Routing and wavelength assignment by partition colouring

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Abstract

The problem of routing and wavelength assignment in all-optical networks may be solved by a combined approach involving the computation of alternative routes for the lightpaths, followed by the solution of a partition colouring problem in a conflict graph. A new tabu search heuristic is also proposed for the partition colouring problem, which may be viewed as an extension of the graph colouring problem. Computational experiments are reported, showing that the tabu search heuristic outperforms the best heuristic for partition colouring by approximately 20% in the average and illustrating that the new approach for the problem of routing and wavelength assignment is more robust than a well established heuristic for this problem.

Keywords: Routing, wavelength assignment, optical networks, WDM, partition colouring, heuristics, tabu search.

1 Motivation

The technology of Optical Networks is increasingly important in telecommunications and appears in a large number of applications such as video-conferences, high performance computing, and grids, among others. The information is transmitted along the optical fibers as optical signals. Each link operates at a speed of the order of terabits per second, which is much faster than the currently available electronic devices for signal reception and transmission. Wavelength Division Multiplexing (WDM) allows more efficient use of the huge capacity of optical fibers, as far as it permits the simultaneous transmission of different channels along the same fiber, each of them using a different wavelength.

In all-optical networks, each signal is converted to the optical domain and reaches the receptor without conversion to the electrical domain. Keeping the signal at the optical domain allows higher transmission speeds, eliminating the overheads imposed by electrical conversions. Such networks require a large number of available wavelengths, especially when wavelength conversion is not possible in the nodes. Routing should be performed so as to minimize the number of wavelengths needed.

We consider all-optical networks in which the nodes make use of wavelength-routing switches. Each signal is routed by an optical switch according to its input port and the wavelength used. Each wavelength at each input port is mapped to a different output port. An all-optical point-to-point connection between two endnodes is called a lightpath. Two lightpaths may use the same wavelength, provided they do not share any common link. In this work, we assume that wavelength conversion along a light-path is not permitted. Therefore, each lightpath should use the same wavelength from the transmitter to the receiver.

The Routing and Wavelength Assignment (RWA) problem consists in routing a set of lightpaths and assigning a wavelength to each of them. Variants of RWA are characterized by different optimization criteria and traffic patterns, see e.g. Choi et al. [6] and Yoo and Banerjee [15]. We consider the min-RWA offline variant, in which all connection requirements are known beforehand and one seeks to minimize the total number of wavelengths used for routing these connections. Erlebach and Jansen [7] showed that min-RWA is NP-hard. Several authors proposed different approximate algorithms for solving min-RWA. Banerjee and Mukherjee [4], Hyytiä and Virtamo [10], and Li and Simha [12] decompose the problem in two phases. In the first phase, one or more possible routes are computed for each lightpath, while in the second one route and one wavelength are assigned to each lightpath. The second phase is often solved as a colouring problem defined on a conflict graph. Manohar, Manjunath, and Shevgaonkar [13] proposed the Greedy-EDP-RWA construction which was used in a multi-start procedure heuristic, in which both subproblems are solved simultaneously. Their algorithm is much faster and finds solutions as good as those found by the algorithm in [4].

We propose a new two-phase heuristic for min-RWA, using the same decomposition scheme described in [12]. In the first phase, a set of possible routes is precomputed for each lightpath. In the second, a route (among those precomputed) and a wavelength are assigned for each lightpath by solving a partition colouring problem, which is formulated in the next section. A tabu search for solving the partition colouring problem is proposed in Section 2. The complete heuristic for min-RWA is presented in Section 3. Computational results comparing the tabu search procedure for partition colouring and the new heuristic for min-RWA with other approaches in the literature are reported in Section 4. Conclusions are drawn in the last section.

2 A Tabu Search Heuristic to the Partition Colouring Problem

2.1 Partition Colouring Problem

Let $G = (V, E)$ be a non-directed graph, where V is the set of nodes and E is the set of edges. Furthermore, let $V_1, V_2, \ldots, V_q$ be a partition of V into q subsets, with $V_1 \cup V_2 \cup \ldots \cup V_q = V$ and $V_i \cap V_j = \emptyset, \forall i, j = 1, \ldots, q$ with $i \neq j$. We refer to $V_1, V_2, \ldots, V_q$ as the *components* of the partition. The *Partition Colouring Problem* (PCP) consists in finding a subset $V' \subset V$ such that $|V' \cap V_i| = 1, \forall i = 1, \ldots, q$ (i.e., V' contains one node from each component V_i), and the chromatic number of the graph induced in G by V' is minimum. This problem is clearly a generalization of the graph colouring problem. Li and Simha [12] have shown that the decision version of PCP is NP-complete.

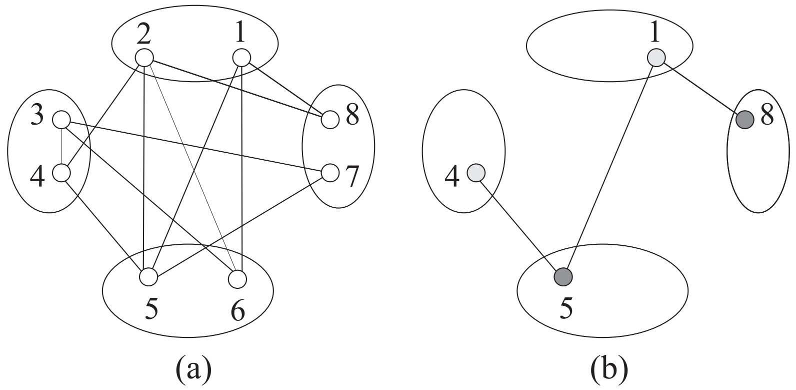


Figure 1: (a) Instance of PCP and (b) optimal solution with two colours.

