consists in finding a double round-robin tournament schedule that takes into account both typical requirements such as conditions on home–away matches and specific requests of the Italian Football Association such as twin-schedules for teams belonging to the same home-town. A solution procedure is presented which is able to derive feasible schedules that are also balanced with respect to additional cable televisions requirements. This procedure adapts the approach of Nemhauser and Trick [129] to schedule a college basketball conference that considers, however, only half of the teams involved here. The proposed procedure is divided into three phases: the first phase generates a pattern set respecting the cable televisions requirements and several other constraints; the second produces a feasible round-robin schedule compatible with the above pattern set; and the third phase generates the actual calendar assigning teams to patterns. The procedure allows to generate within short time several different reasonable calendars satisfying the cable television companies requirements and satisfying various other operational constraints, while minimizing the total number of violations of the home–away matches conditions.

[117] Durán G, Guajardo M, Miranda J, Sauré D, Souyris S, Weintraub A, et al. Scheduling the Chilean soccer league by integer programming. Interfaces 2007;37:539–52.

Since 2005, Chile's professional soccer league has used a game-scheduling system that is based on an integer linear programming model. The Chilean league managers considered several operational, economic, and sporting criteria for the final tournaments' scheduling. Thus, they created a highly constrained problem that had been, in practice, unsolvable using their previous methodology. This led to the adoption of a model that used some techniques that were new in soccer-league sports scheduling. The schedules they generated provided the teams with benefits such as lower costs, higher incomes, and fairer seasons. In addition, the tournaments were more attractive to sports fans. Readers are also referred to Durán et al. [118].

[118] Durán G, Guajardo M, Weintraub A, Wolf R. O.R. & soccer: scheduling the Chilean league using mathematical programming. OR/MS Today 2009;36:42–7.

This paper is a follow up to Durán et al. [117], written for a more accessible audience. If you want an overview of the problem being tackled you might want to read this paper, for more technical details see Durán et al. [117].

[119] Goossens D, Spieksma F. Scheduling the Belgian soccer league. Interfaces 2009;39:109–18.

It is described how the Belgian soccer league is scheduled using an integer linear programming formulation. The objective is to find a double round-robin tournament schedule for 18 teams taking into account constraints on the use of stadiums as well as wishes made by the teams, the police, and the broadcasting companies. The schedules were used in practice for the seasons 2006–2007 and 2007–2008.

[120] Kendall G. Scheduling English football fixtures over holiday periods. Journal of the Operational Research Society 2008;59:743–55.

This paper considers the minimization of the travel distances by English football clubs over the Christmas and New Year period. The results show that fixtures can be generated that adhere to all known constraints and produce total travel distances that are about 25% that of the fixtures that were actually used.

[121] Noronha TF, Ribeiro CC, Duran G, Souyris S, Weintraub A. A branch-and-cut algorithm for scheduling the highly-constrained Chilean soccer tournament. In: Practice and theory of automated timetabling VI. Lecture notes in computer science, vol. 3867. Berlin: Springer; 2007. p. 174–86.

The qualifying phase of the Chilean soccer championship follows the structure of a compact single round-robin tournament. Good schedules are of major importance for the success of the tournament, making them more balanced, profitable, and attractive. The schedules were prepared by ad hoc procedures until 2004, when a rough integer programming strategy was proposed. The original integer programming formulation is improved in this work. Valid inequalities for improving the linear relaxation bound are derived and a new branch-and-cut strategy is developed. Computational results on a real-life instance illustrate the effectiveness of the approach and the improvement in solution quality.

[122] Rasmussen RV. Scheduling a triple round robin tournament for the best Danish soccer league. European Journal of Operational Research 2008;185:795–810.

Scheduling the Danish football league is a highly constrained problem, with many conflicting constraints. The problem is also different to many other leagues as it is a triple round-robin tournament, which leads to an uneven distribution of home and away games. An IP model is presented and the solution methodology utilizes a logic-based Benders decomposition and column generation. The proposed approach is compared against the actual schedules that were used for the 2005/2006 season, as well as for randomly generated instances. The results demonstrate the effectiveness of the approach.

