An application of integer programming to playoff elimination in football championships

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Abstract

Football is the most followed and practiced sport in Brazil, with a major economic importance. Thousands of jobs depend directly from the activity of the football teams. The Brazilian national football championship is followed by millions of people, who attend the games in the stadiums, follow radio and TV transmissions, and check newspapers, radio, TV, and, more recently, the Internet in search of information about the performance and chances of their favorite teams. Teams which are not qualified to the playoffs lose a lot of money and are even forced to dismantle their structure. We comment and compare the complexity of playoff elimination in football and baseball championships. We present two integer programming models which are able to detect in advance when a team is already qualified to or eliminated from the playoffs. Results from these models can be used not only to guide teams and fans, but are also very useful to identify and correct wrong statements made by the press and team administrators. The application and the use of both models in the context of the 2002 edition of the Brazilian national football championship are discussed.

Key words: Football, integer programming, championship, playoff elimination, qualification

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1 Motivation

The Brazilian national football championship is the most important football tournament in Brazil, and possibly the largest in the world in terms of the number of teams in competition. It is followed by millions of people, who attend the games in the stadiums, follow radio and TV transmissions, and check newspapers, radio, TV, and, more recently, the Internet in search of information about the performance and chances of their favorite teams. This tournament is organized in two stages: the first one is the qualification stage, while the second one is the playoff stage. During the qualification stage, each team plays every other in a single game. A team obtains three points for a win and one point for a tie. At the end of the qualification stage, the teams are ranked by the number of points they obtained along the tournament and the first eight teams in the standing table are qualified to the playoffs. Also, the last four teams are moved to a lower category and do not play in the next year. In 2002, this championship was played between August and December by the 26 major Brazilian teams. The 325 games played at the qualification stage were organized in 29 rounds of 13 or fewer games. Not all teams played at every round.

Football is a major economic activity in Brazil. Thousands of jobs in clubs, regional federations, newspapers, radio and TV stations, hotels, stadiums, and air companies depend directly from the activity of the teams. A team which is not qualified to the playoffs loses a lot of money and may be even forced to sell its best players to be able to cover the maintenance expenses during the almost two months of the playoffs, when it will lose all its income from tickets and TV rights.

Besides the obvious desire of its fans, the major goal of each team is to be qualified in one of the eight first positions in the standing table at the end of the qualification stage. For the teams which cannot match this objective, their second goal is, at least, not to finish in the last four positions to remain in the competition next year.

The media offers several statistics to help fans to evaluate the performance of their favorite team. One of this statistics is named the "qualification chance". This number measures, in an obscure way, the probability of each team to be qualified at the end of the first stage. Its computation is based on the performance ratio of each team, which is the fraction of points won with respect to the number of points already played. Early in the qualification stage, it is said that any team will be qualified if it reaches a certain threshold in the number of points won. The estimation of this threshold is an approximation based on the current statistics and on historical information.

When the first team reaches this threshold, the media says that its chance of qualification is 100% and that it is "mathematically qualified". Most often, this information is not correct. For example, this threshold was evaluated as 41 points in 2002. São Paulo reached 43 points at the 24th round on October 31 and the media loudly announced that São Paulo was "mathematically qualified". However, the system developed in this work was able to easily prove that this information was not true, showing a combination of results of the remaining games that led São Paulo out of the eight first positions. Even though the probability of such a combination of results was small, it would be a major surprise and a big failure with strong consequences if this team was not qualified at the end of the first stage.

The first problem we handle in this work is the *Guaranteed Qualification Problem* (GQP). It consists in calculating the minimum number of points any team has to win to be sure it will be qualified, regardless of any other results. This *Guaranteed Qualification Score* (GQS) depends on the current number of points of every team in the standing table and on the still missing games to be played. The second problem we tackle is the *Possible Qualification Problem* (PQP). It consists in computing how many points each team has to win to have any chance to be qualified. This *Possible Qualification Score* (PQS) also depends on the current number of points of every team in the standing table and on the still missing games to be played.