We illustrate an instance of PCP in Figure 1. The associated graph has eight nodes and four components. The optimal solution makes use of two colours: the first colour is used to colour nodes 1 and 4, while the second is used to colour nodes 5 and 8.

2.2 Construction Heuristic

Real life applications of the min-RWA problem are associated with very large instances of the partition colouring problem that cannot be systematically handled in practice by exact methods. Li and Simha [12] proposed six construction algorithms for PCP, based on graph colouring heuristics. Best results were obtained with algorithm `onestepCD` (One Step Colour Degree), which is also used to build initial solutions to our tabu search procedure. Algorithm `onestepCD` described in Figure 2 is a variant of the colour degree heuristic [5] adapted to PCP. We denote by $c(i)$ the component of node $i \in V$ (i.e. $i \in V_{c(i)}$) and by $V' \subseteq V$ the set of nodes of G already coloured in a partial solution under construction to PCP. The colour saturation degree $CD(i)$ of each node $i \in V$ is defined as the number of different colours used to colour the neighbours of i in V'.

procedure `onestepCD`(G)
Remove from G all edges $(i,j) \in E : i,j \in V^k$ for some $k = 1,\ldots,q$;
Set $V' \leftarrow \emptyset$;
while $|V'| < q$ **do**
    Set $X \leftarrow \emptyset$;
    for $k = 1,\ldots,q : V_k \cap V' = \emptyset$ **do**
        Set $X \leftarrow X \cup \text{argmin}\{CD(i) : i \in V^k\}$;
    end-for;
    Set $x \leftarrow \text{argmax}\{CD(i) : i \in X\}$;
    Set $V' \leftarrow V' \cup \{x\}$;
    Assign the minimum possible colour to x;
    Remove from G all nodes in $V_{c(x)} \setminus \{x\}$;
end-while;
end `onestepCD`.

Figure 2: Pseudo-code of the `onestepCD` construction algorithm to PCP.

In line 1, all edges connecting nodes in the same component of the partition are removed. The set of nodes already coloured is initialized in line 2. The loop in lines 3-12 is performed until all partitions have been coloured. In lines 4-7, we build the set X formed by the nodes with least coloured saturation degree from each uncoloured component. The node x with the maximum coloured saturation degree among those in X is selected in 8. Node x is coloured with the minimum possible colour in lines 9 and 10. All uncoloured nodes in the same partition as x are removed from the graph in line 11 and the above steps are repeated. The overall complexity of the algorithm `onestepCD` is $O(m + qn)$.

2.3 Tabu Search for PCP

Algorithm `onestepCD` is applied to create a feasible initial solution S to PCP with $\texttt{max}C$ colours. The tabu search [9] procedure `TS-PCP` described in Figure 3 is based on a first-improving local search strategy using a 1-opt neighbourhood and attempts to find a feasible solution with a smaller number of colours. We define a *colouring conflict* as a pair of adjacent nodes in different components of the graph which are coloured with the same colour. We denote by $\texttt{conflicts}(S)$ the number of colouring conflicts in any solution S.

A new possibly infeasible solution S' using $\texttt{max}C - 1$ colours is built in line 1. The iteration counter and the tabu list are initialized in line 2. The set Q of components involved in colouring conflicts is built in line 3. The loop in lines 4-18 investigates the neighbourhood of the current solution until no further reduction in the number of colouring conflicts is possible. Each neighbour is obtained by (a) recolouring with a different colour exactly one node involved in a colouring conflict or (b) changing (and colouring) the node that is coloured in a component involved in a colouring conflict. A component $k \in Q$ involved in a colouring conflict is randomly selected in line 5. Variable *reduction* is initialized as `.FALSE.` in line 6 and will be used to indicate that an improving solution with a smaller number of conflicts was found. The number `maxConflicts` of colouring conflicts in the neighbour of the current solution with the least number of conflicts is initialized in line 7. The loop in lines 8-17 investigates all neighbours corresponding to the selected component k. The test in line 9 discards all neighbours which belong to the tabu list and do not satisfy an aspiration criterion. A new tentative solution S'' is built in line 11 by colouring a node $i \in V^k$ with a colour $\ell = 1, \ldots, \texttt{max}C - 1$. The best neighbour $\bar{S}$ of the current solution is updated in lines 12-13. If the number of colouring conflicts in S'' is smaller than those in S', then the latter is updated in line 15, together with the set of components involved in colouring conflicts and the flag *reduction* that indicates that an improving solution was found. The iteration counter is also updated in line 15.

If there are no colouring conflicts in the new solution S', the current solution S using fewer colours is updated in lines 21 and 22, and a new attempt to reduce the number of colours is started in line 23. Otherwise, only a non-improving neighbour $\bar{S}$ of the current solution S' was found. We identify in line 25 the node $\bar{i}$ coloured with colour $\bar{\ell}$ in the current solution S'. The pair $\bar{i}$, $\bar{\ell}$ is inserted in the tabu list for `TabuTenure` iterations in line 26. In line 27, the current solution S' moves to the best neighbour $\bar{S}$. The iteration counter is updated in line 28. If the stopping criterion is not yet satisfied, a new iteration resumes in line 29.