[123] Ribeiro CC, Urrutia S. Scheduling the Brazilian soccer tournament with fairness and broadcast objectives. In: Practice and theory of automated timetabling VI. Lecture notes in computer science, vol. 3867. Berlin: Springer; 2007. p. 147–57.

The Brazilian football tournament is yearly organized by the Brazilian Football Confederation (CBF). Its major sponsor is TV Globo, the largest media group and television network in Brazil, who imposes constraints on the games to be broadcast. Scheduling the games of this tournament is a very constrained problem, with two objectives: breaks minimization (fairness) and the maximization of the revenues from TV broadcasting. An integer programming decomposition strategy to solve this problem to optimality is proposed. Numerical results obtained for the 2005 and 2006 editions of the tournament are reported and compared.

Other papers dealing with scheduling problems for football leagues are already reviewed in other sections, see e.g. Griggs and Rosa [18] in Section 2.1, and Schreuder [23] in Section 2.1. A scheduling problem for football referees is described in Yavuz et al. [156] in Section 4.7.

4.2. Basketball

[124] Ball BC, Webster DB. Optimal scheduling for even-numbered team athletic conferences. AIIE Transactions 1977;9:161–9.

This paper handles a scheduling problem for a basketball conference similar to that in Campbell and Chen [126]. The problem of minimizing travel distances is modeled using an integer programming formulation, but solved by a heuristic very similar to the method developed by Campbell and Chen.

[125] Bean JC, Birge JR. Reducing travelling costs and player fatigue in the national basketball association. Interfaces 1980;10:98–102.

For the National Basketball Association (NBA) in the United States, schedules for 22 teams are constructed where each team plays 82 games and resting times and building availabilities have to be taken into account. Methods based on heuristics for the traveling salesman problem are proposed in order to reduce the airline travelling costs. Computational results are presented for the seasons 1978/1979 and 1979/1980, showing that the calculated schedules reduce the costs by about 20% compared with the official NBA schedules.

[126] Campbell RT, Chen D-S. A minimum distance basketball scheduling problem. In: Machol RE, Ladany SP, Morrison DG, editors. Management science in sports. Studies in the management sciences, vol. 4. Amsterdam: North-Holland; 1976. p. 15–25.

This is the first paper considering the problem of scheduling a basketball conference of 10 teams, corresponding to a relaxed double round robin tournament. The teams are allowed to play at most two consecutive away games without returning home. In the first phase, optimal trips for each team are derived. This is shown to be equivalent to pairing the teams two by two such that the distances between the paired teams are minimized. In the second phase, the optimal pairing is used to build a number of feasible sequences using a constructive approach attempting to minimize the total traveled distance.

[127] Fronček D. Scheduling the Czech national basketball league. Congressus Numerantium 2001;153:5–24.

Schedules are constructed for the Czech national basketball league using graph models.

[128] Henz M. Scheduling a major college basketball conference—revisited. Operations Research 2001;49(1):163–8.

A much faster constraint programming approach is provided for the same problem as in Nemhauser and Trick [129]. The 179 solutions from Nemhauser and Trick [129] were found in less than 1 min, whereas Nemhauser and Trick reported an overall running time of about 24 h.

[129] Nemhauser GL, Trick MA. Scheduling a major college basketball conference. Operations Research 1998;46:1–8.

The nine universities in the Atlantic Coast Conference (ACC) have a basketball competition in which each school plays home and away games against each other over a nine-week period. The creation of a suitable schedule is a very difficult problem with a myriad of conflicting requirements and preferences. The paper presents an approach that uses a combination of integer programming and enumerative techniques. It yields reasonable schedules very quickly and gave a schedule that was accepted by the ACC for play in 1997–1998.

[130] van Voorhis TV. Highly constrained college basketball scheduling. Journal of the Operational Research Society 2002;53:603–9.

Considers the scheduling of three basketball conference leagues: The Big 12, The Southeastern Conference and Conference—USA. Each of these leagues has 12 teams, partitioned into 6-team divisions. The scheduling problem is modeled as an integer program. The constraints that are catered for include that each team plays at home during two of the last four rounds, each team plays two of its first four games at home, each team is given four or five weekend home games, and each pair of teams cannot meet twice within four consecutive rounds. A customized depth-first algorithm is presented. The results show that the results are not only superior to the 2001 schedules, but that the system can also create many alternative schedules which can be used by the schedulers to consider other aspects such as TV coverage.