Of course, $PQS \leq GQS$ for any team at any time of the qualification stage. The value of GQS for any team cannot increase along the competition, while that of PQS cannot decrease. Winning a minimum of GQS (resp. PQS) points is a sufficient (resp. necessary) condition for qualification. A team is *mathematically qualified* to the playoffs if and only if its number of points won is greater than or equal to its GQS. Only at this point its qualification to the playoffs can be announced without any risk of misinformation. Analogously, a team can be said to be *mathematically eliminated* from the playoffs when the total number of points it still has to play (i.e., the number of remaining games multiplied by three) plus the number of points it already won (MNP) is less than its PQS. The formulations and the computer codes used in the solution of these two problems can be easily adapted to deal with the second goal of not finishing in the last four positions. It is just a matter of considering scores for remaining in the 22 first positions in the standing table, while in the first case it was a matter of standing in the eight first positions.

This paper is organized as follows. In the next section, we study the complexity of the Guaranteed Qualification Problem and we review previous work on similar problems in other sports such as baseball and basketball. The general formulation of qualification problems is discussed in Section 3. The integer programming models associated with problems GQP and PQP are given in Section 4. Numerical results and issues involving the application of these models to the 2002 Brazilian national football championship are discussed in Section 5. Concluding remarks are made in the last section.

2 Complexity

To illustrate the *Guaranteed Qualification Problem*, we consider a small example in which three teams are in competition for the last two places in the playoffs. The three teams are Flamengo, Cruzeiro, and Bahia, which have respectively 37, 37, and 36 points in the standing table:

Team	points
Flamengo	37
Cruzeiro	37
Bahia	36

Flamengo still has to play against Vasco and Bahia in the last two rounds, Cruzeiro against Grêmio and Santos, and Bahia against Flamengo and Fluminense. The question is: "How many points Cruzeiro has to obtain to be sure to be qualified to one of the two remaining places in the playoffs?" We first consider what happens if Cruzeiro wins one game and loses the other, obtaining 40 points at the end. In this case, if Flamengo beats Vasco and Bahia wins its two games, Bahia will have 42 points, Flamengo 40, and Cruzeiro 40. Flamengo and Cruzeiro will end up in a tie. Cruzeiro may not be qualified, depending on the tie breaking rule applied. Now, if Cruzeiro obtains four points, it will have 41 points at the end of the qualification stage. Depending on the result of the game between Flamengo and Bahia, there are three possible scenarios for the best possible final scores for these two teams: Flamengo 43 and Bahia 39 (Flamengo beats Bahia, which is eliminated), Flamengo 41 and Bahia 40 (there is a tie in the game between Flamengo and Bahia, and the latter is eliminated), or Flamengo 40 and Bahia 42 (Bahia beats Flamengo, which is eliminated). Cruzeiro is qualified to the playoffs in any of these three scenarios. This small example illustrates the kind of situation which may happen even for a small number of teams playing for qualification. The problem is certainly much more complex and difficult to analyze when there are many more teams in competition for more places in the playoffs.

Almost all the previous work dealing with this kind of problem concerns the Major League Baseball (MLB) in the United States. According with Gusfield and Martel's notation [3], we say that MLB follows the {(1,0)} rule: since a game cannot end up with a tie, one team will necessarily obtain one point and the other one zero. The Brazilian football championship follows the {(3,0),(1,1)} rule: if one team wins it makes three points and the other zero, if the game ends up with a tie, both teams obtain one point each. It is showed that qualification problems are easier for tournaments following the {(1,0)} rule than for those under the {(3,0),(1,1)} rule [2]. Another characteristic which makes MLB computations easier, is that until a few years ago the only goal in the MLB was to finish in the first place of small groups of at most six teams each, leading to much fewer possible combinations of results.

Schwartz [7] solved the problem of determining if a team is eliminated in the MLB with a maximum flow algorithm. Also for the MLB, Hoffman and Rivlin [4] described necessary and sufficient conditions for a team not to be qualified in the first k positions. Robinson [6] gave a linear integer programming model to calculate the maximum difference in the number of points a team can win by at the end of baseball season, with respect to that in second place. McCormick [5] showed that to decide if a team cannot finish among the first k positions is NP-Complete. Adler et al. [1] showed how integer programming can be used to compute the *Guaranteed Qualification Score* and the *Possible Qualification Score* in the case of the MLB. Wayne [8] showed that there exists a number of points such that every team is eliminated from the quest for the first place in the MLB if and only if it cannot reach this threshold. Gusfield and Martel [3] generalized Wayne's result, showing that this threshold exists for every tournament with certain characteristics, including football leagues following the {(3,0),(1,1)} rule. Bernholt et al. [2] showed that deciding football elimination from the first place under the {(3,0),(1,1)} rule is NP-Complete. Thus, football elimination from the first k positions (and, consequently, problem GQP) is also NP-Complete.

