Optimal Solutions for Fault-Tolerant Topology Control in Wireless Ad Hoc Networks

Renato E. N. Moraes, Celso C. Ribeiro and Christophe Duhamel

***Abstract*—Topology control is one of the most important techniques used in wireless ad hoc and sensor networks to reduce energy consumption. Algorithms for topology control attempt to reduce the number of links and the power consumption in a network subject to connectivity constraints. We show that the related optimization problems may be classified into four main variants, regarding the topology of the input graph (symmetric or asymmetric) and of the solution (unidirectional or bidirectional). We present three mixed integer programming formulations for the k-connected minimum power consumption problem, which consists in finding a power assignment to the nodes of a wireless network so as that the resulting network topology be k-vertex connected (i.e., k-fault tolerant) and the total power consumption be minimum. These formulations are sufficiently general to encompass all four problem variants. We report computational experiments comparing the formulations. Optimal solutions for moderately sized networks are obtained using a commercial solver.**

***Index Terms*—Wireless networks, ad hoc networks, topology control, k-connectivity, fault tolerance, energy consumption optimization, mixed integer programming.**

I. INTRODUCTION

AN *ad hoc network* consists of a collection of transceivers, in which a packet may have to traverse multiple consecutive wireless links to reach its destination. They have become an increasingly common and important object of study due to their applications in battlefield communication, disaster relief communication, and sensor networks, among others.

Ad hoc networks can be represented by a set V of transceivers (nodes), numbered $0, 1, \ldots, |V|-1$, together with their locations or the distances between them. A transmission power p_u is associated with each node $u \in V$. For each ordered pair (u, v) of transceivers, with $u, v \in V$, we are given a non-negative arc weight $e(u, v)$ such that a signal transmitted by the transceiver u can be received at node v if and only if the transmission power of u is at least equal to $e(u, v)$, i.e. if $p_u \geq e(u, v)$.

Wireless networks face a variety of constraints that do not appear in wired networks. Nodes in a wireless network are typically battery-powered, and it is expensive and sometimes even infeasible to recharge the device. We focus on radio power consumption, since radios tend to be the major source of power dissipation in wireless networks [1]. Instead of transmitting with maximum power, the topology control algorithm adjusts the transmission power of each node.

There are also increasing fault-tolerance requirements, due to the evolving critical application domains and to the large number of failures that may result from mobility, fading or obstructions [2]. A connected graph is usually assumed as the minimum connectivity requirement by the algorithms running in different layers of the network, such as routing protocols [3]. However, if there is only one path between a pair of nodes, failure of a single node (or link) between them will result in a disconnected graph. Therefore, topologies with multiple, alternative disjoint paths between any pair of nodes are often required [4].

The *transmission graph* $G(p) = (V, E(p))$, where $E(p) = \{(u, v) : u \in V, v \in V, p_u \geq e(u, v)\}$ is said to be k-vertex connected if for any two nodes $u, v \in V$ there exist k vertex-disjoint paths connecting u to v. In other words, the graph $G(p)$ is k-vertex connected if it remains connected after the removal of any subset of up to $k-1$ vertices. Since a k-vertex connected graph is also k-edge connected, but the converse is not necessarily true, we say that a graph is k-connected if it is k-vertex connected.

Given the node set V, non-negative arc weights $e(u, v)$ for any $u, v \in V$, and a parameter $k \geq 1$, the *k-connected minimum power consumption problem* consists in finding an optimal assignment of transmission powers $p : V \rightarrow R+$ to every node $u \in V$, such that the total power consumption $\sum_{u \in V} p_u$ is minimized and the resulting transmission graph $G(p) = (V, E(p))$ is k-connected. This problem was proved to be NP-hard for $k = 1$ in [5]. Calinescu and Wan [6] established its NP-hardness for $k = 2$. Since the *minimum cost k-connected spanning subgraph problem* is known to be NP-hard even for $k = 2$ [7], the k-connected minimum power consumption problem is conjectured to be NP-hard as well [8] for any positive integer k.

Only one specific variant of the k-connected minimum power consumption problem has been tackled to date by exact integer programming approaches, and this for the particular case where $k = 1$ (i.e., only a connected graph is required) [9, 10, 11]. In this work, we present mixed integer programming formulations that apply to all variants of the problem and to every value of the connectivity parameter k. The paper is organized as follows. The system model is described in detail in the next section. Previous work is reviewed in Section III, in which we show that problem variants can be organized into four different categories, regarding the topologies of the input graph (symmetric or asymmetric) and

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of the solution (unidirectional or bidirectional). Three mixed integer programming formulations are proposed in Section IV. Comprehensive computational results illustrating the behavior of the integer programming formulations are reported and discussed in Section V. Concluding remarks are made in the last section.

II. System Model

We are given a set V of transceivers, with $|V| = n$, equipped with an omnidirectional antenna which is responsible for sending and receiving signals. An ad hoc network is established by assigning a transmission power p_u to each transceiver $u \in V$.

Each node can (possibly dynamically) adjust its transmitting power, based on the distance to the receiving nodes and on the background noise. In the most common power attenuation model [12], the signal power falls with $1/d^{\varepsilon}$, where d is the distance from the transmitter and ε is the path loss exponent (typical values of ε are between 2 and 4). Under this model, the power requirement at node u for supporting the transmission through a link from u to v is given by

$$p_u \geq d_{uv}^{\varepsilon} . q_v, \tag{1}$$

where d_{uv} is the Euclidean distance between the transmitter u and the receiver v, and q_v is the receiver's power threshold for signal detection, which is usually normalized to 1.

We first define the *symmetric input* version of the k-connected minimum power consumption problem. In this case, we assume that the power requirement (also referred to as the weight of arc (u, v)) for supporting a transmission between nodes u and v separated by a distance d_{uv} becomes $e(u, v) = e(v, u) = d_{uv}^{\varepsilon}$. Although the symmetric version is widely accepted as reasonable, equation (1) holds only for free-space environments with non-obstructed lines of sight. It does not consider the possible occurrence of reflections, scattering, and diffraction caused e.g. by buildings and terrains.

In practice, power requirement values for two nodes u and v may be asymmetric because of many reasons. For example, asymmetric arc weights can be used to model batteries with different power levels [13] and heterogeneous nodes [14]. Also, the ambient noise levels of the regions containing the two nodes may be different [15]. Therefore, we also study the more general *asymmetric input* version of the k-connected minimum power consumption problem. Under this model, there may be pairs of transceivers $u, v \in V$ such that $e(u, v) \neq e(v, u)$.

Communication from node u to node v will be enabled whenever $p_u \geq e(u, v)$. Therefore, the transmission graph associated with a power assignment p_u to each transceiver $u \in V$ is defined as the direct graph $G(p) = (V, E(p))$, where $E(p) = \{(u, v) : u \in V, v \in V, p_u \geq e(u, v)\}$.