[131] Wright M. Scheduling fixtures for basketball New Zealand. Computers & Operations Research 2006;33:1875–93.

Describes a real-life problem in scheduling basketball fixtures in New Zealand. The problem has only a few constraints, but many objectives. The problem is described in detail and a simulated annealing variant is used to produce high quality schedules. The system was used to produce the 2004 schedule.

4.3. Cricket

[132] Armstrong J, Willis RJ. Scheduling the cricket World Cup—a case-study. Journal of the Operational Research Society 1993;44:1067–72.

This paper describes the scheduling of the 1992 Cricket World Cup tournament, which was co-hosted by Australia and New Zealand. The initial stage of the competition required that each of the nine teams had to play each other once (36 matches) over 2 days. A variety of constraints had to be respected, which included satisfying local populations and worldwide TV audiences and other practical and logistical considerations. An integer programming formulation was presented, which was not utilized due to the large number of constraints. The solution methodologies were developed in Lotus 1-2-3 (a spreadsheet package). One of these methods allowed interaction with the user and was more useful, but took about 4 h to run. None of the schedules produced were ideal and a problem had to be decomposed so that the 13 matches to be played in New Zealand were produced first.

[133] Willis RJ, Terrill BJ. Scheduling the Australian state cricket season using simulated annealing. Journal of the Operational Research Society 1994:45:276–80.

Considers the scheduling of domestic cricket in Australia, including both first class and one day matches. Various constraints had to be respected, including scheduling around international fixtures. Simulated annealing was utilized and, after some manual amendments, the schedule was used for the 1992–1993 season.

[134] Wright M. A fair allocation of county cricket opponents. Journal of the Operational Research Society 1992;43:195–201.

Presents a case study that produces a four year schedule (1992–1995) for English county cricket. The nature of the scheduling problem is such that some teams meet each other once during a season, but some teams play each other twice. This could be considered unfair if the teams playing twice were considered as weak. However, certain teams would like to play each other twice (whether they are considered strong or weak), due for example to local rivalries and for maximizing gate receipts. The schedule has to be as fair as possible (and to be seen to be fair), whilst respecting the various constraints. The solution approach used is based on simulated annealing.

[135] Wright M. Timetabling county cricket fixtures using a form of tabu search. Journal of the Operational Research Society 1994;45:758–70.

Describes a system that was developed to automate the production of English county cricket schedules. The constraints consider aspects such as the times of international matches and knock out competitions, travel considerations between matches, and questionnaires that are sent to each club where they are able to make certain requests. An initial solution is created, which takes into account the constraints but is free to ignore them. A two phase local search based on tabu search is then applied to the initial solution. The algorithm runs overnight (or perhaps a weekend) with the best solutions being presented to the cricket authorities who, after making a few manual changes, present it to the various teams.

[136] Wright M. Scheduling fixtures for New Zealand cricket. IMA Journal of Management Mathematics 2005;16:99–112.

Describes the methodology that was used to produce the 2003–2004 cricket fixtures for New Zealand cricket. Subcost-guided simulated annealing is utilized paying particular attention to the neighborhood moves as even simple moves can lead to an infeasible solution.

Additionally, Wright [155] in Section 4.7 deals with the scheduling problem for cricket umpires.

4.4. Baseball

[137] Cain WO. The computer-aided heuristic approach used to schedule the major league baseball clubs. In: Ladany SP, Machol RE, editors. Optimal strategies in sports. Amsterdam: North-Holland; 1977. p. 33–41.

For the National American Baseball League, schedules for 12 teams divided into two divisions have to be found, where each team plays 162 games (18 times against each team of its own division and 12 times against each team of the other division). The objective is to determine a schedule regarding fairness aspects, maximizing attendance and minimizing travel costs. The author describes how schedules for this problem are created in practice assisted by a computer. Results are reported for the seasons 1969 and 1975.

[138] Russell RA, Leung JM. Devising a cost effective schedule for a baseball league. Operations Research 1994;42:614–25.