3 Problem formulation

In this section, we state the problems introduced in the previous section, in the context of the Brazilian national football championship under the {(3,0),(1,1)} rule. Let n be the number of teams in the championship and m the number of teams which qualify to the playoffs.

We denote by p_i the total number of points accumulated by each team $i = 1, \ldots, n$ at any particular moment of the championship, with $p_i = 0$ at the beginning. For any pair (i, j) of teams, with $i, j = 1, \ldots, n$ and $i \neq j$, let $g_{ij} = g_{ji}$ be the number of remaining games still to be played between teams i and j. Since the rules of the Brazilian national football championship establish that each team plays against every other team exactly once, then either $g_{ij} = 0$ in case the game between teams i and j was already played, or $g_{ij} = 1$ otherwise. At any time, a *valid assignment* is a set of triples $A(i, j) = (p_1(i, j), p_2(i, j), p_3(i, j))$ of non-negative integers for each pair (i, j) of different teams, such that $p_1(i, j) + p_2(i, j) + p_3(i, j) = g_{ij}$, $p_1(i, j) = p_3(j, i)$, and $p_2(i, j) = p_2(j, i)$, where $p_1(i, j)$, $p_2(i, j)$, and $p_3(i, j)$ represent respectively a possible number of victories of team i over team j, a possible number of games between team i and j which end up with a tie, and a possible number of victories of team j over team i along the remaining g_{ij} games.

Given a valid assignment, the total number of points accumulated by each team $i = 1, \ldots, n$ at the end of the championship is

$$t_i = p_i + \sum_{j \neq i} 3 \cdot p_1(i,j) + \sum_{j \neq i} p_2(i,j).$$

For every valid assignment and in the context of the *Guaranteed Qualification Problem*, the final position of team $i = 1, \ldots, n$ in the standing table is defined as $P_i = |\{j : 1 \leq j \leq n, j \neq i, t_j \geq t_i\}| + 1$. Then, GQP for any team k consists in finding the minimum integer GQS^k such that for every valid assignment if $t_k \geq GQS^k$ then $P_k \leq m$.

Similarly, in the context of the *Possible Qualification Problem*, the final position of team $i = 1, \ldots, n$ in the standing table is defined as $P_i = |\{j : 1 \leq j \leq n, j \neq i, t_j > t_i\}| + 1$. Then, PQP for any team k consists in finding the minimum integer PQS^k such that there exists at least one valid assignment leading to $t_k = PQS^k$ and $P_k \leq m$.

The definitions of the position of a team in the standing table in the contexts of problems GQP and PQP are different. In the context of problem GQP, to insure that team k is qualified despite any tie breaking rule, every other team with the same number of points as k is considered as qualified before the latter. Contrarily, in the context of problem PQP, we just consider the possibility (i.e., not the certainty) of qualification. In this case, team k has a chance to be qualified even if there is a tie with other teams in the first positions.

4 Integer programming model

The notation defined in the previous section is used to formulate an integer programming model for the *Guaranteed Qualification Problem*.

For any team $k = 1, \ldots, n$, let $\underline{GQS}^k$ be the maximum number of points such that there exists a valid assignment leading to $t_k \geq \underline{GQS}^k$ and $P_k > m$ at the end of the championship. Then, $\underline{GQS}^k$ is the maximum number of points a team can make and still not be qualified. Therefore, $GQS^k = \underline{GQS}^k + 1$ is the minimum number of points team i has to obtain to ensure its qualification among the first m teams. We define the following variables:

$$x_{ij} = \begin{cases} 1, \text{ if team } i \text{ wins over team } j \\ 0, \text{ otherwise;} \end{cases}$$

$$y_j = \begin{cases} 1, \text{ if } t_j \geq t_i \text{ (i.e. if team } i \text{ is not ahead } j) \\ 0, \text{ otherwise.} \end{cases}$$