Two different graph topology structures may be used to enforce k-connectedness. In a *unidirectional topology*, all arcs established by the power settings in the transmission graph $G(p) = (V, E(p))$ are considered to enforce the connectivity constraints. In a *bidirectional topology*, the bidirectional edge $[u, v]$ (instead of the unidirectional arc (u, v)) is used as a communication link to enforce k-connectedness if v is within the transmission range of u and u is also within the transmission range of v. In this case, the arc set considered to enforce the connectivity constraints in the transmission graph $G(p) = (V, E(p))$ is restrained to $B(p) = \{(u, v) : u \in V, v \in V, p_u \geq e(u, v), p_v \geq e(v, u)\} \subseteq E(p)$. The bidirectional k-connected minimum power consumption problem is also NP-hard [6, 16, 17, 18].

III. Previous Work

We may consider four versions of the k-connected minimum power consumption problem:

- Symmetric input with unidirectional topology,
- Symmetric input with bidirectional topology,
- Asymmetric input with unidirectional topology, and
- Asymmetric input with bidirectional topology.

A. Symmetric Input with Unidirectional Topology

The symmetric version of the minimization of power consumption while establishing a unidirectional 1-connected transmission graph ($k = 1$) was proved to be NP-hard by Chen and Huang [5], who presented a 2-approximation algorithm based on minimum spanning trees. Kirousis et al. [19] gave an $O(n^4)$ dynamic programming algorithm for the case where the nodes are co-linear, proved that the problem is NP-hard in the three-dimensional Euclidean space, and described the same 2-approximation algorithm based on minimum spanning trees also presented in [5]. Clementi et al. [20] gave a reduction proving that the same problem is also NP-hard in the two-dimensional Euclidean space.

Calinescu and Wan [6] discussed algorithms for the symmetric input with unidirectional topology version of the bi-connected ($k = 2$) minimum power consumption problem and established its NP-hardness. They also described a 4-approximation algorithm for the problem.

Shpungin and Segal [21] addressed the symmetric input with unidirectional topology version of the k-connected minimum power consumption problem in wireless ad-hoc networks. They presented an exact solution method for radio networks with uniformly spaced nodes on a line and provided fast constant factor approximation algorithms for the more general case of linear networks. They also gave an $O(k^2)$-approximation algorithm for the planar case. Carmi et al. [22] presented a polynomial-time $O(k)$-approximation algorithm based on minimum spanning trees for the two dimensional instance of the same k-connected problem version.

B. Symmetric Input with Bidirectional Topology

Although implementing wireless unidirectional links is technically feasible [23], and imposing the requirement of symmetry incurs in a considerable additional cost, the advantage of using unidirectional links is questionable. There is a potential for packet loss and error in realistic networks, and thus acknowledgments and retransmissions are required [5]. Therefore, to improve the network performance, link bidirectionality is implicitly assumed in many routing protocols [24]. Marina and Das [25] have shown that the overhead needed to handle unidirectional links in routing protocols outweighs the benefits that they can provide, and that better performance can be achieved by simply avoiding them.

The minimum power consumption problem with bidirectional 1-connected subgraph ($k = 1$) from symmetric inputs was proposed in [16, 17], where it is proved to be NP-complete. Blough et al. [16] gave asymptotic bounds on the solution cost for random instances and for the so called (Δ, δ) Euclidean instances. Cheng et al. [18] showed the importance of the problem in the case of sensor networks, proved its NP-completeness, and proposed two approximate algorithms.

The 2-approximation algorithm in [19] solves the symmetric version of the minimum power consumption problem with bidirectional 1-connectivity. Calinescu et al. [17] pushed the approximation ratio of the latter to below 2 by exploiting similarities with the minimum Steiner tree problem. In particular, they gave a fully polynomial $7/4 + \epsilon$ approximation scheme and a more practical 15/8 approximation. These approximation factors have been improved by Althaus et al. [9] to $5/3 + \epsilon$ and 11/6, respectively. They also gave an exact branch-and-cut algorithm based on a new integer programming formulation. Another exact algorithm was presented in [10]. Das et al. [11] developed a mixed integer programming model for the problem with sectored antennas, presented a centralized heuristic based on Kruskal's algorithm for the minimum spanning tree problem, and discussed a simple branch exchange heuristic to improve the topology generated by the Kruskal-like algorithm.

Lloyd et al. [26] studied the symmetric input with bidirectional topology version of the biconnected minimum power consumption problem ($k = 2$). They gave an algorithm with approximation ratio of at most $2(2 - 2/n)(2 + 1/n)$. Calinescu and Wan [6] proved it NP-hardness and developed a 4-approximation algorithm.

For the case of general values of k, algorithms with an approximation factor of $O(k)$ for the symmetric input with bidirectional k-connected minimum power consumption problem were presented in [27]. This approximation factor was improved from $O(k)$ to $O(\log^4 n)$ in [28]. Jia et al. [8] presented, among others results, a $3k$-approximation algorithm for $k \geq 3$ and a 6-approximation for $k = 3$. Das and Mesbahi [29] proposed a heuristic procedure applying an algebraic view of graph connectivity, defined as the second smallest eigenvalue of the Laplacian matrix of a graph.

C. Asymmetric Input with Unidirectional Topology

While the symmetric version of the k-connected minimum power consumption problem has received significant attention in recent years, only a few approximation algorithms have been proposed for the case with asymmetric power requirements.

Krumke et al. [15] considered the asymmetric version of the unidirectional 1-connected minimum power consumption problem ($k = 1$). They showed that an $\Omega(\log n)$-approximation algorithm cannot exist unless $P = NP$ and presented an $O(\log n)$-approximation algorithm. Independently, Calinescu et al. [13] achieved a similar approximation bound by an algorithm which incrementally constructs a tree. Caragiannis et al. [30] also obtained an $O(\log n)$-approximation algorithm.

Wang et al. [31] presented an approximation algorithm for the the asymmetric input with bidirectional topology version of the k-connected minimum power consumption problem. The algorithm has an approximation factor of $O(k + \Delta^-)$, where Δ^- is the maximum out-degree of a minimum power k-outconnected subgraph, i.e., a subgraph with k node disjoint paths from the root node r to every other node.

D. Asymmetric Input with Bidirectional Topology

Althaus et al. [9] obtained an inapproximability result within a factor of $O(\log n)$ for the asymmetric version of the bidirectional 1-connected minimum power consumption problem ($k = 1$). Caragiannis et al. [30] developed an $O(1.35 \ln n)$-approximation algorithm for the same problem. A slightly inferior $O(\ln n)$-approximation algorithm has been independently obtained in [13] by different techniques.

There seems to be no further results for the asymmetric version of the k-connected minimum power consumption problem with bidirectional and unidirectional topology for $k \geq 2$.

The algorithms discussed until now are centralized approaches, mainly designed to static ad hoc wireless networks. Their major advantage is the fact that they have provable approximation factors. Distributed algorithms for energy-efficient power assignments can be found in [32, 33, 34, 35, 36]. Non-centralized algorithms have the clear advantage of being localized. However, the power consumption assignments of the resulting solutions can be arbitrarily worse than those of the optimal solutions [27].

In the following, we present three mixed integer programming formulations for the four variants of the k-connected minimum power consumption problem, together with computational results obtained with a commercial integer programming solver. The more interesting fault-tolerant case in practice, corresponding to $k = 2$, is investigated in more detail.

IV. Integer Programming Formulations

In this section, we give three mixed-integer programming (MIP) formulations based on multicommodity flows for the k-connected minimum power consumption problem, where k is the required number of node-disjoint paths between any pair of vertices. The reader is also referred to [37] for similar formulations for a class of single commodity network design optimization problems.