This paper discusses the complexity, and constraints, in producing a schedule for a baseball league. Two heuristics are presented to enable a low cost schedule to be found. The Texas Baseball League is used as an example instance, with an improved schedule being produced.

Additionally, Evans [149] in Section 4.7 deals with a scheduling problem for baseball umpires.

4.5. Hockey

[139] Ferland JA, Fleurent C. Computer aided scheduling for a sport league. INFOR 1991;29:14–25.

Describes a support system to help scheduling the National Hockey League, a relaxed tournament with 21 teams. It is divided into two conferences and each conference is divided into two divisions. The schedules are subject to a number of constraints involving aspects such as the places where the games take place, how often teams can play, the minimum time between two games with the same opponents, and the traveling distances. A number of procedures to be used while the schedule is created manually are presented.

[140] Fleurent C, Ferland JA. Allocating games for the NHL using integer programming. Operations Research 1993;41:649–54.

In response to the National Hockey League expanding from 21 teams, the authors devised an integer linear programming formulation for the problem to investigate how the increase in the number of teams would add to the complexity of generating schedules. The paper considers various scenarios (such as 23 or 24 teams and 80, 82, or 84 games). The solution accepted by the NHL managers for a 24-team problem is shown. It was used as the basis for a schedule, with other matches being manually added.

Additionally, Costa [110] in Section 3.4 deals with a hockey scheduling problem.

4.6. Tennis and table tennis

[141] Della Croce F, Tadei R, Asioli P. Scheduling a round robin tennis tournament under courts and players availability constraints. Annals of Operations Research 1999;92:349–61.

A tennis tournament is considered where courts and players are not always available. The objective is to find a schedule in which the number of scheduled matches is maximized. The problem is modeled as a maximum matching problem in a bipartite graph with additional constraints. It is solved by a two-stage heuristic. In the first step pairings are determined which are assigned to courts afterwards using local search.

[142] Knust S. Scheduling non-professional table-tennis leagues. European Journal of Operational Research, 2007, in press.

Timetables for a time-relaxed scheduling problem in a non-professional table-tennis league are calculated, where especially constraints on team availabilities have to be taken into account. The problem is modeled as an integer linear program and a multi-mode resource-constrained project scheduling problem. Based on the second model a heuristic solution algorithm is proposed, which proceeds in two stages using local search and genetic algorithms.

[143] Schönberger J, Mattfeld DC, Kopfer H. Memetic algorithm timetabling for non-commercial sport leagues. European Journal of Operational Research 2004;153:102–16.

Timetables for a time-relaxed scheduling problem in a non-professional table-tennis league are constructed by memetic algorithms and constraint programming.

Additionally, Farmer et al. [151] in Section 4.7 study the scheduling of tennis umpires.

4.7. Referee assignment

The assignment of officials (or crews of officials) to the already scheduled games of a competition is possibly the mostly studied scheduling problem in sports after that of game scheduling. Applications to professional leagues of sports such as baseball, cricket, football, and tennis appear in the literature. Fairness issues, such as avoiding frequent assignments of the same referee to games of the same team, are among the most important constraints and motivations for these applications.

The referee assignment problem is also common to many amateur leagues of different sports such as soccer, baseball, and basketball. In this problem, a limited number of referees with different qualifications and availabilities should be assigned to a set of games already scheduled. Typical applications involve leagues where hundreds of referees should be assigned to hundreds of games taking place in a short period of time such as a weekend. The assignment of judges in some non-sporting competitions has some features that are common to referee assignment in sport leagues.

[144] Dinitz JH, Stinson DR. On assigning referees to tournament schedules. Bulletin of the Institute of Combinatorics and its Applications 2005;44:22–8.

It is shown how assigning referees to tournament schedules can be done by using so-called room squares known from combinatorial designs.

[145] Duarte AR, Ribeiro CC. Referee assignment in sports leagues: approximate and exact multi-objective approaches. In: 19th international conference on multiple criteria decision making, Auckland, 2008. p. 58–60.