The maximum number of iterations used as the stopping criterion is set as $\texttt{maxIter} = q \cdot (\texttt{max}C - 1) \cdot F_{end}$, where q is the number of components in the graph, $\texttt{max}C - 1$ is the

procedure `TS-PCP`($G, S,$ `max`C)

1. Build a new solution S' by randomly recolouring with any of the colours $1, \ldots,$ `max`$C - 1$ every node originally coloured with colour `max`C;
2. Free the tabu list and set $iter \leftarrow 0$;
3. Let Q be the set formed by all components involved in colour conflicts in S';
4. **while** $Q \neq \emptyset$ **do**
5.   Randomly select $k \in Q$ and update $Q \leftarrow Q \setminus \{k\}$;
6.   Set $reduction \leftarrow$ `.FALSE.`;
7.   Set `maxConflicts` $\leftarrow \infty$;
8.   **for each** $i \in V_k$ **and for each** colour $\ell = 1, \ldots,$ `max`$C - 1$ **while** `.NOT.`$reduction$ **do**
9.   **if** the pair i, ℓ is not in the tabu list or if it satisfies the aspiration criterion
10.   **then do**;
11.   Obtain a tentative solution S'' by recolouring node i with colour ℓ;
12.   **if** `conflicts`$(S'') <$ `maxConflicts`
13.   **then** `maxConflicts` $\leftarrow$ `conflicts`(S''), $\bar{S} \leftarrow S''$, and $\bar{k} \leftarrow k$;
14.   **if** `conflicts`$(S'') <$ `conflicts`(S')
15.   **then** $S' \leftarrow S''$, $reduction \leftarrow$ `.TRUE.`, $iter \leftarrow iter + 1$, and update Q;
16.   **end-if**;
17.   **end-for**;
18. **end-while**;
19. **if** `conflicts`$(S') = 0$
20. **then do**;
21. $S \leftarrow S'$;
22.   `max`$C \leftarrow$ `max`$C - 1$;
23.   **return to** step 1;
24. **else do**
25.   Let $\bar{i}$ be the node currently coloured with colour $\bar{\ell}$ in component $V_{\bar{k}}$ of S';
26.   Insert the pair $\bar{i}, \bar{\ell}$ in the tabu list for `TabuTenure` iterations;
27. $S' \leftarrow \bar{S}$;
28. $iter \leftarrow iter + 1$;
29.   **if** $iter <$ `maxIter` **then return to** step 3;
30. **end-if**;
31. **return** S;

end `TS-PCP`.

Figure 3: Pseudo-code of the `TS-PCP` local search algorithm to PCP.

tentative number of colours, and F_{end} is a parameter to be tuned, as described in Section 4.1.

3 A New Heuristic to min-RWA

Let $N = (X, A)$ be a graph representing the physical topology of an all-optical-network, where X is the set of switches (nodes) and A is the set of links (edges) connecting the switches. Furthermore, we denote by T the set of connections to be established. Each connection is defined by a pairs of endnodes in X.

In this section we describe a new two-phase heuristic for min-RWA. First, we compute a set of alternative possible routes for each connection using the k-`EDR` construction procedure summarized below. Next, we build a conflict graph, in which the nodes correspond to routes and there is an edge between each pair of nodes whose associated routes share a common link. All alternative routes associated to the same connection are placed in the same component of the conflict graph. In the second phase, the tabu search heuristic `TS-PCP` is applied to solve the above defined instance of PCP. The coloured nodes define the routes and the colours define the selected wavelengths.

The pseudo-code of the construction procedure k-`EDR` is given in Figure 4. Its main component is the heuristic `BGAforEDP` proposed by Kleinberg [11] for the *maximum edge disjoint path problem*, which finds the largest subset of connections that can be routed by edge-disjoint paths with a maximum number d of edges. We notice that routing with as many disjoint paths as possible allows a better use of the available wavelengths. The maximum number d of edges in each route is initialized in line 1 as suggested in [11]. The set R of routes is initialized in line 2. The loop in lines 3-19 attempts to build at most k alternative routes for each connection in T. A copy T' of the set of connections to be established is made in line 4. The loop in lines 5-18 computes a single route for each connection. A copy N' of the network is made in line 6. In line 7 we build a list L defining a random permutation of the order in which the connections still in T' will be scanned. The loop in lines 8-17 finds a set of disjoint routes with at most d edges for as many connections in T' as possible. The next connection t to be investigated is selected in line 9. The shortest path s in the network N' between the endnodes of connection t is computed in line 10. Line 11 checks if this path has at most d edges. In this case, the set T' is updated in line 13, the set of routes is updated in line 14, and the residual network N' is updated in line 15. The set R of routes is returned in line 20.

procedure k-`EDR`(N, T, k)
1. Set $d \leftarrow max(diameter(N), \sqrt{|A|})$;
2. Set $R \leftarrow \emptyset$;
3. **for** $j = 1, \ldots, k$ **do**
4.   Set $T' \leftarrow T$;
5.   **while** $T' \neq \emptyset$ **do**
6.   Set $N' \leftarrow N$;
7.   Build a list L with a random permutation of the connections in T';
8.   **for** $i = 1, ..., |T'|$ **do**
9.   Let t be the i-th connection in L;
10.   Compute the shortest path s in N' between the endnodes of t;
11.   **if** s has no more than d edges
12.   **then do**
13. $T' \leftarrow T' \setminus \{t\}$;
14. $R \leftarrow R \cup \{(t, s)\}$;
15.   Remove the edges in path s from N';
16.   **end-if**;
17.   **end-for**;
18.   **end-while**
19. **end-for**;
20. **return** R;
end k-`EDR`

Figure 4: Pseudo-code of the k-`EDR` construction heuristic.

4 Computational Results

Algorithms `onestepCD`, `TS-PCP`, and k-`EDR` described in the previous sections, as well as the heuristic proposed by Manohar, Manjunath, and Shevgaonkar [13] for min-RWA, were implemented in C++ and compiled with version v2.96 of the Linux/GNU compiler. All experiments were performed on a Pentium IV machine with a 2.0 GHz clock and 512 Mbytes of RAM memory. Numerical results concerning the partition colouring problem are reported in Section 4.1. Computational results regarding the heuristics for min-RWA are reported in Section 4.2.