A. Continuous Power Model

A simple, naive way to formulate the k-connected minimum power consumption problem consists in defining k commodities with a unit demand which have to be sent from each of the $|V|$ nodes to every one of the remaining $|V| - 1$ nodes. Such a formulation would therefore involve $k|V|(|V|-1)$ commodities and would be very large. However, Raghavan [38] has shown in the context of the network design problem with connectivity requirements [39], that a

$$\min \sum_{i \in V} p_i \tag{2}$$

subject to:

$$\sum_{j \in V} f_{ji}^c - \sum_{l \in V} f_{il}^c = D_c(i), \qquad \forall c \in C, \forall i \in V \tag{3}$$

$$\sum_{j \in V} f_{ij}^c \leq 1, \qquad \forall c \in C, \forall i \in V : i \neq o(c), i \neq d(c) \tag{4}$$

$$p_i \geq e(i,j) \cdot f_{ij}^c, \qquad \forall i, j \in V, \forall c \in C \tag{5}$$

$$f_{ij}^c \in \{0, 1\}, \qquad \forall i, j \in V, \forall c \in C \tag{6}$$

$$p_i \geq 0, \qquad \forall i \in V \tag{7}$$

Fig. 1. Continuous power model (CP).

more compact model can be formulated using a k-connected undirected requirement graph $G^k = (V, E^k)$ with a minimum number $|E^k| = \lceil k|V|/2 \rceil$ of edges [40] built as follows:

- If k is even, there is an edge $[i, j]$ in E^k for $i, j \in V$ whenever $(i - j) \mod |V| \leq k/2$.
- If k is odd and $|V|$ is even, first build graph G^{k-1}. Next, obtain E^k from E^{k-1} by adding to the latter, edges $[i, i + |V|/2]$ for $i = 0, \ldots, |V|/2$.
- Otherwise, build graph G^{k-1} and obtain E^k from E^{k-1} by adding to the latter, edges $[0, (|V|-1)/2)]$, $[0, (|V| + 1)/2]$, and $[i, i + (|V| + 1)/2]$ for $i = 1, \ldots, (|V| - 1)/2$.

The set C of commodities is built as follows. Let $[i, j]$ be any edge in E^k. If the problem calls for an unidirectional topology, then create one commodity from node i to j and another from node j to i, both with a demand of k units. Otherwise, create only one commodity between nodes i and j with a demand of k units, arbitrarily choosing any of them as the origin and the other as the destination. This procedure entails a multicommodity flow model for the k-connected minimum power consumption problem with unidirectional topology using only $2 \cdot \lceil k|V|/2 \rceil$ commodities, which is smaller than $k|V|(|V| - 1)$. The bidirectional solution uses half the number of commodities as the unidirectional case.

For each commodity $c \in C$, we represent by $o(c)$ its origin and by $d(c)$ its destination. For any node $i \in V$ and any commodity $c \in C$, let $D_c(i) = -k$ if $i = o(c)$, $D_c(i) = +k$ if $i = d(c)$, $D_c(i) = 0$ otherwise. The discrete variable f_{ij}^c and the continuous variable p_i represent, respectively, the flow of commodity c through arc (i, j) and the power assignment to node i. The binary variable f_{ij}^c is equal to one if the arc (i, j) is used by commodity c for communication from node i to j, zero otherwise.

The mixed integer program CP defined by the objective function (2) and constraints (3)-(7) presented in Figure 1 is a valid formulation for the unidirectional topology case for both the symmetric (i.e. $e(u, v) = e(v, u)$) and asymmetric ($e(u, v)$ not necessarily equal to $e(v, u)$) versions of the k-connected minimum power consumption problem. Constraints (3) are the flow conservation equations. Inequalities (4) ensure node-disjointness. Inequalities (5) state that arc (i, j) should be used if there is a positive flow through it. If arc (i, j) is used, then

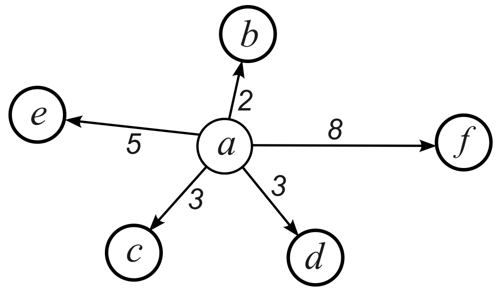


Fig. 2. Formulation DP: $P_a = [2, 3, 5, 8]$ and $S_a^1 = \{b\}$, $S_a^2 = \{b, c, d\}$, $S_a^3 = \{b, c, d, e\}$, $S_a^4 = \{b, c, d, e, f\}$; formulation IP: $Q_a = [2, 1, 2, 3]$ and $T_a^1 = \{b\}$, $T_a^2 = \{c, d\}$, $T_a^3 = \{e\}$, $T_a^4 = \{f\}$.

the power p_i assigned to node i should be at least as large as the requirement $e(i, j)$. Constraints (6) and (7) express the integrality and non-negativeness requirements on the variables.

Whenever a bidirectional topology is sought, constraints

$$p_i \geq e(i,j) \cdot f_{ji}^c, \qquad \forall i, j \in V, \forall c \in C \tag{8}$$

are added, ensuring that an edge $[i, j]$ is used if there is flow from i to j or from j to i.

B. Discrete Power Model

Let $P_i = [p_i^1, \ldots, p_i^{\phi(i)}]$ be a finite list of increasing power levels that can be assigned to node $i \in V$. We denote by p_i^1 the minimum power p_i such that transmissions from node i reach at least one node in $V \setminus \{i\}$. Furthermore, $\phi(i) \leq |V| - 1$ and $p_i^{\ell+1} > p_i^\ell$ for any $\ell = 1, \ldots, \phi(i) - 1$. We define S_i^ℓ as the set of nodes reachable from node i with the power assignment $p_i = p_i^\ell$, for any $\ell = 1, \ldots, \phi(i)$, as illustrated in Figure 2. We remark that $\cup_{\ell=1}^{\ell=\phi(i)} S_i = V \setminus \{i\}$. For ease of notation, we define $S_o = \emptyset$.