The bi-objective referee assignment problem consists in assigning referees to all games of a tournament scheduled to a given time interval, minimizing the sum over all referees of the absolute value of the difference between the target and the actual number of games assigned to each referee and the sum over all referees of the idle times between consecutive games assigned to the same referee. A heuristic based on extensions of the three-phase strategy proposed in Duart et al. [146] originally developed for the single objective problem version is proposed to find good approximations of the Pareto set for real-size instances. Numerical results show that the biobjective heuristic gives a good approximation of the exact Pareto frontier.

[146] Duarte AR, Ribeiro CC, Urrutia S. A hybrid ILS heuristic to the referee assignment problem with an embedded MIP strategy. In: Burke EK, Rudová H, editors. Hybrid metaheuristics. Lecture notes in computer science, vol. 4771. Berlin: Springer; 2007. p. 82–95.

This paper extends and improves a previous three-phase approach for this problem proposed by Duart et al. [147], based on a constructive heuristic, a repair heuristic to make the initial solutions feasible, and an iterated local search improvement heuristic. A new construction algorithm based on a greedy criterion is proposed to build low-cost initial solutions. Also proposed is an innovative hybridization strategy, in which a mixed integer programming exact algorithm replaces the original neighborhood-based local search within the ILS heuristic. The use of time-to-target-solution-value plots is emphasized in the evaluation of the numerical results obtained for large realistic instances, illustrating the efficiency and the robustness of the new approach. The proposed hybridization of mixed integer programming with local search can be extended to other metaheuristics and applications, opening a new research avenue to more robust algorithms.

[147] Duarte AR, Ribeiro CC, Urrutia S, Haeusler EH. Referee assignment in sports leagues. In: Burke EK, Rudová H, editors. Practice and theory of automated timetabling VI. Lecture notes in computer science, vol. 3867. Berlin: Springer; 2007. p. 158–73.

This paper introduces a novel problem in sports management, in which a limited number of referees with different qualifications and availabilities should be assigned to a set of games already scheduled. The authors describe a number of rules and objectives that should be taken into account when referees are assigned to games. The problem has applications in many amateur sport leagues of different sports. A basic problem variant common to leagues of sports such as soccer, baseball, and basketball is addressed. It is formulated by integer programming and its decision version is proved to be NP-complete. To tackle real-life large instances of the referee assignment problem, the authors developed a three-phase heuristic approach based on a constructive procedure, a repair heuristic to make solutions feasible, and a local search heuristic to improve feasible solutions. Numerical results on realistic instances with up to 500 games and 875 referees are given, illustrating in particular the effectiveness of the construction and repair procedures and the importance of starting from good initial solutions.

[148] Ernst AT, Jiang H, Krishnamoorthy M, Owens B, Sier D. An annotated bibliography of personnel scheduling and rostering. Annals of Operations Research 2004;127:21–144.

This paper focusses on personnel scheduling and does not explicitly mention sports scheduling. However, it contains references to over 700 papers, many of which could be applicable to areas such as referee assignment or other personnel scheduling problems related to sports. One section of the paper also refers to venue management, which might also have relevance to sports scheduling.

[149] Evans JR. A microcomputer-based decision support system for scheduling umpires in the American Baseball League. Interfaces 1988;18:42–51.

Umpire scheduling in the American Baseball League is a complex, multicriteria problem, suited for an interactive decision support system that incorporates optimization techniques, heuristic rules, and human judgement. In assigning officials to games, one objective typically is to minimize total travel cost while satisfying a set of frequency-oriented constraints. These constraints limit the number of times an official or crew of officials is exposed to each team, balance home and away game exposures, and balance exposures to teams over the course of a season. The American Baseball League is composed of 14 professional teams. The season lasts about 26 weeks and each team plays 168 games.

[150] Evans JR, Hebert JE, Deckro RF. Play ball—the scheduling of sports officials. Perspectives in Computing 1984;4:18–29.

Considers the assignment of umpires for the American League of Professional Baseball Clubs and the for the Atlantic Coast Conference (football and basketball). These two are examples of fixed-crew and variable-crew, fixed-position scheduling. In the first case an official is assigned to a crew and that crew remains intact for the entire season. The crews are then assigned to games. In the second case, an individual always works in the same position but the crew changes from one game to another. The assignment algorithms are described.

[151] Farmer A, Smith JS, Miller LT. Scheduling umpire crews for professional tennis tournaments. Interfaces 2007;37:187–96.