4.1 Tabu Search for PCP

For the first experiment, we generated a small set of four PCP instances with 500 components each. These instances were built from node colouring instances DSJC250.5 and DSJC500.5 [3] available from `http://mat.gsia.cmu.edu/COLOR/instances.html`. They are used exclusively for tuning once for all the parameters of the tabu search heuristic `TS-PCP`. PCP instance DSJC500.5-1 has exactly one node in each component and coincides with the original DSJC500.5 vertex colouring instance. Further PCP instances were generated by adding 1, 2, or 3 new nodes to each component of instance DSJC500.5-1. Edges adjacent to the new nodes were randomly generated with the same probability 0.5, as in the original node colouring instance. The main characteristics of these instances are summarized in Table 1.

Instance	Nodes	Components	Nodes per component	Density
DSJC500.5-1	500	500	1	0.5
DSJC500.5-2	1000	500	2	0.5
DSJC500.5-3	1500	500	3	0.5
DSJC500.5-4	2000	500	4	0.5

Table 1: Characteristics of the PCP instances used for tuning the tabu search heuristic.

The tabu search heuristic `TS-PCP` was applied to instances DSJC500.5-1 to DSJC500.5-4 with different values for the parameters `TabuTenure` and F_{end} which defines the stopping criterion. Numerical results are reported in Tables 2 to 5. For each combination of parameters, we report the minimum, average, and maximum number of colours over 10 runs using different seeds for each instance. Best average results are underlined and were always obtained with the tabu tenure parameter randomly generated from a uniform distribution in the interval $[0, \frac{1}{2}C']$, with $C' = \texttt{max}C - 1$. The parameter $F_{end} = 5$ was set at the intermediary value. This combination of parameters will be used in all remaining experiments.

In the next experiment we investigate the behaviour of heuristics `onestepCD` and `TS-PCP` (with the parameter values previously set) for randomly generated graphs with 100 to 900 components. Each component has exactly two nodes and the edge density is 0.5. Ten instances were generated for each number of nodes and algorithms `onestepCD` and `TS-PCP` were run 10 times with different seeds. The average number of colours found by each heuristic over 100 runs for each graph size is plotted in Figure 5. We observe that for both algorithms the average number of colours increases linearly with the number of components. For the largest graphs with 900 nodes, the number of colours used by the new tabu search heuristic is approximately

	$F_{end} = 1$			$F_{end} = 5$			$F_{end} = 10$		
`TabuTenure`	min	avg	max	min	avg	max	min	avg	max
$\mathrm{U}[\frac{1}{4}C', \frac{3}{4}C']$	53	53.5	54	52	52.7	53	52	52.5	53
$\mathrm{U}[0, C']$	53	53.7	55	52	52.9	53	52	52.3	53
$\mathrm{U}[0, \frac{1}{2}C']$	52	$\underline{53.1}$	54	51	$\underline{52.2}$	53	51	$\underline{51.3}$	52
$\mathrm{U}[\frac{1}{2}C', C']$	54	54.2	55	53	53.3	54	53	53.0	53
$\mathrm{U}[\frac{1}{4}C', C']$	53	53.8	54	52	53.0	54	52	52.8	53
$\mathrm{U}[0, \frac{3}{4}C']$	52	53.3	54	51	52.5	53	51	52.2	53

Table 2: Results for instance DSJC500.5-1, with $C' = \mathtt{max}C - 1$.

	$F_{end} = 1$			$F_{end} = 5$			$F_{end} = 10$		
`TabuTenure`	min	avg	max	min	avg	max	min	avg	max
$\mathrm{U}[\frac{1}{4}C', \frac{3}{4}C']$	47	47.8	49	46	46.8	47	46	46.7	47
$\mathrm{U}[0, C']$	46	47.5	48	46	46.8	47	46	46.7	47
$\mathrm{U}[0, \frac{1}{2}C']$	47	$\underline{47.3}$	48	45	$\underline{46.1}$	47	45	$\underline{45.9}$	46
$\mathrm{U}[\frac{1}{2}C', C']$	48	48.1	49	47	47.7	48	47	47.3	48
$\mathrm{U}[\frac{1}{4}C', C']$	47	47.9	49	47	47.3	48	46	46.9	47
$\mathrm{U}[0, \frac{3}{4}C']$	46	47.5	48	46	46.6	47	46	46.2	47

Table 3: Results for instance DSJC500.5-2, with $C' = \mathtt{max}C - 1$.

	$F_{end} = 1$			$F_{end} = 5$			$F_{end} = 10$		
`TabuTenure`	min	avg	max	min	avg	max	min	avg	max
$\mathrm{U}[\frac{1}{4}C', \frac{3}{4}C']$	44	44.8	45	44	44.4	45	44	44.0	44
$\mathrm{U}[0, C']$	45	45.4	46	44	44.7	45	44	44.2	45
$\mathrm{U}[0, \frac{1}{2}C']$	44	$\underline{44.6}$	45	43	$\underline{43.7}$	44	43	$\underline{43.2}$	44
$\mathrm{U}[\frac{1}{2}C', C']$	45	45.8	46	44	44.9	45	44	44.8	45
$\mathrm{U}[\frac{1}{4}C', C']$	44	45.5	46	44	44.7	45	44	44.2	45
$\mathrm{U}[0, \frac{3}{4}C']$	44	44.8	45	43	44.0	45	43	43.9	44

Table 4: Results for instance DSJC500.5-3, with $C' = \mathtt{max}C - 1$.