The discrete variable f_{ij}^c is defined as before and represents the flow of commodity c through arc (i, j). For any $\ell = 1, \ldots, \phi(i)$, the binary variable w_i^ℓ is equal to one if there is a node $j \in S_i^\ell$ such that arc (i, j) is used for communication from i to j, zero otherwise. We define $\bar{\ell}(i) \in \{1, \ldots, \phi(i)\}$ such as that $|S_i^{\bar{\ell}(i)-1}| < k \leq |S_i^{\bar{\ell}(i)}|$. Then, for any node i, $|S_i^{\bar{\ell}(i)}|$ gives the minimum number of nodes needed to

$$\min \sum_{i \in V} \sum_{\ell=1}^{\phi(i)} p_i^\ell \cdot w_i^\ell \quad (9)$$

subject to:

$$\sum_{j \in V} f_{ji}^c - \sum_{l \in V} f_{il}^c = D_c(i), \qquad \forall c \in C, \forall i \in V \quad (10)$$

$$\sum_{j \in V} f_{ij}^c \leq 1, \qquad \forall c \in C, \forall i \in V : i \neq o(c), i \neq d(c) \quad (11)$$

$$\sum_{\ell=\bar{\ell}(i)}^{\phi(i)} w_i^\ell = 1, \qquad \forall i \in V \quad (12)$$

$$w_i^\ell = 0, \qquad \forall i \in V, \ell = 1, \ldots, \bar{\ell}(i) - 1 \quad (13)$$

$$\sum_{\ell=\max(\ell'(i,j),\bar{\ell}(i))}^{\phi(i)} p_i^\ell \cdot w_i^\ell \geq e(i,j) \cdot f_{ij}^c, \qquad \forall c \in C, \forall i, j \in V \quad (14)$$

$$f_{ij}^c \in \{0,1\}, \qquad \forall i, j \in V, \forall c \in C \quad (15)$$

$$w_i^\ell \in \{0,1\}, \qquad \forall i \in V, \ell = 1, \ldots, \phi(i). \quad (16)$$

Fig. 3. Discrete power model (DP).

establish the k-connectivity requirement from node i. We also define $\ell'(i,j) = 1$ if $p_i^1 = e(i,j)$; $\ell'(i,j) \in \{2, \ldots, \phi(i)\}$ if $p_i^{\ell'(i,j)-1} < e(i,j) \leq p_i^{\ell'(i,j)}$, i.e., $p_i^{\ell'(i,j)}$ is the lowest power level required to setup communication from node i to j.

The mixed integer program DP defined by the objective function (9) and constraints (10)-(16) presented in Figure 3 is also a valid formulation for the unidirectional topology case for both the symmetric and asymmetric versions of the k-connected minimum power consumption problem. Constraints (10), (11), and (14) in this formulation are the same as (3), (4), and (5) in the previous formulation, respectively. A node with a null power assignment cannot transmit or forward any message. Since the transmission graph $G(V, E(p))$ is required to be k-connected, each node must be able to communicate with at least k other nodes. Therefore, the power assigned to each node must be enough to reach at least the k closest nodes to it. Constraints (12) ensure that one single power level is assigned to each node. Furthermore, they establish that this power level is capable to reach at least the k closest nodes. Constraints (13) complement constraints (12), setting to zero the power levels incapable of reaching, at least, the k closest nodes. Since constraints (12) ensure that only one single power level is assigned to each node, inequalities (14) state that only the power levels which are greater than the power requirement $e(i,j)$ are acceptable. Constraints (15) and (16) express the integrality requirements.

This formulation gives an exact solution for the unidirectional topology case. If a bidirectional topology is sought, it suffices to add constraints

$$\sum_{\ell=\max(\ell'(i,j),\bar{\ell}(i))}^{\phi(i)} p_i^\ell \cdot w_i^\ell \geq e(i,j) \cdot f_{ji}^c, \qquad \forall c \in C, \forall i, j \in V \quad (25)$$

ensuring the existence of one arc in each direction.

C. Incremental Power Model

Let $Q_i = [q_i^1, \ldots, q_i^{\phi(i)}]$ be a finite list of successive cumulative increments in the power setting that can be assigned to node i, for any $i \in V$. Furthermore, let T_i^ℓ be the set of new nodes reachable from node i if an additional increment q_i^ℓ is added to its current power assignment. With respect to the notation defined in the previous section, $q_i^1 = p_i^1$, $T_i^1 = S_i^1$, $q_i^\ell = p_i^\ell - p_i^{\ell-1}$ and $T_i^\ell = S_i^\ell - S_i^{\ell-1}$ for any $\ell = 2, \ldots, \phi(i)$, as illustrated in Figure 2.

The discrete variable f_{ij}^c represents the flow of commodity c through arc (i,j). The binary variable x_i^ℓ takes the value one if there is a node $j \in T_i^\ell$ such that (i,j) is used for communication from i to j, zero otherwise.

The mixed integer program IP defined by the objective function (17) and constraints (18)-(24) presented in Figure 4 is also a valid formulation for the unidirectional topology case for both the symmetric and asymmetric versions of the k-connected minimum power consumption problem. Constraints (18), (19), and (23) in this formulation are the same as (3), (4), and (5) in the first formulation, respectively. Inequalities (20) state that x_i^ℓ must be set to one if there is a node $j \in T_i^\ell$ such that arc (i,j) is used for communication from node i to j by commodity c. Constraints (21) enforce $x_i^{\ell+1}$ to be equal to zero if the previous increment was not used, i.e. if $x_i^\ell = 0$. Constraints (22) set to one the incremental

$$\min \sum_{i \in V} \sum_{\ell=1}^{\phi(i)} q_i^\ell \cdot x_i^\ell \tag{17}$$

subject to:

$$\sum_{j \in V} f_{ji}^c - \sum_{l \in V} f_{il}^c = D_c(i), \qquad \forall c \in C, \forall i \in V \tag{18}$$

$$\sum_{j \in V} f_{ij}^c \leq 1, \qquad \forall c \in C, \forall i \in V : i \neq o(c), i \neq d(c) \tag{19}$$

$$x_i^\ell \geq f_{ij}^c, \qquad \forall i \in V, \forall c \in C, \forall j \in T_i^\ell, \ell = 1, \ldots, \phi(i) \tag{20}$$

$$x_i^{\ell+1} \leq x_i^\ell, \qquad \forall i \in V, \ell = 1, \ldots, \phi(i) - 1 \tag{21}$$

$$x_i^\ell = 1, \qquad \forall i \in V, \ell = 1, \ldots, \bar{\ell}(i) \tag{22}$$

$$f_{ij}^c \in \{0,1\}, \qquad \forall i,j \in V, \forall c \in C \tag{23}$$

$$x_i^\ell \in \{0,1\}, \qquad \forall i \in V, \ell = 1, \ldots, \phi(i). \tag{24}$$

Fig. 4. Incremental power model (IP).

powers necessaries to reach at least the k closest nodes of each node i. Constraints (23) and (24) express the integrality requirements.

Whenever a bidirectional topology is sought, it suffices to add constraints

$$x_i^\ell \geq f_{ji}^c, \qquad \forall i \in V, \forall c \in C, \forall j \in T_i^\ell, \ell = 1, \ldots, \phi(i) \tag{26}$$

to ensure the existence of one arc in each direction.

In the above case, we notice that the power assigned to node i must be enough to establish bidirectional links whenever a bidirectional topology solution is sought. Inequalities

$$x_j^m \geq x_i^\ell - x_i^{\ell+1}, \qquad \forall i,j \in V, \ell = 1, \ldots, \phi(i) - 1, \quad m = 1, \ldots, \phi(j) : i \in T_j^m, j \in T_i^\ell, |T_i^\ell| = 1 \tag{27}$$

imply that the transmission power of node i is set to reach node $j \in T_i^\ell$ as the farthest one, i.e. $x_i^\ell = 1$ and $x_i^{\ell+1} = 0$. They imply in the existence of the bidirectional edge $[i,j]$. Therefore, they enforce x_j^m to be set to one, for $i \in T_j^m$.