The following integer linear programming model computes $\underline{GQS}^k$ for team $k = 1, \ldots, n$, i.e., the maximum number of points team k can make and still not be qualified:

$$
\text{GQP}(k): \begin{cases}
\underline{GQS}^k = \quad \text{maximum } t_k & & \\
\text{subject to:} & & \\
\qquad x_{ij} + x_{ji} \leq g_{ij} & \forall\, 1 \leq i < j \leq n & (1) \\
\qquad t_j = p_j + 3\sum_{i \neq j} x_{ji} + \sum_{i \neq j}[1 - (x_{ij} + x_{ji})] & & \\
 & \forall\, 1 \leq j \leq n & (2) \\
\qquad t_k - t_j \leq M(1 - y_j) & \forall\, 1 \leq j \leq n, j \neq k & (3) \\
\qquad \sum_{j \neq k} y_j \geq 8 & & (4) \\
\qquad x_{ij} \in \{0,1\} & \forall\, 1 \leq i \leq n, 1 \leq j \leq n, i \neq j & \\
\qquad y_j \in \{0,1\} & \forall\, 1 \leq j \leq n\,, j \neq k. & \\
\qquad t_j \geq 0 & \forall\, 1 \leq j \leq n &
\end{cases}
$$

The objective function consists in determining the maximum number of points $\underline{GQS}^k$ team k can make and still not be qualified. Constraints (1) determine that only one team can win a game. Constraints (2) establish the total number of points obtained by each team at the end of the championship. For each team $j = 1, \ldots, n$, the sum $\sum_{i \neq j} x_{ji}$ gives the number of games won by team j among those still remaining to be played (three points each), while $\sum_{i \neq j}[1-(x_{ij}+x_{ji})]$ corresponds to the number of games involving team j that end up with a tie (one point each).

Let M be an upper bound to the maximum difference between the number of points obtained by any pair of teams. Since there are 26 teams in the championship and each of them plays exactly once against every other, $M \geq 3 \cdot 25 = 75$ is a valid upper bound to $|t_j - t_k|$ for any pair (j, k) of teams, with $j, k = 1, \ldots, n$ and $j \neq k$. Constraints (3) state that if $t_j < t_k$, then $y_j = 0$ (i.e., team k is ahead j in the standing table). Constraint (4) enforces that team k is not qualified among the first $m = 8$ teams.

Recall that GQP(k) determines the maximum number of points a team can make and still not be qualified in the first m positions. Then, team k is mathematically qualified when GQP(k) turns out to be infeasible. The tighter is the value of the upper bound M, the faster will be the solution time of any solver applied to GQP(k). A tighter, and consequently better, upper bound to replace M in constraint (3) is given by $M_j = p_j - p_k + 3\sum_{i \neq j} g_{ji}$. This tighter upper bound is used in the actual implementation.

We now address the integer programming formulation to the *Possible Qualification Problem* for team $k = 1, \ldots, n$. Let PQS^k be the minimum number of points such that there exists at least one set of valid assignments leading to $t_k = PQS^k$ and $P_k \leq m$ at the end of the championship. We define the following additional variables:

$$z_j = \begin{cases} 1, \text{ if } t_j > t_k \text{ (i.e. if team } j \text{ is ahead } k) \\ 0, \text{ otherwise.} \end{cases}$$

The previous model can be reformulated as follows to deal with the new situation:

$$\text{PQP}(k): \begin{cases} PQS^k = \quad \text{minimum } t_k \\ \text{subject to:} \\ \qquad x_{ij} + x_{ji} \leq g_{ij} \quad \forall\, 1 \leq i < j \leq n & (1) \\ \qquad t_j = p_j + 3\sum_{i \neq j} x_{ji} + \sum_{i \neq j} [1 - (x_{ij} + x_{ji})] \\ \qquad\qquad \forall\, 1 \leq j \leq n & (2) \\ \qquad t_j - t_k \leq M z_j \quad \forall\, 1 \leq j \leq n, j \neq k & (3') \\ \qquad \sum_{j \neq k} z_j \leq 7 & (4') \\ \qquad x_{ij} \in \{0,1\} \qquad \forall\, 1 \leq i \leq n, 1 \leq j \leq n, i \neq j \\ \qquad z_j \in \{0,1\} \qquad \forall\, 1 \leq j \leq n\,, j \neq k. \\ \qquad t_j \geq 0 \qquad \forall\, 1 \leq j \leq n \end{cases}$$