	$F_{end} = 1$			$F_{end} = 5$			$F_{end} = 10$		
`TabuTenure`	min	avg	max	min	avg	max	min	avg	max
$\mathrm{U}[\frac{1}{4}C', \frac{3}{4}C']$	43	43.5	44	42	42.8	43	42	42.4	43
$\mathrm{U}[0, C']$	43	43.6	44	42	42.7	43	42	42.7	43
$\mathrm{U}[0, \frac{1}{2}C']$	42	$\underline{42.8}$	43	42	$\underline{42.0}$	42	42	$\underline{42.0}$	42
$\mathrm{U}[\frac{1}{2}C', C']$	43	43.9	44	43	43.0	43	43	43.0	43
$\mathrm{U}[\frac{1}{4}C', C']$	43	43.6	44	42	42.9	43	42	42.8	43
$\mathrm{U}[0, \frac{3}{4}C']$	42	43.0	44	42	42.4	43	42	42.2	43

Table 5: Results for instance DSJC500.5-4, with $C' = \mathtt{max}C - 1$.

80% of that used by the original construction heuristic `onestepCD` .

Average computation times for the `TS-PCP` heuristic are plotted in Figure 6. Computation times increase quadratically with the number of components. We notice that the average times to find the best solutions are much smaller than the average times to finish.

In the last experiment in this section, we randomly generated graphs with 1000 nodes distributed over 500 components with two nodes each. The edge density ranges from 0.1 to 0.9. Ten graphs were generated for each density. Algorithms `onestepCD` and `TS-PCP` were run 10 times for each instance with different seeds. We observe in Figure 7 that the average number of colours increases with edge density. Figure 8 shows that once again the computation times increase quadratically with the edge density.

4.2 Application to RWA

In this section we report computational results concerning the proposed approach of combining the heuristics k-`EDR` with $k = 2$ and `TS-PCP` to solve problem min-RWA of routing and wavelength assignment in all optical networks.

We considered five realistic networks with arbitrary topologies, whose main characteristics are summarized in Table 6. We also give the references from where they were extracted. All links are bidirectional and there are connections to be established between all pairs of nodes. Lightpaths are directional, i.e. the lightpath from node $i \in X$ to node $j \in X$ may be different from that connecting j to i. Accordingly, there are $|X| \cdot (|X| - 1)$ lightpaths to be established. Lower bounds were obtained as the solution of the linear relaxation of an integer programming formulation of min-RWA.

Name	Nodes	Links	Connections	Lower bound
NSFnet [12]	14	21	182	13
USA [2]	32	50	992	69
Brazil	27	70	702	24
Finland [10]	31	51	930	46

Table 6: Characteristics of min-RWA instances.

The two approaches were applied to the four instances described in Table 6. In the first experiment, each approach was applied 20 times to each instance, with an upper bound of 10 minutes on the computation time. For each instance, Table 7 displays the minimum, average, and maximum solution values found by `2-EDR` + `TS-PCP` and `Greedy-EDP-RWA` over the 20 runs. Both algorithms found the same solution value for instances NSFnet and Brazil in all runs. The picture is not the same for the largest instances USA and Finland. In the case of instance USA, the new combined heuristic found the minimum solution value 69 in all but one run, while `Greedy-EDP-RWA` failed to find the minimum solution value in 18 out of the 20 runs. In the case of instance Finland, the new approach `2-EDR` + `TS-PCP` found solutions as good as 47 in all but two runs, while the best solution value obtained by `Greedy-EDP-RWA` in the same computation time was only 49.

In the final experiment, two hundred independent runs for each approach were performed

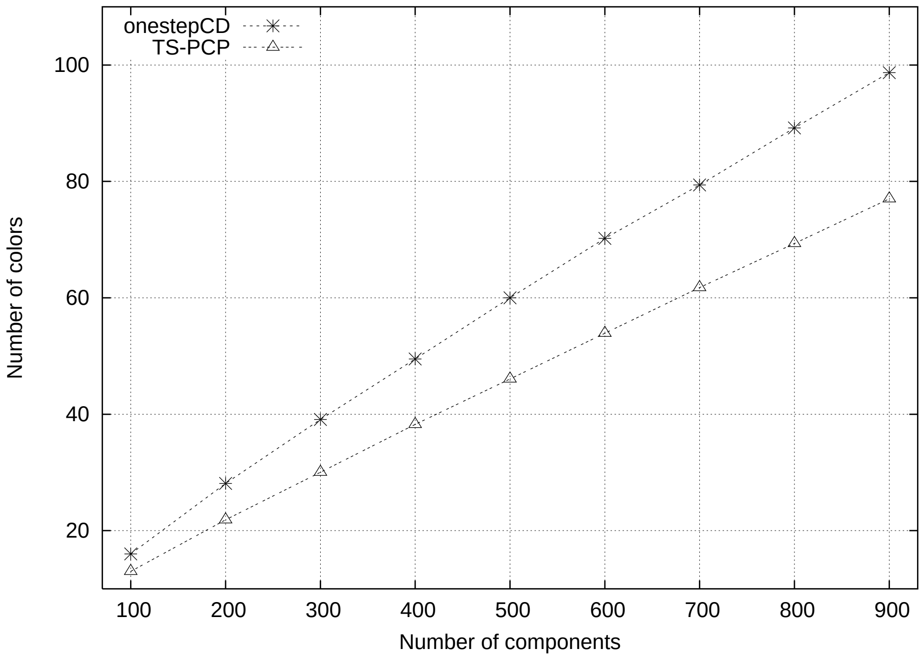


Figure 5: Average number of colours vs. number of components.