We also can replace the unidirectional constraints (20) and (26) by the bidirectional constraints

$$x_i^\ell \geq f_{ij}^c + f_{ji}^c, \qquad \forall i \in V, \forall c \in C, \forall j \in T_i^\ell, \ell = 1, \ldots, \phi(i). \tag{28}$$

Every solution satisfying constraints (28) clearly also satisfies (20) and (26). In order to show that the reverse is also valid, we have to prove that both f_{ij}^c and f_{ji}^c cannot be simultaneously equal to one. This cannot be true, because otherwise there would be a cycle of commodity c through nodes i and j.

The IP model extended with the set of inequalities (27) and (28) is referred to as the *incremental power bidirectional model* (IP-B).

V. Computational Results

Computational experiments have been carried out on a set of random moderately sized asymmetric instances with $|V| \in [10, 30]$ nodes uniformly distributed in the unit square grid. The weight of the arc between nodes u and v is set as $e(u,v) = F \cdot d_{u,v}^\varepsilon$, where $d_{u,v}$ is the Euclidean distance between nodes u and v, the path loss exponent ε is set at 2, and $F \in [0.8, 1.2]$ is a random uniform perturbation. Symmetric instances were built from their original asymmetric counterparts by assigning to edge $[u,v]$ the highest of the weights among $e(u,v)$ and $e(v,u)$. All generated instances are represented as complete graphs.

An Intel Core 2 Quad machine with a 2.40 GHz clock and 8 Gbytes of RAM memory running under GNU/Linux 2.6.24 was used in the experiments. ILOG CPLEX 11.0 was used as the MIP solver with parallel features disabled.

A. First Experiment: k-connected Solutions

In the first set of experiments, we compare the computation times observed with the different models for different values of the parameter k ranging from 2 to $|V| - 1$. Tables I and II show the computation times in seconds and the optimal solution values for asymmetric and symmetric instances with 15 nodes, respectively. Tables III and IV show the same results for instances with 20 nodes. All values in these tables are average results over one run of fifteen randomly generated Euclidean instances with the same size.

Instances whose optimal solutions were not found by a given formulation within three hours of computations were discarded. In this case, Tables I to IV display in brackets the number of instances exactly solved and used to calculate the averages for the respective formulation. Formulation CP was the only one which was not able to find optimal solutions within this time limit to both unidirectional and bidirectional topology instances (see Tables I and II). The DP formulation failed to find optimal solutions within this time limit for $k = 9$ and $k = 11$ in the case of unidirectional topologies (see Table III) and for $k = 11$ in the case of unidirectional topologies (see Table IV). Formulations IP and IP-B always found the optimal solution within three hours of computations.

TABLE I
COMPUTATION TIMES (IN SECONDS) AND AVERAGE OPTIMAL SOLUTION VALUES: ASYMMETRIC INPUTS WITH $|V| = 15$.

	Unidirectional topology				Bidirectional topology				
	Time (s)			Optimal	Time (s)				Optimal
k	CP	DP	IP	value	CP	DP	IP	IP-B	value
2	(12) 2909.30	29.72	20.98	1.52	400.55	20.54	10.28	7.55	1.57
3	(7) 5056.50	76.20	34.46	2.30	2137.02	54.14	30.01	17.78	2.40
4	(3) 7640.35	103.22	37.02	3.17	(12) 2471.24	63.72	34.38	17.12	3.27
5	(1) 8564.12	142.30	65.34	4.08	(9) 5309.48	85.43	39.08	27.16	4.22
6	(0) –	109.55	41.06	5.03	(4) 6766.61	81.92	34.02	23.14	5.18
7	(0) –	105.35	38.22	5.99	(7) 3906.37	76.01	35.36	22.08	6.18
8	(0) –	48.28	18.59	6.99	(1) 2871.09	39.46	20.28	12.71	7.20
9	(0) –	19.16	11.17	7.93	(1) 5368.02	34.05	11.05	9.29	8.18
10	(0) –	12.91	7.12	8.87	(2) 8209.58	16.96	7.19	4.70	9.15
11	(1) 5580.70	6.03	2.44	9.91	(4) 5278.28	12.49	4.78	5.46	10.14
12	(3) 6391.45	7.23	1.59	10.91	(9) 4466.20	10.35	1.87	2.05	11.10
13	6.19	1.67	0.59	11.99	4.88	1.36	0.36	0.32	12.10
14	0.23	0.33	0.23	13.06	0.15	0.28	0.16	0.12	13.06

TABLE II
COMPUTATION TIMES (IN SECONDS) AND AVERAGE OPTIMAL SOLUTION VALUES: SYMMETRIC INPUTS WITH $|V| = 15$.