Professional tennis organizations, such as the United States Tennis Association (USTA), the Association of Tennis Professionals (ATP), the International Tennis Federation (ITF), and the Women's Tennis Association (WTA), host tennis tournaments throughout the world. At these tournaments, chief umpires assign and schedule line umpires for every match. This task is performed manually for most tournaments. For large tournaments, such as the US Open, they can use software developed to facilitate scheduling, which often creates suboptimal or infeasible schedules that must be manually adjusted. The authors developed a program based on optimization that automates the scheduling procedure. They claim that their program consistently provides high-quality schedules in as little as 25% of the time taken with other methods. The problem is formulated by integer programming. The objective includes components for which the user can use a subjective weighting scheme to set priorities. The user can thus assign umpires with lower skill ratings to some lines for training purposes instead of umpires with better skill ratings. Since all feasible schedules may have gender shortages, an arbitrarily high penalty factor is used in the objective function to heavily penalize shortages.

[152] Lamghari A, Ferland JA. Heuristic techniques to assign judges in competitions. In: Prahlad V, Tan WW, Loh AP, editors. Proceedings of the 3rd international conference on computational intelligence, robotics and autonomous systems, Singapore, 2005.

Heuristic techniques are introduced to assign the judges for the John Molson International Case Competition. This judge assignment problem can be seen as that of forming maximally diverse groups, which consists in partitioning a number of entities into a fixed number of groups having the same size and maximizing diversity within groups. The complexity of the integer programming mathematical formulation accounting for the rules to be followed in assigning the judges lead to the use of heuristic techniques to solve the problem. Three different techniques are introduced and compared numerically.

[153] Lamghari A, Ferland JA. Structured neighborhood tabu search for assigning judges to competitions. In: Proceedings of IEEE symposium on computational intelligence in scheduling, Honolulu. IEEE; 2007. p. 238–45.

A metaheuristic approach including three different stages is introduced to assign the judges for the John Molson International Case Competition. The complexity of the mathematical formulation accounting for the rules to be followed in assigning the judges inspired the use of heuristics. Two different tabu search methods in the first two stages are combined with a diversification strategy. Numerical results on problems with up to 500 individual competitions and

2530 judges are provided to illustrate the efficiency of the approach to generate good solutions.

[154] Ordonez R, Knowles TW. Solving the American League umpire crew scheduling problem as a constraint satisfaction problem. In: Proceedings of the 29th annual meeting of the decision sciences institute, vol. 2, Las Vegas, 1998. p. 1058–61.

This scheduling problem involves assigning seven crews of the American Baseball League to officiate games over an 8-series schedule segment. The schedule must satisfy a number of travel rules and restrictions, while at the same time meeting a number of goals. The associated integer programming set partitioning problem is reformulated as a constraint satisfaction problem and solved by the ILOG Solver.

[155] Wright MB. Scheduling English cricket umpires. Journal of the Operational Research Society 1991;42:447–52.

This paper concerns a computer system which was devised to schedule umpires for the major cricket games played in England. The Test and County Cricket Board needs to allocate umpires to matches every year, in such a way as to satisfy various constraints and meet various objectives. Every cricket match requires two umpires. A number of professional umpires are employed full-time to cover major cricket matches. In addition, there are a few other umpires who are used occasionally. Results are reported for the 1990 season, involving 614 matches. Of these 478 required two topclass umpires, while the other 136 required only one, hence the total number of allocations to be made was 1092. These games were not all of the same length: their duration was 1, 3, 4, or 5 days. There were 26 full-time umpires involved and five reserves, who could be used on a prespecified number of occasions for particular types of games. An outline of the solution method is given, rather than exact details of the algorithms used.

[156] Yavuz M, Inan UH, Figlali A. Fair referee assignments for professional football leagues. Computers & Operations Research 2008;35:2937–51.

This paper presents an integer programming model for the fair assignment of referees to football games, avoiding frequent assignments of the same referee to games of the same team. A constructive heuristic and a local search procedure are developed for its solution.

4.8. Miscellaneous

In this section, we review some applications of optimization methods to other miscellaneous problems in sports.