Constraints (3') plays the same role as (3) in the formulation of GQP(k). It enforces that if $t_j > t_k$, then $z_j = 1$ (i.e., team j is ahead team k in the standing table). The same tighter upper bound M_j replaces M in the actual implementation. Constraint (4') states that there are at most $m-1 = 7$ teams ahead k in the standing table. The infeasibility of PQP(k) means that this team is mathematically eliminated, i.e., it cannot be qualified anymore to the playoffs.

A nice feature of the above models is that they can be easily extended to accommodate some of the most usual tie breaking rules. In the case of the Brazilian national football championship, the first tie breaking rule to be applied is the number of wins obtained by each team: for any two teams with the same number of points at the end of the championship, the one with more wins is qualified before the other. A very slight modification in the models can handle this rule: replace constraints (2) by $t_j = p_j + (3+\epsilon)\sum_{i \neq j} x_{ji} +$

$\sum_{i \neq j}[1-(x_{ij}+x_{ji})]$. With this modification, each win gives $3+\epsilon$ points instead of only 3. Since each team plays 25 games, if we choose $\epsilon < 1/25$ the order determined by the number of points obtained at the end of the championship is not affected by this modification. However, if two teams obtain the same number of points, that with more wins will be ahead the other. Further tie breaking rules based e.g. on the number of goals scored for and against cannot be embedded into a 0-1 integer programming model.

5 Results and applications

The 2002 edition of the Brazilian national football championship started on August 10. Problems GQP and PQP were considered in two contexts. In the first one, we were interested in observing statistics concerning the quest for the first eight positions in the standing table, which define the eight teams qualified to the playoffs. In the second case, we were interested in observing the teams in the four last positions of the standing table, since these teams will not be allowed to play in the first division in the next year. Not being qualified among the first eight teams to the playoffs is a major loss, since these are the games with more attendance and higher TV rights. Being qualified among the last four teams brings enormous difficulties for these teams, which will possibly have to dismantle their structure due to the smaller importance of the games to be played in the next year in the second division. The press and fans of every team speculate and follow the standing table every day.

Models GQP(k) and PQP(k) were solved immediately after the games in each round were completed, for every team $k = 1, \ldots, n$. Since there were 26 teams in the championship and we were interested in computing the guaranteed qualification score and the possible qualification score for each of them in the two above contexts (qualification to the playoffs in the first eight positions - i.e., $m = 8$ - and not finishing in the last four positions - i.e., $m = 22$ -), 104 integer programming problems had to be solved immediately after every round was completed. We used CPLEX 5.0 as the integer programming solver. *GQS* and *PQS* scores for each team, together with other statistics, were immediately made available at the web site of the *FutMax* project at the URL `http://www.inf.puc-rio.br/brasileirao2002`. Computation times are high in the very initial rounds of the tournament. However, after some rounds towards the end of the qualification stage, when the interest for the outcome of the models is maximum, the computation times amount to just a few seconds on a standard Pentium IV processor. Table 1 displays the processing times (in seconds) for computing the *GQS* scores for Fluminense at every round.

On September 29 the authors were interviewed by sport journalists from *Globo* radio station. This interview generated a huge debate about the quality and

Round	seconds	Round	seconds	Round	seconds
1	871.2	11	14.6	21	0.9
2	856.8	12	16.0	22	0.7
3	743.4	13	15.6	23	0.7
4	697.0	14	15.3	24	0.7
5	710.3	15	1.3	25	0.6
6	677.8	16	4.7	26	0.5
7	757.5	17	2.9	27	0.1
8	457.1	18	2.6	28	<0.1
9	888.6	19	2.1	29	<0.1
10	39.7	20	2.1		

Table 1
Processing times for computing the GQS scores.

the use by the press of the GQS and PQS scores and other statistics. The *FutMax* web site was reviewed by *Jornal do Brasil*, one of the major newspapers in Brazil, on September 30.