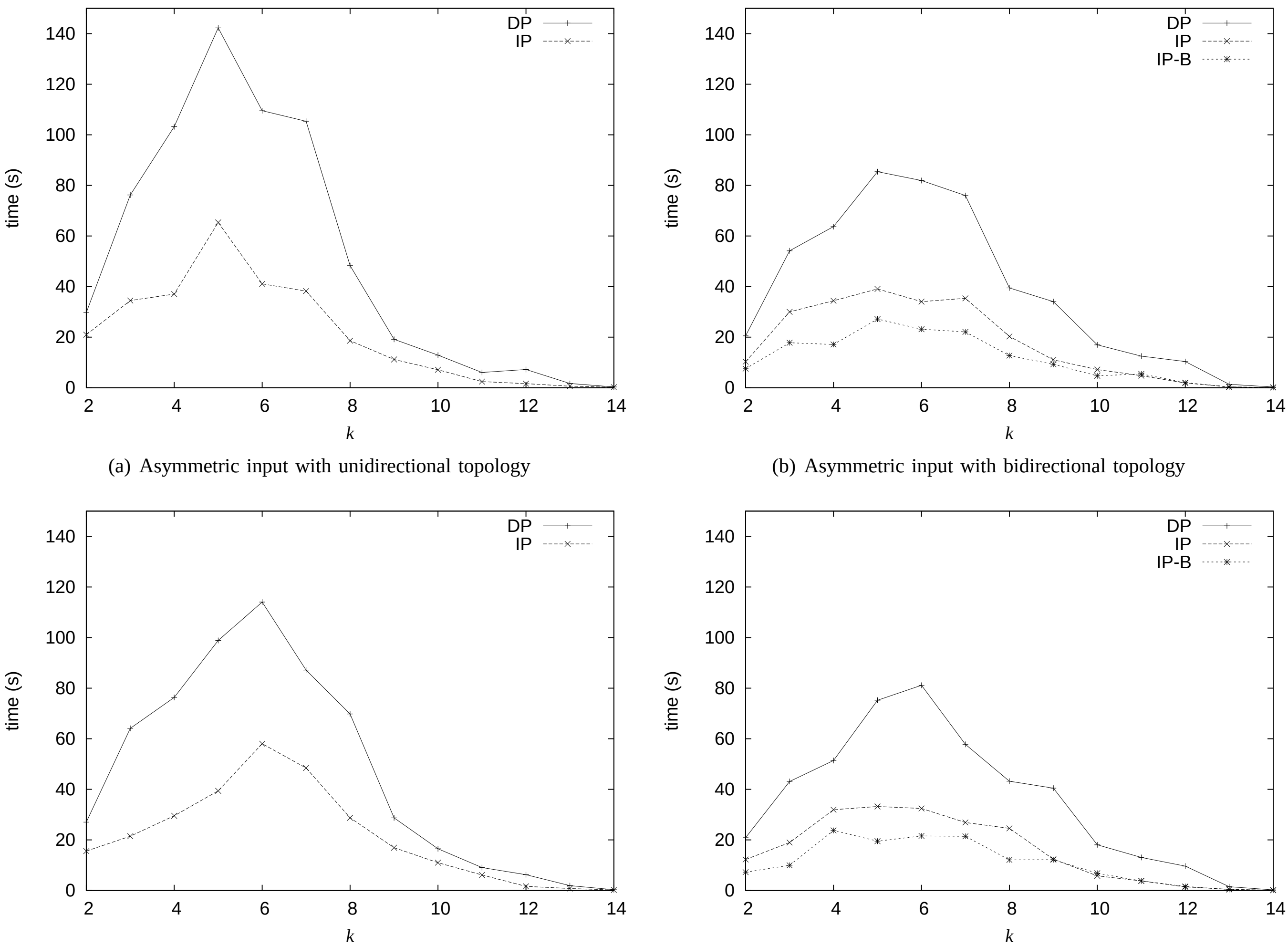
	Unidirectional topology				Bidirectional topology				
	Time (s)			Optimal	Time (s)				Optimal
k	CP	DP	IP	value	CP	DP	IP	IP-B	value
2	(12) 2298.42	27.04	15.57	1.63	410.63	20.88	12.25	7.24	1.67
3	(5) 3052.69	64.11	21.46	2.47	1901.08	43.12	18.95	9.95	2.54
4	(1) 3414.39	76.32	29.53	3.40	(12) 2616.42	51.40	31.94	23.74	3.48
5	(0) –	98.84	39.38	4.40	(7) 4621.02	75.22	33.20	19.51	4.49
6	(0) –	114.04	58.04	5.43	(2) 6286.75	81.15	32.43	21.56	5.50
7	(0) –	87.15	48.43	6.45	(2) 4644.15	57.77	26.90	21.42	6.54
8	(0) –	69.79	28.76	7.53	(2) 6841.71	43.20	24.56	12.10	7.64
9	(0) –	28.72	16.92	8.57	(0) –	40.45	12.36	12.17	8.67
10	(0) –	16.51	11.01	9.62	(2) 4688.78	18.06	5.78	6.83	9.74
11	(0) –	9.07	6.16	10.71	(6) 4997.13	13.08	3.79	3.78	10.78
12	(4) 3917.60	6.24	1.63	11.76	(6) 4975.63	9.66	1.43	1.59	11.82
13	10.16	1.92	0.77	12.85	6.37	1.49	0.45	0.39	12.89
14	0.19	0.31	0.20	13.85	0.14	0.24	0.13	0.11	13.85

TABLE III
COMPUTATION TIMES (IN SECONDS) AND AVERAGE OPTIMAL SOLUTION VALUES: ASYMMETRIC INPUTS WITH $|V| = 20$.

	Unidirectional topology			Bidirectional topology			
	Time (s)		Optimal	Time (s)			Optimal
k	DP	IP	value	DP	IP	IP-B	value
2	268.79	177.59	1.35	160.83	72.89	67.79	1.39
3	1090.84	779.51	2.08	1018.91	426.89	290.34	2.17
4	1828.26	1104.11	2.89	1458.10	671.47	420.07	2.98
5	1818.49	1479.14	3.73	1760.31	1051.78	641.60	3.89
6	1757.40	1287.72	4.61	1263.74	680.89	562.82	4.82
7	1928.14	895.80	5.56	1579.53	626.97	391.30	5.80
8	2067.12	720.45	6.48	1627.56	571.64	380.74	6.69
9	(14) 1866.11	397.01	7.32	2047.16	598.10	473.43	7.65
10	1405.65	421.43	8.32	1609.07	377.42	269.88	8.63
11	(14) 1326.30	317.64	9.23	2300.35	473.91	388.87	9.68
12	952.18	208.39	10.37	1257.34	169.95	140.80	10.71
13	412.91	122.25	11.38	1248.65	186.67	142.22	11.76
14	158.40	65.66	12.42	377.77	44.29	51.74	12.74
15	107.51	36.71	13.45	131.35	36.26	24.10	13.78
16	79.16	30.06	14.47	140.39	35.86	32.85	14.80
17	45.10	8.65	15.66	73.56	5.45	5.78	15.91
18	11.50	3.34	16.79	9.34	1.97	1.59	16.92
19	1.24	0.73	17.92	0.98	0.50	0.38	17.92

TABLE IV
COMPUTATION TIMES (IN SECONDS) AND AVERAGE OPTIMAL SOLUTION VALUES: SYMMETRIC INPUTS WITH $|V| = 20$.

	Unidirectional topology			Bidirectional topology			
	Time (s)		Optimal	Time (s)			Optimal
k	DP	IP	value	DP	IP	IP-B	value
2	277.16	179.02	1.46	226.05	54.82	47.26	1.48
3	615.13	464.32	2.23	592.78	236.07	160.49	2.30
4	1200.44	732.42	3.09	1030.55	404.07	310.09	3.17
5	1564.31	1405.94	4.02	1645.62	779.98	649.54	4.13
6	1761.35	1154.83	4.98	1409.22	605.27	492.14	5.11
7	2042.92	795.34	5.97	1279.66	577.23	276.39	6.11
8	1933.01	499.87	6.92	774.83	325.02	309.01	7.06
9	1440.98	524.45	7.92	1413.48	390.23	245.79	8.09
10	2031.82	495.39	8.95	1645.91	412.01	288.60	9.11
11	(13) 1018.45	184.48	9.93	2196.05	356.81	291.57	10.21
12	1745.10	299.14	11.14	1094.11	234.45	181.36	11.33
13	986.46	187.22	12.24	1163.28	173.41	121.32	12.42
14	367.52	119.45	13.29	605.58	68.08	50.82	13.44
15	89.85	60.20	14.40	138.23	25.48	22.89	14.56
16	73.23	34.18	15.54	105.32	18.20	14.85	15.68
17	58.82	16.18	16.74	85.87	8.92	13.00	16.83
18	11.51	3.47	17.88	9.31	2.05	1.62	17.92
19	1.23	0.72	18.95	1.00	0.50	0.36	18.95



(a) Asymmetric input with unidirectional topology

(b) Asymmetric input with bidirectional topology

(c) Symmetric input with unidirectional topology

(d) Symmetric input with bidirectional topology

Fig. 5. Computation times for $|V| = 15$.

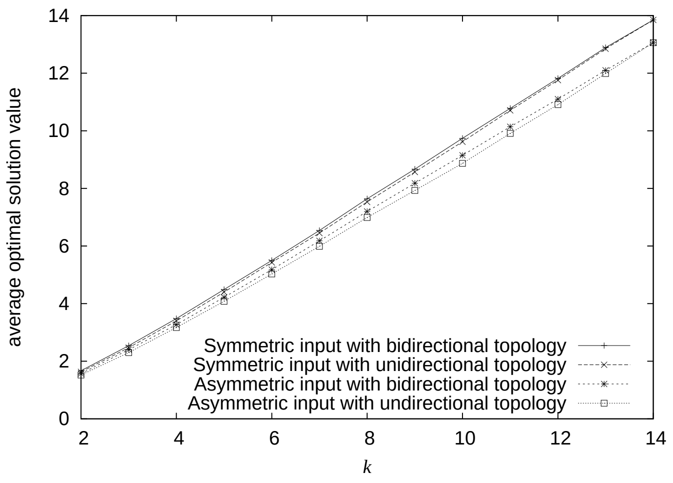


Fig. 6. Optimal solution values for $|V| = 15$ for $k = 2, \ldots, 14$.