[157] Lucena CJP, Noronha TF, Ribeiro CC, Urrutia S. A multi-agent framework to build integer programming applications to playoff elimination in sports tournaments. International Transactions in Operational Research 2008;15:739–53.

Describes a multi-agent framework for developing autonomous applications of integer programming to playoff qualification and elimination problems in sports tournaments. These applications involve collecting results from several sources, processing them, and publishing a report on the situation of each team taking part in the competition, regarding qualification and elimination statistics. An instance of this framework was created to follow the Brazilian national football tournament, showing very good results in practice in terms of ease of use; speed of update, processing and publication; and attractiveness to the users.

[158] Nowak M, Epelman M, Pollock SM. Assignment of swimmers to dual meet events. Computers & Operations Research 2006;33:1951–62.

Describes a problem where a swimming coach has to assign swimmers across a number of swim meets, with the aim of maximizing the number of points won by the team. It is essentially a rostering problem. A binary integer model is presented to capture the variety of constraints that have to be respected. The model was tested on a real world problem instance consisting of 17 swimmers and 11 events. CPLEX found solutions in approximately 3.5 s.

[159] Ribeiro CC, Urrutia S. OR on the ball: applications in sports scheduling and management. OR/MS Today 2004;31:50–4.

Two major sporting competitions—Major League Baseball in the United States and the national soccer championship in Brazil—are season-long tournaments divided in two stages. In the first stage, classification for the playoffs is determined. Baseball fans in the United States are much like soccer fans in Brazil: they love statistics almost as much as they love the game itself. The paper describes the use of two integer programming models developed to detect in advance when a teal is qualified for, or eliminated from, the playoffs.

[160] Ribeiro CC, Urrutia S. An application of integer programming to playoff elimination in football championships. International Transactions in Operational Research 2005;12:375–86.

Football is the most followed and practiced sport in Brazil. Thousands of jobs depend directly from the activity of the football teams. Teams which are not qualified to the playoffs of the national championship loose a lot of money and sometimes are even forced to dismantle their structure due to the lack of resources. Two integer programming models are presented. Applied together, they are able to detect in advance when a team is already qualified to, or eliminated from, the playoffs. Both problems are NP-hard. Necessary conditions for qualification are also established. Results from these models can be used not only to guide teams and fans, but are also very useful to identify and correct wrong statements made by the press and team administrators.

[161] Russell T, van Beek P. Mathematically clinching a playoff spot in the NHL and the effect of scoring systems. In: Advances in artificial intelligence. Lecture notes in computer science, vol. 5032. Berlin: Springer; 2008. p. 234–45.

The problem of determining when a National Hockey League team has clinched a playoff spot is considered. An exact and fast approach based on constraint programming is proposed. Dominance constraints and special-purpose propagation algorithms are introduced. The approach was experimentally evaluated on two seasons of the NHL. It could show qualification before the results posted in the Globe and Mail, a widely read newspaper which uses a heuristic approach. Each instance was solved within seconds. The solver was used to examine the effect of scoring models on elimination dates.

[162] Scarf P, Yusof MM, Bilbao M. A numerical study of designs for sporting contests. European Journal of Operational Research 2009;198:190–8.

This paper proposes a number of tournament metrics that can be used to measure the success of a sporting contest or tournament, and describe how these metrics may be evaluated for a particular tournament design. Knowledge of these measures can then be used to compare competing designs, such as round-robin, pure knockout, and hybrids of these designs. It is shown, for example, how the design of the tournament influences the outcome uncertainty of the tournament and the number of unimportant matches within the tournament. The implications of these designs may be explored within a modeling paradigm.

5. Concluding remarks

This annotated bibliography references over 160 papers, with the earliest being published in 1968.

In the following tables we provide a brief analysis of the papers that we reference in this paper. Table 1 details how many papers have been published each year. Table 2 provides an analysis of where the papers have been published. The *Others* heading categorizes publications such as books, book chapters, and conference proceedings.

There has been a steady increase in sports scheduling articles in recent years. This is due not only to the intrinsic computational

Table 1
Analysis of cited papers by year.