The models presented in Section 4 and the web site of the *FutMax* project showed their usefulness several times during the championship. After the 11th round, on September 15, Vasco da Gama's (one of the major teams in Brazil) coach, Antonio Lopes, said to the media in a press interview that his team would be qualified to the playoffs if it could win ten out of the remaining 14 matches to be played. Our work showed that his statement was not true and we were able to show that there existed a set of possible results for the remaining games that led Vasco da Gama outside the qualification zone even if it had won all of its remaining games.

São Paulo reached 43 points at the 24th round on October 31. Using performance estimations, the press announced it was the first team mathematically qualified to the playoffs. Once again, we have been able to show this was not true. Table 2 (a) displays the standing table after the 24th round, while Table 2 (b) illustrates a possible scenario for the standing table at the end of the championship, in which São Paulo could finish beyond the 8th position and not be qualified to the playoffs.

Figures 1, 2, and 3 displays plots illustrating the evolution of the GQS and PQS scores round after round for three different teams. Besides these two scores, we also plot the number of points P obtained after each round and the maximum number of points MNP that could be obtained if the team won all remaining games (three additional points for each of them). We recall that PQS and P cannot decrease, while GQS and MNP cannot increase along the

	Team	points	wins
1	**São Paulo**	43	13
2	São Caetano	39	12
3	Corinthians	38	11
4	Juventude	37	11
5	Santos	36	10
6	Atlético MG	33	10
7	Grêmio	33	9
8	Fluminense	31	9
9	Coritiba	30	9
10	Ponte Preta	30	9
11	Goiás	30	8
12	Vitória	28	8
13	Guarani	28	8
14	Figueirense	27	8
15	Atlético PR	27	7
16	Portuguesa	26	7
17	Bahia	25	7
18	Internacional	25	6
19	Cruzeiro	24	6
20	Vasco da Gama	23	7
21	Paraná	23	7
22	Palmeiras	23	5
23	Paysandú	22	7
24	Gama	22	6
25	Flamengo	22	6
26	Botafogo	22	5

(a)

	Team	points	wins
1	Corinthians	48	14
2	Fluminense	46	14
3	Juventude	46	14
4	Coritiba	45	14
5	Atlético MG	45	14
6	São Caetano	45	14
7	Grêmio	45	13
8	Santos	45	13
9	**São Paulo**	45	13
10	Guarani	40	12
11	Vitória	38	11
12	Ponte Preta	32	9
13	Goiás	32	8
14	Figueirense	29	8
15	Vasco da Gama	29	8
16	Portuguesa	29	7
17	Atlético PR	29	7
18	Cruzeiro	28	6
19	Internacional	27	6
20	Paraná	26	7
21	Bahia	26	7
22	Paysandú	25	7
23	Flamengo	25	6
24	Palmeiras	25	5
25	Botafogo	25	5
26	Gama	23	6

(b)

Table 2
(a) The standing table after the 24th round and (b) a possible final standing table at the end of the championship.

championship. A team is mathematically qualified to the playoffs as soon as $P \geq GQS$. Analogously, a team is mathematically eliminated from the playoffs when $MNP < PQS$. A not yet qualified team depends only on its own to be qualified to the playoffs if $MNP \geq GQS$. Similarly, a not yet qualified team depends from results of the other teams to be qualified if $MNP < GQS$.