TABLE V
NUMBER OF INSTANCES SOLVED IN THREE HOURS OF RUNNING TIME FOR EACH FORMULATION CP, DP, IP, AND IP-B, WITH $k = 2$.

		Unidirectional			Bidirectional			
	$\|V\|$	CP	DP	IP	CP	DP	IP	IP-B
Asymmetric	10	15	15	15	15	15	15	15
	15	12	15	15	15	15	15	15
	20	0	15	15	12	15	15	15
	25	–	15	15	–	15	15	15
	30	–	5	11	–	9	12	12
Symmetric	10	15	15	15	15	15	15	15
	15	12	15	15	15	15	15	15
	20	0	15	15	12	15	15	15
	25	–	15	15	–	15	15	15
	30	–	6	10	–	9	13	12

Tables I to IV show that all formulations become progressively harder to solve with the increase in k. Figure 5 displays the average computation times in seconds for increasing values of k, according to the numbers in Tables I and II. This behavior is explained by the growth in the number of variables, since the number of commodities is given by $|C| = k \cdot O(|V|)$ and the number of constraints and variables by $|C| \cdot O(|V|^2)$.

The computation times decrease as k increases up to $|V|-1$. In fact, the solution graph is fully connected for $k = |V| - 1$. In wired networks, the power assigned to node i should be greater than or equal to $e(i, u) + e(i, v)$ if both nodes u and v should be reached by transmissions from node i. In wireless networks, however, the power assignment should be greater than or equal to $\max\{e(i, u), e(i, v)\}$ in the same situation, which is possibly smaller than $e(i, u) + e(i, v)$. This reduction in the power that has to be assigned to each node with respect to wired networks is called the *wireless multicast advantage* [41]. Whenever the parameter k increases, each node requests a greater power assignment to be able to transmit through at least k arcs to ensure that k-node disjoint paths exists. Therefore, the wireless multicast advantage enforces an increasing number of arcs to enter into the solution when k increases, reducing the number of free decision variables and speeding up the solver. In particular, the unique, trivial feasible solution to $k = |V| - 1$ consists in assigning the maximum power $\max_{j \in V, j \neq i}\{e(i, j)\}$ to each node $i = 1, \ldots, |V|$. The solver is very quick not only for $k = |V| - 2$ for all formulations, but also for $k = |V| - 3$ in the case of formulations DP, IP and IP-B, because the solution of their linear relaxations in the root of the search tree is already integer.

All models are affected and benefit from the wireless multicast advantage property. It is explicitly explored in formulation DP by inequalities (12) and (13), and in formulation IP by inequalities (22).

Figure 6 presents the growth in the optimal solution values for all types of test instances with $|V| = 15$ when k increases, according to Tables I and II. The average power value also increases as k grows. In the next section, we focus our analysis into the biconnected case ($k = 2$), since it gives the smallest fault tolerant minimum power assignments.

B. Second Experiment: Biconnected Solutions

The most important metric for performance evaluation is the computation time taken by each formulation to solve to optimality the biconnected minimum power consumption problem. Table V shows the number of instances solved by CPLEX using each model in less than three hours of computation time. Cells in blank correspond to instance types and sizes for which the weakest formulation CP could not be applied.

Since some formulations do not solve all instances in three hours, the numbers in Tables VI and VII are average results over all instances solved to optimality by all formulations. For each problem dimension $|V| = 10, 15, 20, 25, 30$ and each formulation, Tables VI and VII display the average computation time in seconds taken by CPLEX, the average relative MIP gap M in percent between the first integer solution found and the linear relaxation value at that time, and the average relative duality gap D in percent between the linear relaxation value and that of the optimal integer solution.

The log-scale plots in Figure 7 summarize the results in Tables VI and VII, regarding the behavior of the exact formulations in terms of their average computation times when the number of nodes increases from 10 to 30. These results show that the CP formulation takes very long computation times and is very difficult to be solved, becoming unpractical for $n > 15$ (resp. $n > 20$) in the asymmetric (resp. symmetric) case. The main drawback of this formulation is the inexistence of valid inequalities to further explore the wireless multicast advantage property. Formulations DP and IP are much stronger and lead to significantly smaller computation times and to linear relaxation values that are very close to those of the optimal integer solutions, for every instance size.

Formulation IP achieves smaller computation times and becomes progressively better than DP with the increase in the number of nodes. This is due to the fact that constraints (22) of formulation IP lead to the fixation of more variables to one, while contraints (13) of formulation DP allow the fixation of fewer variables to zero. In consequence, the solver is more effective for formulation IP than for DP, as illustrated by the

TABLE VI
COMPUTATIONAL RESULTS OBTAINED WITH EACH FORMULATION (CP, DP, IP, AND IP-B) FOR THE UNIDIRECTIONAL PROBLEM VARIANTS WITH $k = 2$

		CP formulation			DP formulation			IP formulation		
	$\|V\|$	time (s)	M (%)	D (%)	time (s)	M (%)	D (%)	time (s)	M (%)	D (%)
Asymmetric	10	51.73	84.67	61.69	0.75	24.82	11.06	0.89	27.53	11.06
	15	2909.30	92.98	68.87	23.72	35.49	13.75	16.20	43.60	13.75
	20	–	–	–	268.79	61.67	13.40	177.59	57.07	13.40
	25	–	–	–	3011.59	72.75	11.96	1563.94	66.93	11.96
	30	–	–	–	7186.82	77.04	7.47	2837.09	64.58	7.47
Symmetric	10	41.02	83.88	61.73	0.79	20.13	10.90	0.78	28.64	10.90
	15	2298.42	92.09	68.72	23.48	33.21	14.23	16.03	40.62	14.23
	20	–	–	–	277.16	61.20	12.80	179.02	58.21	12.80
	25	–	–	–	2405.44	74.57	12.15	1600.28	76.05	12.15
	30	–	–	–	7009.86	89.04	11.51	4875.97	82.02	11.51

TABLE VII
COMPUTATIONAL RESULTS OBTAINED WITH EACH FORMULATION (CP, DP, IP, AND IP-B) FOR THE BIDIRECTIONAL PROBLEM VARIANTS WITH $k = 2$.