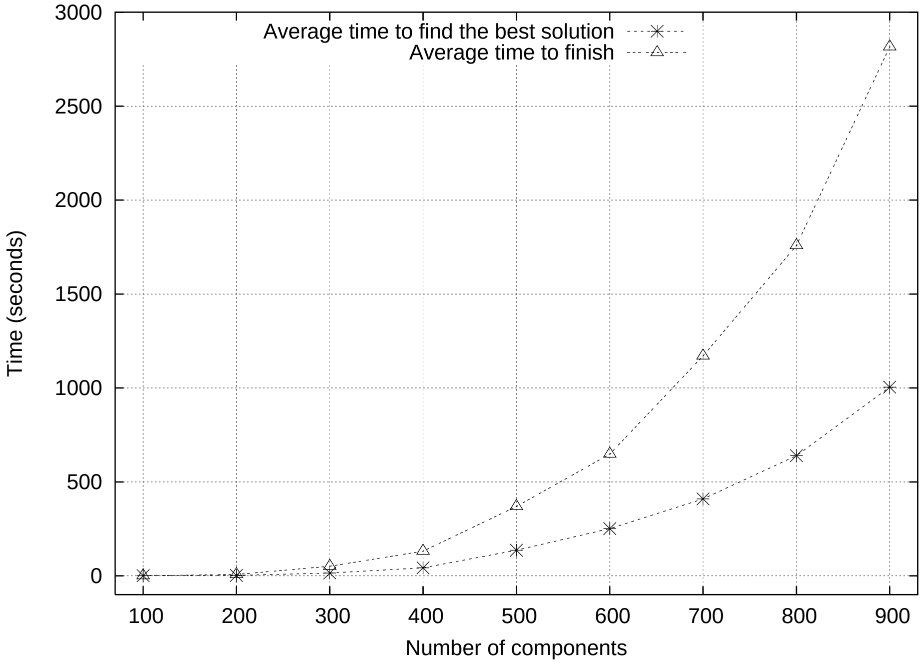


Figure 6: Average computation times for TS-PCP vs. number of components.

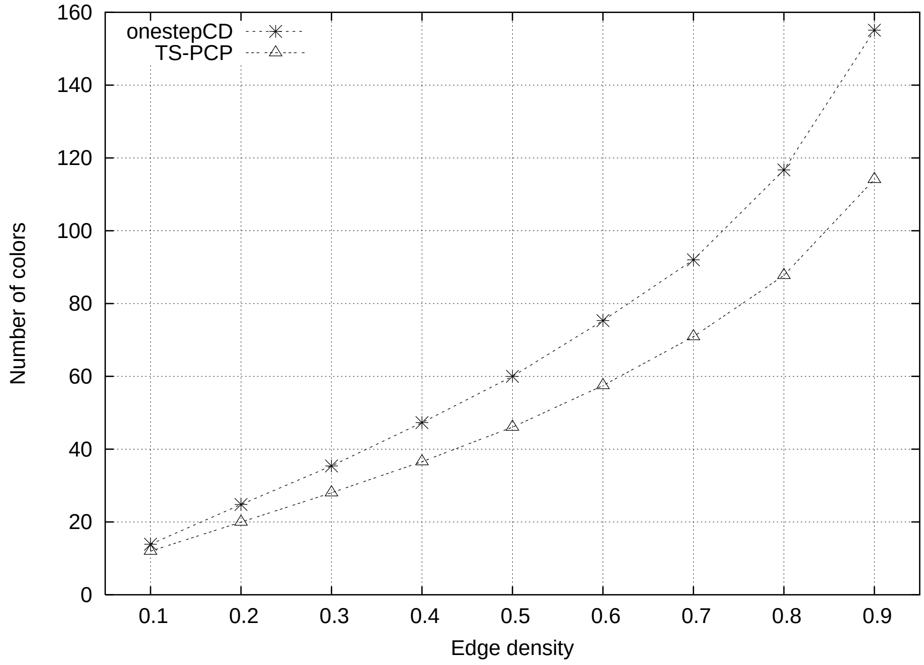


Figure 7: Average number of colours vs. edge density.

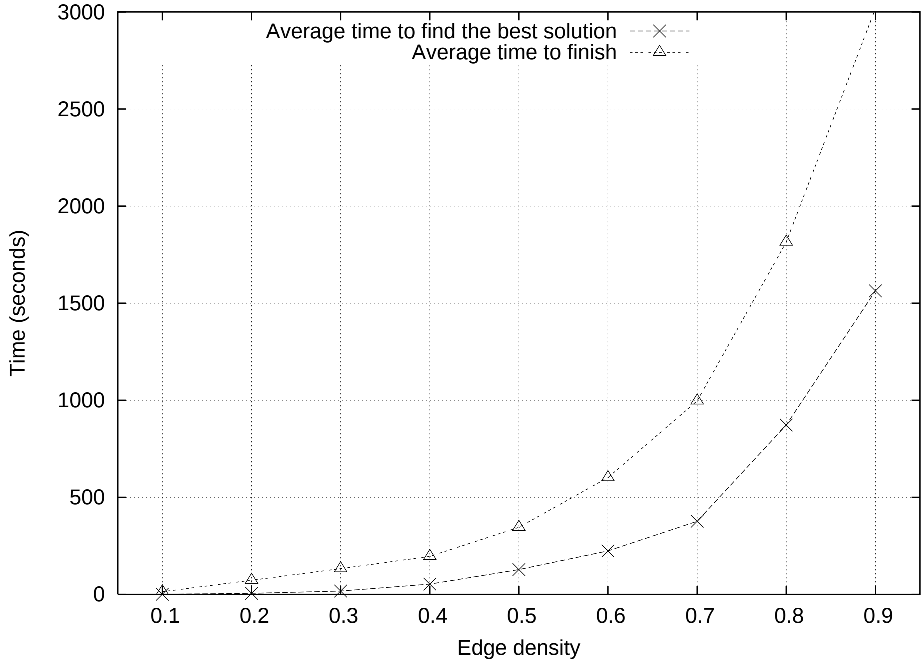


Figure 8: Average computation times for `TS-PCP` vs. edge density.

