Scheduling the Brazilian soccer tournament by integer programming maximizing audience shares under fairness constraints

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Motivation

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Mathematical programming in sports scheduling

- Mathematical programming techniques have been applied to a number of theoretical and real sport scheduling problems.

- Techniques: integer programming, constraint programming, heuristics, metaheuristics, etc.

- Schedule of a competition may have a great influence in its success, in terms of interest, income, and fairness.

- Fans and team administrators strongly believe on the role played by some constraints.
About Brazil and its soccer tournament

- Soccer is the most important sport in Brazil.
- Brazilian soccer tournament is by far the most important national sport event in Brazil.
- Two biggest cities in Brazil are São Paulo (21 millions) and Rio de Janeiro (14 millions): largest audience shares.
- Teams from São Paulo and Rio de Janeiro are traditionally among the best and those with more fans.
- 20 teams play in the first division of the 2009 edition of the Brazilian soccer tournament.
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Round robin tournaments

- Round robin tournaments: any team play against every other a given number of times in a given number of rounds.
- Single (SRR) and double round robin (DRR) tournaments are the most common.
- Common restriction: any team can play at most one game per round.
- Compact schedule: number of rounds is minimum.
- A compact SRR schedule for $n$ (even) teams has $n - 1$ rounds and every team plays exactly one game per round.
Round robin tournaments

- Each team is usually linked to a certain location (facility, stadium, etc.).
- Each game is played in the location of one of the opponents.
- Home games vs. away games
- A home-away pattern (or simply a pattern) is associated with each team: HAHAHAHA
- A break occurs whenever a team plays two consecutive home games or two consecutive away games.
- The set of patterns associated with the teams is called the pattern set.
Pattern set feasibility

- Problem: given a pattern set, find a schedule where each team plays according to its pattern.
- Necessary condition for feasibility: every team has a different pattern in a feasible pattern set.
- At every round, half of the teams play at home and half away.
- More complex necessary conditions exist.
- No polynomial time characterization for pattern set feasibility seems to exist for general SRR schedules.
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Brazilian soccer tournament

- Brazilian soccer tournament is a compact mirrored DRR:
  - Two consecutive compact SRR phases.
  - Rounds involve the same games in each phase, but teams have complementary patterns.
- Tournament may be seen as a SRR, with constraints on both phases of the schedule.
- Scheduling the Brazilian soccer tournament may be seen as a constraint satisfaction problem.
- Three groups of constraints: pattern constraints, pattern set constraints, and timetable constraints.
Pattern constraints

- **Number of breaks is minimum:**
  - There are only two teams with patterns without breaks: HAHAH... and AHAHA...
  - All other teams will have at least three breaks each (one in the first phase, one in the second, and one between the two phases.
  - Minimum number of breaks becomes $3(n - 2)$

- **All teams must have the same number of breaks:**
  - Then, the minimum number of breaks is $3n$.

- **Every team playing the first game at home should play the last away (and vice versa):**
  - Every team must have a even number of breaks.
  - In this case, the minimum number of breaks is $4n = 80$. 
Further pattern constraints

- For any team, the number of games still to be played at home and away at any moment of the second phase should differ by at most one unity:
  - Then, breaks must occur only at even rounds.
- No consecutive breaks.
- No breaks in the first four or last two rounds.
- Therefore, there are only 42 feasible patterns.
Pattern set constraints

- All teams are paired.
- Teams in the same pair have complementary patterns.
- Teams in cities with exactly two teams are paired together.
- Some teams can only take a subset of the feasible patterns due to the timetable constraints.
Timetable constraints

- Games in fixed dates:
  - Important games at the first round, classical regional games in important dates, etc.
- No regional games in midweek rounds.
- No more than five consecutive games against strong teams.
- No games between teams in the same pair before the seventh round.
- No team plays two classic games in consecutive rounds.
- Many others...
Timetable constraints: TV constraints

- There must be at least one attractive game to be broadcast to São Paulo at every round, involving a team from São Paulo playing away against one of the strong G-13 teams.
- There must be at least one attractive game to be broadcast to Rio de Janeiro at every round, involving a team from Rio de Janeiro playing away against one of the strong G-13 teams.
- Lower and upper bounds on the number of games played between strong teams in any round.
- Number of games that can be broadcast by pay-per-view channels.
- Many others...
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- Integer programming approach

### Decomposition scheme

- Same approach already used in the literature.
- Three stages:
  1. Pattern creation (complete enumeration)
  2. Pattern assignment (randomly, but observing the constraints)
  3. Timetable creation (integer programming)
- Key strategy: consider the constraints as early as is possible.
Pattern creation

- 42 patterns are created by (explicit) complete enumeration.
- 21 pairs of complementary patterns.
- Every pattern satisfies the pattern constraints.
Pattern assignment

- The assignment is random but ...
- ... paired teams have complementary patterns.
- ... there might be no feasible pattern for some teams.
- All pattern assignments satisfy the pattern set constraints.
- No pattern set feasibility test is performed.
Timetable creation

- Integer programming model generated considering the pattern assignments.
- Binary variable $x_{ijk} = 1$ if the game between teams $i$ and $j$ take place at the stadium of team $i$ in round $k$.
- Timetable constraints are easily modeled: for example, $x_{1,2,10} + x_{2,1,10} = 1$ sets the game between teams 1 and 2 at round 10.
Timetable creation

- Solve the model using an integer programming solver (e.g. CPLEX).
- Infeasibility is usually determined in less than one second.
- If the problem is infeasible, return to the pattern assignment stage.
- If the problem is feasible, a feasible schedule was found.
- Stop or continue searching for alternate schedules.
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Implementation and tests

- CPLEX 9.0 used as the linear and integer programming solver.
- Software coded in C++ runs on a standard Pentium IV processor.
- Approach applied to the 2005 and 2006 editions of the Brazilian tournament:
Results for 2005 and 2006

- Schedules optimizing both the broadcast and the fairness objectives obtained in less than ten minutes of running time.
- Official schedules used in 2005 and 2006 violated some of the problem requirements: new schedules satisfy all constraints.
- Feasible solutions produced by integer programming (with all soft constraints satisfied) were much better than infeasible schedules produced by the current scheduler.
- Integer programming approach lead to schedules in which all 56 more attractive games could be broadcast by open TV channels, while the ad hoc rules used before made it possible to broadcast only 43 and 47 games, respectively.
Results for 2005 and 2006

<table>
<thead>
<tr>
<th></th>
<th>Official schedule</th>
<th>Optimized schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>All constraints</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Breaks</td>
<td>156</td>
<td>80 (optimal)</td>
</tr>
<tr>
<td>Broadcast</td>
<td>43</td>
<td>56 (optimal)</td>
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</tbody>
</table>

**Tabela:** 2005 edition of the Brazilian tournament (22 teams).

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**Tabela:** 2006 edition of the Brazilian tournament (20 teams).
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Conclusions

- Due to the good results obtained on historical data, a software system implementing this approach was developed and made operational through a partnership with the Brazilian Soccer Confederation (CBF).

- System was used for the first time in 2009 as the official scheduler to build the fixtures of the first and second divisions of the Brazilian national soccer tournament.

- Several alternative fixtures were provided to the users, who selected the best choice.

- Fixtures obtained with this system for both the first and second divisions of the 2009 edition of the Brazilian national soccer tournament were announced last January.