		CP formulation			DP formulation			IP formulation			IP-B formulation		
	$\|V\|$	time (s)	M (%)	D (%)	time (s)	M (%)	D (%)	time (s)	M (%)	D (%)	time (s)	M (%)	D (%)
Asymmetric	10	10.83	80.07	58.16	0.83	25.45	8.93	0.87	30.22	8.93	0.47	25.48	7.51
	15	400.55	91.67	66.75	20.54	32.11	11.51	10.28	31.27	11.51	7.55	28.07	10.34
	20	6878.64	94.61	68.74	141.12	40.45	9.40	62.69	45.37	9.40	66.61	72.26	8.10
	25	–	–	–	2357.44	55.05	9.20	543.38	41.16	9.20	298.53	63.73	7.71
	30	–	–	–	5393.10	53.16	5.42	1983.42	38.05	5.42	1351.98	56.92	4.56
Symmetric	10	9.52	80.18	58.61	0.63	24.48	8.54	0.73	19.44	8.54	0.48	23.61	7.25
	15	410.63	91.02	66.65	20.88	44.35	11.42	12.25	26.64	11.42	7.24	42.58	10.14
	20	5837.51	94.75	94.75	226.05	56.60	9.62	54.82	33.69	9.62	47.26	52.80	8.27
	25	–	–	–	1679.72	49.74	9.05	703.74	37.10	9.05	509.83	57.71	7.70
	30	–	–	–	5263.26	71.83	5.41	2436.90	52.81	5.41	1373.72	82.79	4.20

better results reported in Tables VI and VII for the average relative gap M between the value of the first integer solution found and the linear relaxation bound.

The IP-B formulation for bidirectional solutions is obtained by reinforcing formulation IP with inequalities (27) and (28). The computation times and the duality gaps D observed with formulation IP-B are improved with respect to formulation IP, in spite of the slightly increase in the relative MIP gap M obtained by CPLEX. The linear relaxation bound is improved, but the additional constraints lead to an increase in the objective function value of the first integral solution found and, consequently, to an increase in the relative gap M.

The results in Tables V, VI and VII also show that unidirectional topology problems are harder to solve. This can be explained by the number of commodities, since the number of commodities in the case of formulations for unidirectional topologies is twice the number of commodities for bidirectional topologies.

VI. CONCLUDING REMARKS

We have shown that the variants of the minimum power k-fault tolerant topology control optimization problem can be organized into four different categories, regarding the topologies of the input graph (symmetric or asymmetric) and of the solution (unidirectional or bidirectional). We also presented a literature survey according with this classification.

We have proposed three integer programming formulations for the minimum power k-fault tolerant topology control optimization problem in wireless ad hoc networks. All formulations are sufficiently general to encompass all four problem variants. The formulations are also flexible enough to handle any value of the parameter k associated with the connectivity requirements. Stronger formulations with tighter lower bounds were proposed for bidirectional solution topologies by the addition of valid cut inequalities.

All conclusions were supported by comprehensive computational experiments comparing the formulations. The numerical results showed that the more elaborate discrete formulations, which explicitly incorporate the ad hoc multicast advantage as constraints, lead to much better computation times and linear relaxation bounds than the continuous formulation. Moderate-size problem instances with up to 30 nodes could be solved to optimality by commercial solvers. We are currently working on approximate algorithms based on metaheuristics for extending this limit and solving larger problem instances. The exact formulations are also important in this context, since they provide exact solutions and their linear and Lagrangean relaxations give lower bounds that are useful to address and compare the behavior of such heuristics.

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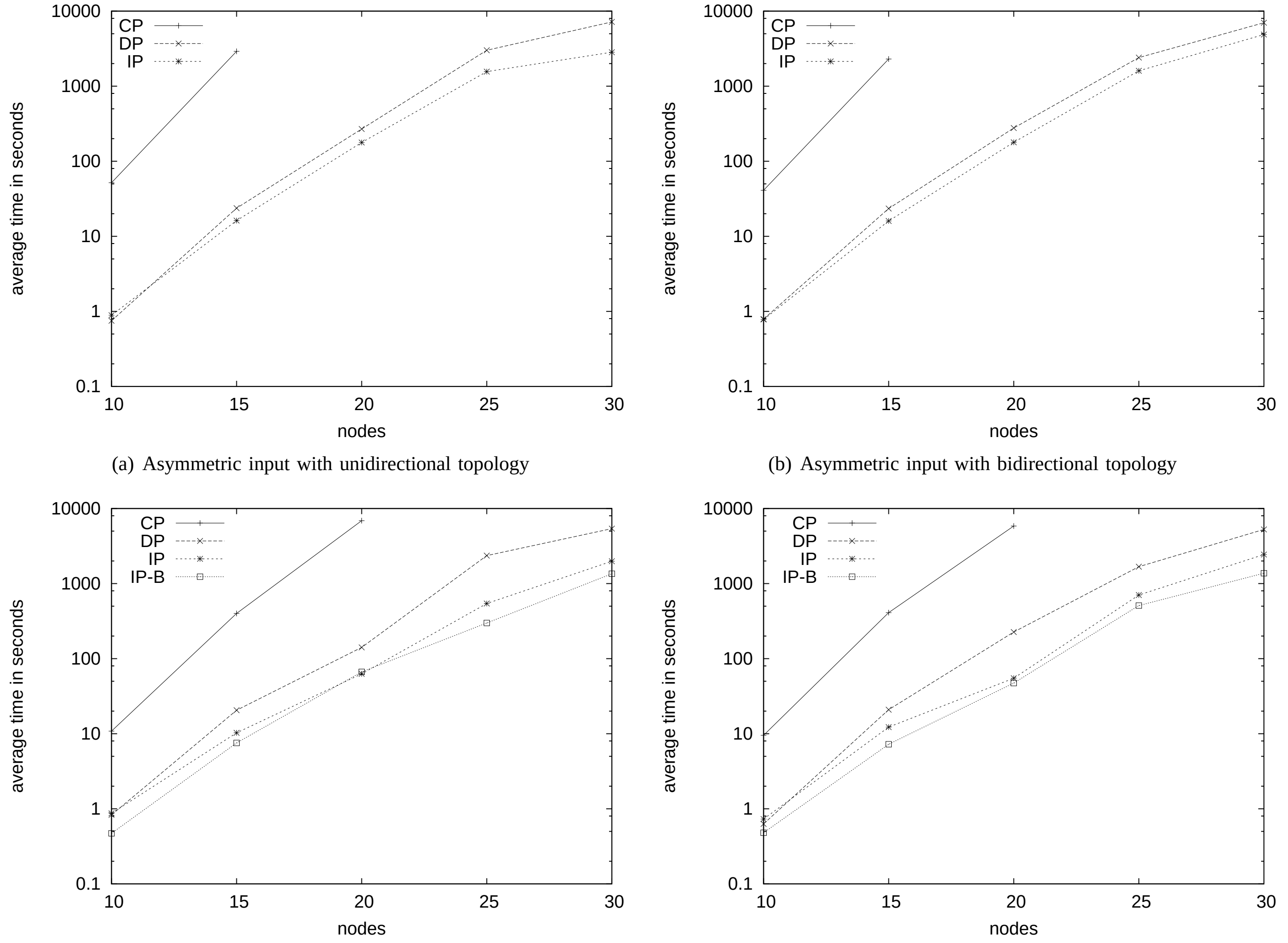

(a) Asymmetric input with unidirectional topology

(b) Asymmetric input with bidirectional topology

(c) Symmetric input with unidirectional topology

(d) Symmetric input with bidirectional topology

Fig. 7. Computation times with different exact formulations for $|V|$ ranging from 10 to 30.

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