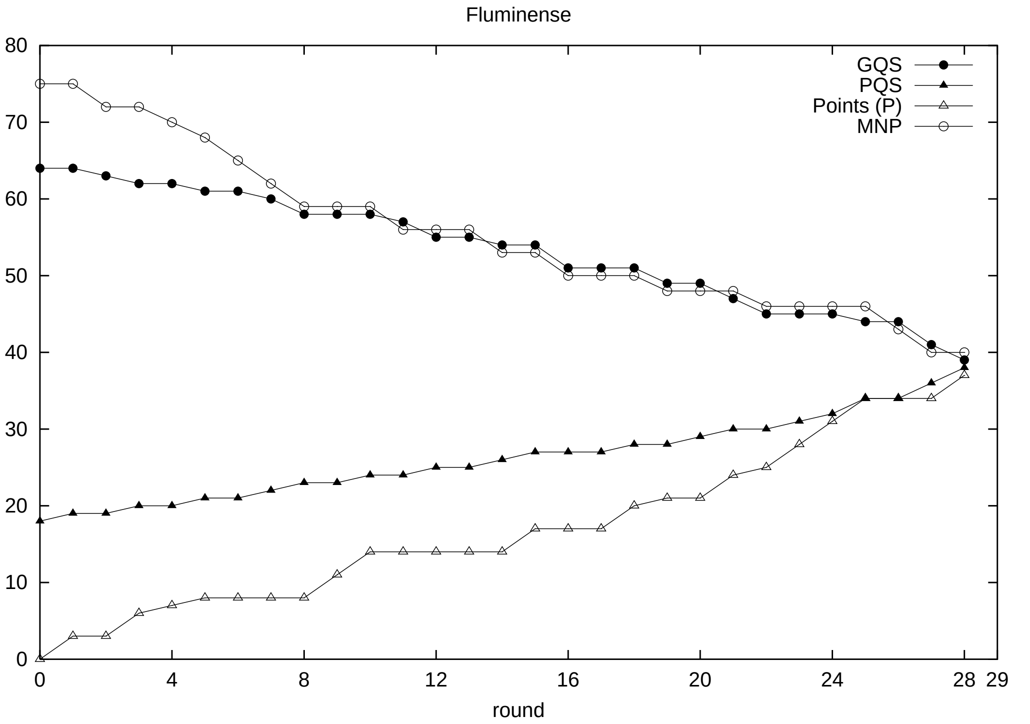

Fig. 1. Fluminense in the 2002 Brazilian national football championship

Figure 1 illustrates that MNP got smaller than GQS for Fluminense at the 11th round, showing that at this time Fluminense was not on its own: even if it obtained the maximum number of possible points by wining all its remaining games, there existed at least one set of results leading Fluminense outside of the qualification zone. At the 25th round, the number of points made by Fluminense reached its PQS, meaning that it had a chance to qualify even if it lost all remaining games. Indeed, due to a sequence of favorable results, Fluminense did qualified at the 8th position only at the last round.

The plot in Figure 2 illustrates the scores for Palmeiras. This team was mathematically eliminated from the playoffs at the 25th round. In fact, Palmeiras was one of the teams in the last four positions and was indeed eliminated from the 2003 edition of the championship. The plot in Figure 3 displays the same scores for São Caetano. This team was always on its own and guaranteed its qualification at the 26th round.