		2-EDR + TS-PCP	Greedy-EDP-RWA
NSFnet	min	13	13
	avg	13	13
	max	13	13
USA	min	69	69
	avg	69.05	69.90
	max	70	70
Brazil	min	24	24
	avg	24	24
	max	24	24
Finland	min	47	49
	avg	47.10	49.10
	max	48	50

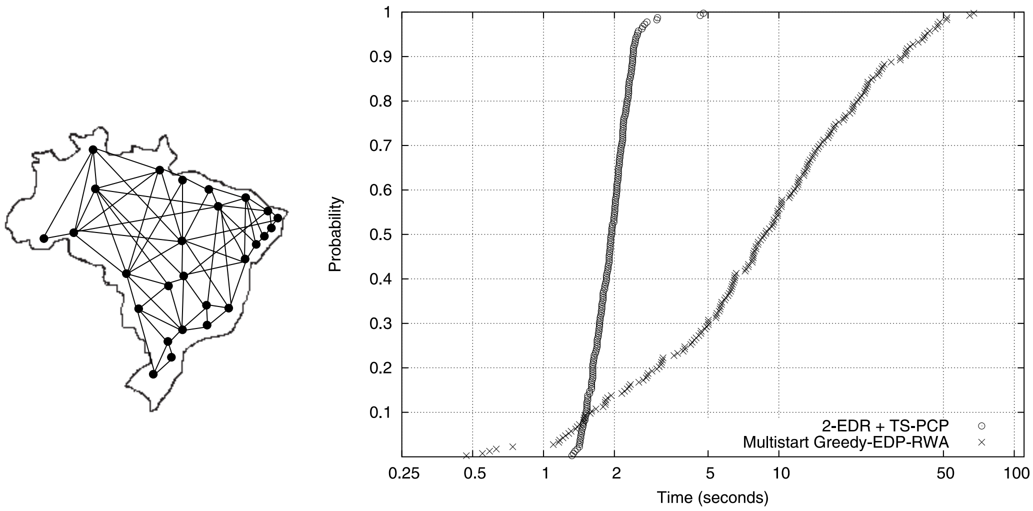
Table 7: Comparative results for 2-EDR + TS-PCP and Greedy-EDP-RWA on 10 minutes of CPU.

for each instance. Execution was terminated when a solution of value less than or equal to some target value was found. We used target values of 13 for instance NSFnet, 70 for instance USA, 24 for instance Brazil, and 50 for instance Finland. For each instance, these targets correspond to the best sub-optimal values listed in Table 7 which were found by both heuristics in all 20 runs within the upper bound of 10 minutes on the computation time. Empirical probability distributions for the time-to-target-solution-value are plotted. To plot the empirical distribution for each approach and each instance, we follow the methodology described in [1] and successfully applied to several other problems, see e.g. [8, 14]. We associate with the i-th smallest running time t_i a probability $p_i = (i - \frac{1}{2})/200$, and we plot the points $z_i = (t_i, p_i)$, for $i = 1, \ldots, 200$. The results obtained by the combined approach 2-EDR + TS-PCP proposed in this work are compared with those obtained by the multi-start heuristic Greedy-EDP-RWA proposed in [13].

Figures 9 to 12 illustrate the topology of each network and display the time-to-target-solution-value plot for each approach. The new combined strategy 2-EDR + TS-PCP is much more robust than the multi-start heuristic Greedy-EDP-RWA. The former is more robust and finds target solutions much faster than the latter. 2-EDR + TS-PCP clearly outperforms Greedy-EDP-RWA for instances Brazil and NSFnet. In the case of the largest instances USA and Finland, the combined approach 2-EDR + TS-PCP is much more stable than the multi-start heuristic, whose times to target solution values are much more spread and unpredictable.

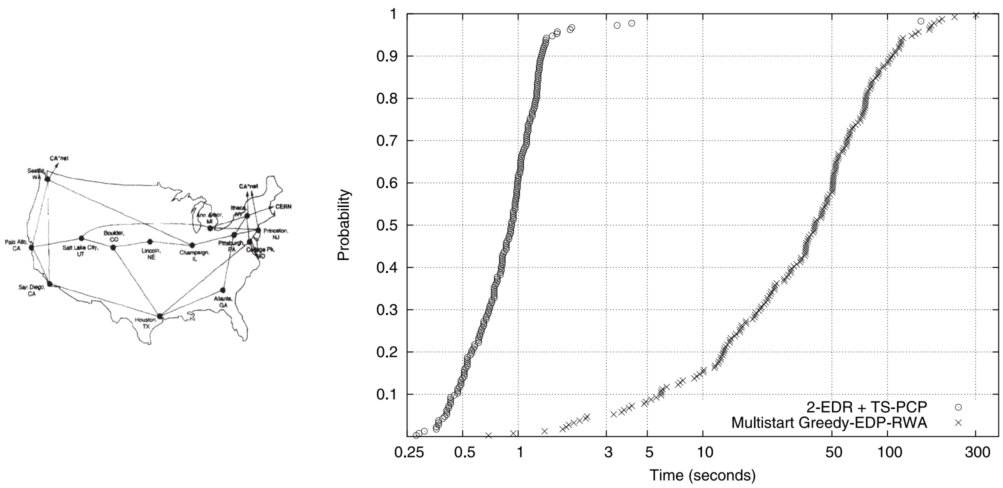
5 Concluding Remarks

In this work, we considered the partition colouring problem (PCP) in graphs and the problem of routing and wavelength assignment in all-optical networks (min-RWA). The former may be viewed as an extension of the graph colouring problem. The latter can be solved by a combined two-phase approach, involving the computation of alternative routes for the lightpaths by a heuristic for finding edge-disjoint paths, followed by the solution of a partition colouring problem in a conflict graph.