Year	Publications
1968	1
1969	0
1970	0
1971	1
1972	0
1973	0
1974	1
1975	0
1976	2
1977	4
1978	0
1979	0
1980	4
1981	1
1982	3
1983	0
1984	1
1985	3
1986	0
1987	0
1988	4
1989	2
1990	1
1991	2
1992	2
1993	3
1994	4
1995	2
1996	2
1997	3
1998	5
1999	4
2000	3
2001	9
2002	1
2003	11
2004	8
2005	10
2006	17
2007	19
2008	17
2009	12
Total	162

Table 2
Analysis of cited papers by journal.

Journal	Publications
Others	37
Lecture Notes in Computer Science	15
European Journal of Operational Research	14
Discrete Applied Mathematics	13
Computers & Operations Research	9
Journal of the Operational Research Society	8
Ars Combinatoria	5
Bulletin of the Institute of Combinatorics and its Applications	5
Interfaces	5
Operations Research Letters	5
Operations Research	4
Discrete Optimization	4
Annals of Operations Research	2
Congressus Numerantium	2
Discrete Mathematics	2
INFOR	2
International Transactions in Operational Research	2
Journal of Combinatorial Mathematics and Combinatorial Computing	2
Journal of Scheduling	2
Management Science in Sports	2
OR/MS Today	2
AIIE Transactions	1
American Mathematical Monthly	1
Australasian Journal of Combinatorics	1
Biometrika	1
Constraints	1
DIMACS Series in Discrete Mathematics and Theoretical Computer Science	1
IEEE Intelligent Systems	1
IEEE Transactions on Systems Science and Cybernetics	1
IMA Journal of Management Mathematics	1
INFORMS Transactions on Education	1
Journal of Combinatorial Theory	1
Journal of Graph Theory	1
Journal of Heuristics	1
Lecture Notes in Artificial Intelligence	1
Lecture Notes in Economics and Mathematical Systems	1
Mathematical Programming Study	1
Networks	1
Omega	1
Pacific Journal of Optimization	1
Perspectives in Computing	1
Total	162

difficulty of the problems in the area and to their challenging nature, but is also motivated by a large number of innovative applications in practice. In fact, sporting events continue to generate a lot of public interest, with fairness and technical criteria to be respected and logistic issues to be optimized.

For researchers and practitioners who are interested in this area, the articles that are discussed in this work provide access to all the important contributions over the past 40 years. They may also provide inspiration for teachers of Operations Research courses, who may be able to use successful case studies to motivate students and to illustrate in classroom the use of different formulation tools and solution methods (see Trick [10] in Section 1).

The hardness of the optimization problems in sports scheduling has led to the use of different techniques in their solution: decomposition strategies, integer programming, column generation, Benders cuts, constraint programming, heuristic search, metaheuristics, and their hybrids. The best results are often obtained by methods derived from the hybridization of integer programming, constraint programming and metaheuristics. Optimal solutions can be found by exact methods for some medium-size problems in professional leagues.

Interest by problems in this area has risen, in particular, since the introduction of the traveling tournament problem and its benchmark instances in the seminal paper by Easton et al. [4], discussed in Section 2.8. TTP benchmark instances and real-life instances are intrinsically difficult to solve: for many years, the largest TTP instance exactly solved to optimality had only six teams. Only recently, in June 2008, the 8-team NL8 benchmark TTP instance was solved to optimality by an intricate branch-and-price algorithm described in Irnich [70] (see Section 2.8).

The hardness of optimization problems in sports scheduling is also driving research on new algorithmic directions for clusters and computational grids, as outlined by Araújo et al. [62] (see Section 2.8). A hierarchical distributed implementation of cooperative metaheuristics such as GRASP and ILS on a medium-size grid lead to improved results for several TTP benchmark issues. However, in spite of its hardness, it is noticeable that the decision version of the traveling tournament problem is not proved to be NP-complete at the time of writing.

The literature reports real-life applications of optimization methods to the scheduling of baseball, basketball, and hockey tournaments in the North America, as well as to soccer leagues in Austria, Chile, Denmark, England, Germany and Japan, to volleyball tournaments in Argentina and The Netherlands, and to the rugby World Cup. The Brazilian Soccer Confederation also used an optimized fixture constructed by a decision support system powered by an integer programming model for the 2009 edition of its annual tournament.