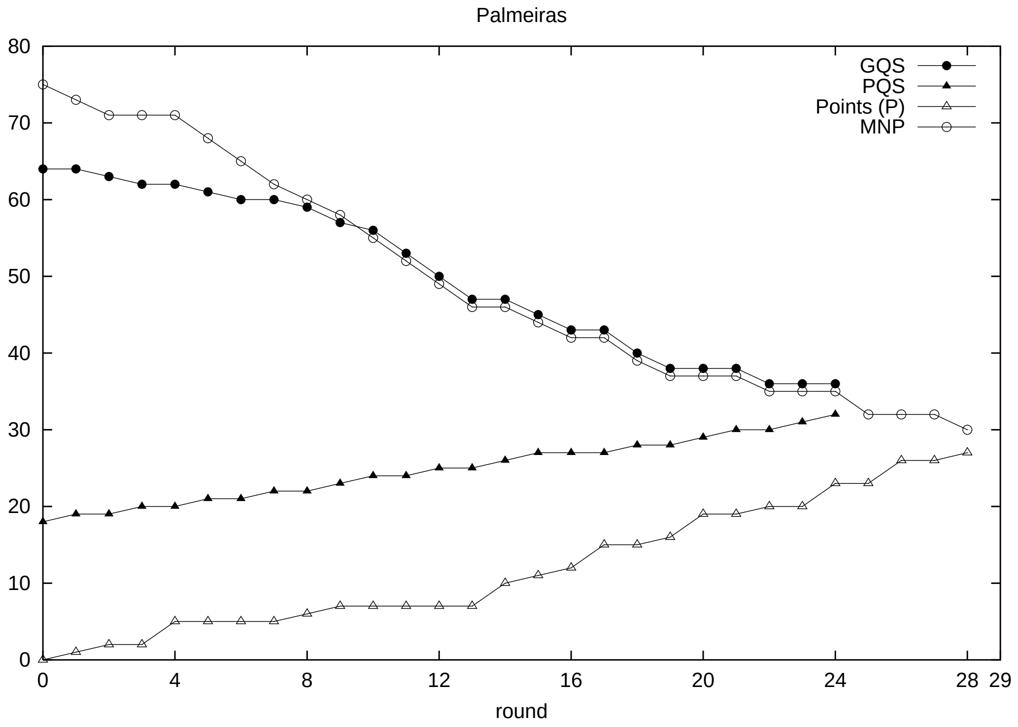

Fig. 2. Palmeiras in the 2002 Brazilian national football championship

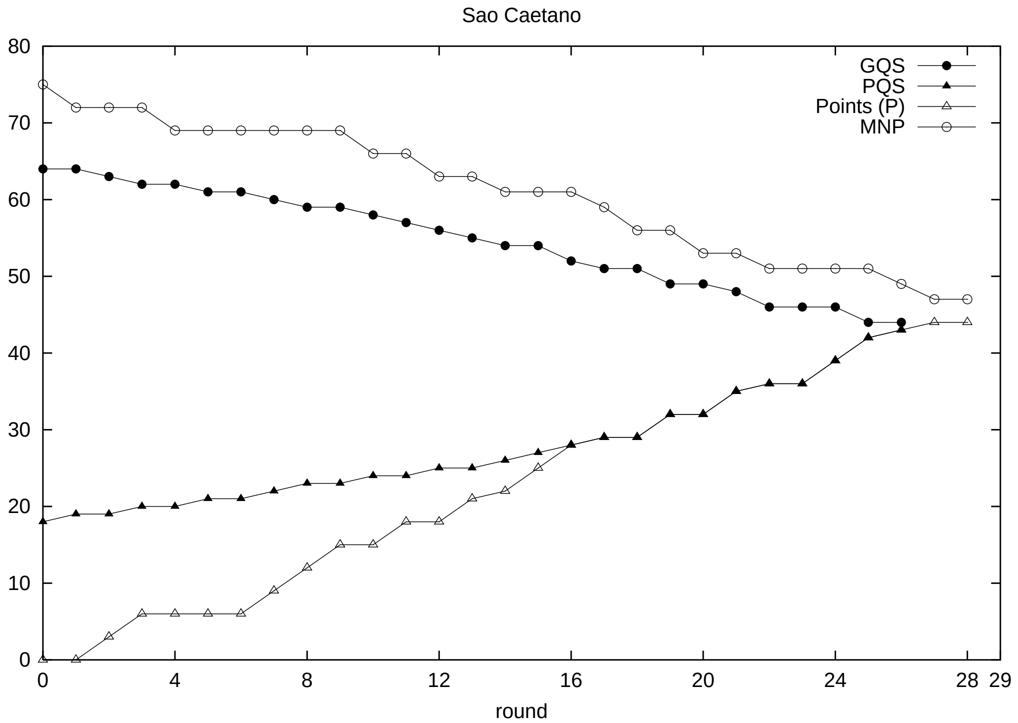

Fig. 3. São Caetano in the 2002 Brazilian national football championship

6 Concluding remarks

We considered in this work an application of integer programming to playoff elimination in football tournaments. The two models presented give necessary and sufficient conditions to ensure the qualification of a team in the first m positions.

Football is the most practiced sport and a major economic activity in countries such as Brazil. The models presented in this work are very useful tools for team administrators, the press, and the fans. In particular, they are quite effective to correct common misleading statements made by the press and team managers, indicating the exact number of points a team has to obtain to ensure its qualification, contrarily to probabilistic models which estimate chances of qualification.

The two models were applied in the context of the 2002 edition of the Brazilian national football championship and showed their usefulness in several occasions, as commented in the previous section. In addition, the web site created to make these statistics available to the public was also a useful tool to make this information available to the fans. The web site of the *FutMax* project was reactivated in the second half of 2003 at the URL `http://www.futmax.org` to follow the 2003 edition of the Brazilian national football championship and the Latin America qualification round for the XVIII World Cup to be played in 2006. This site will remain active following other football competitions. The study reported in this paper and the two proposed models constitute a motivating application of Operations Research techniques in the area of sports scheduling and management. Other applications developed by our research group are permanently reviewed in our web site `http://www.esportemax.org`.

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