(a) (b)

Figure 9: Instance Brazil.



(a) (b)

Figure 10: Instance NSFnet.

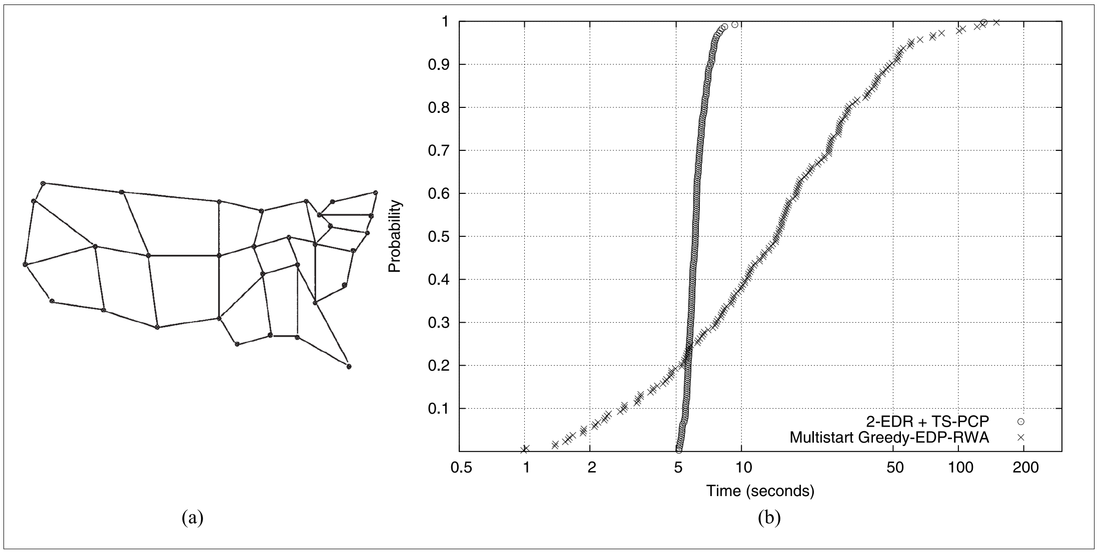


Figure 11: Instance USA.

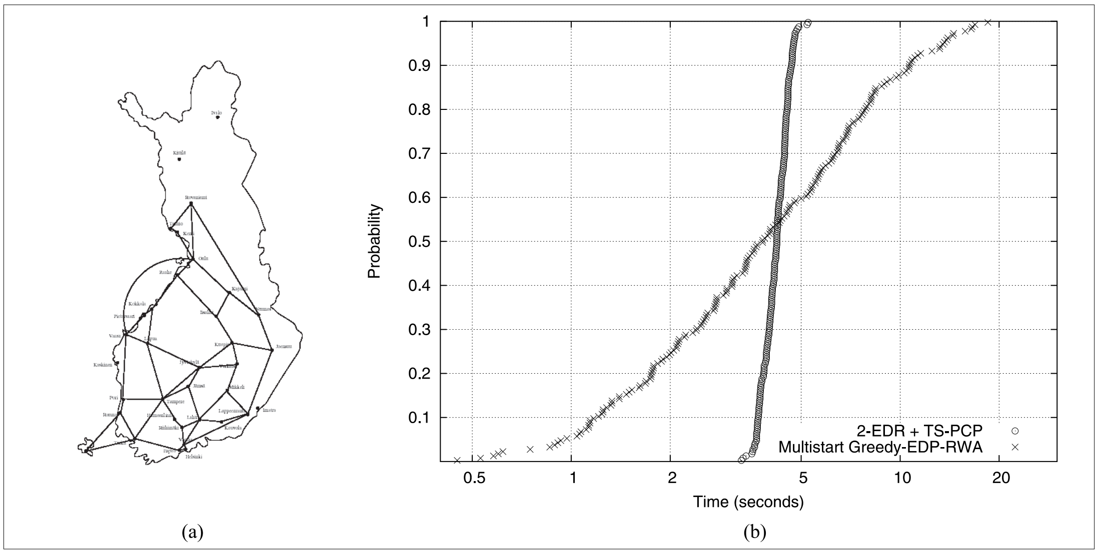


Figure 12: Instance Finland.

The main contribution of this paper is twofold. First, the tabu search heuristic `TS-PCP` for solving the partition colouring problem is proposed. Computational experiments are reported, showing that the tabu search algorithm outperforms the best previously known heuristic for partition colouring, reducing by approximately 20% in the average the number of colours used by algorithm `onestepCD`. The new heuristic was able to significantly improve the solutions obtained by the latter with a few tabu search iterations.

Next, we compared a combined two-phase approach for solving the problem of routing and wavelength assignment with a well-established multi-start heuristic. This new combined approach involves building alternative routes for the lightpaths using heuristic `2-EDR` for finding disjoint paths, followed by the solution of a partition colouring problem in a conflict graph using the tabu search heuristic `TS-PCP`. Numerical experiments have shown that the new strategy is able to find better solutions for larger instances using the same computation times. Moreover, the new `2-EDR` + `TS-PCP` approach for the problem of routing and wavelength assignment is much more robust and stable than the multi-start heuristic `Greedy-EDP-RWA`, whose times to target solution values are much more spread and unpredictable.

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