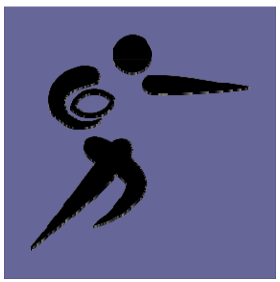
A hybrid heuristic for minimizing weighted carry-over effects in round robin tournaments




Allison C. B. **Guedes**

Celso C. **Ribeiro**

Summary

- Optimization problems in sports
- Preliminary definitions
- The carry-over effects minimization problem
  - Weighted version
- Approaches
  - Proposed algorithm
- Computational results
  - Weighted instances
- Conclusion and future research

Optimization problems in sports

- Multidisciplinary area
  - Combinatorial optimization
  - Operations Research
  - Schedulling theory
  - Graph theory
  - Integer programming
  - Constraint programming
  - Applied math
- Increasing importance in the last two decades
  - A sub-area on its own

Optimization problems in sports

- Sports competitions organization
  - Multiple decision makers
    - Organizers
    - Media
    - Public security authorities
  - Big investments
    - Athletes
    - Merchandising
    - Others

Optimization problems in sports

- Sports competitions organization
  - Different conflicting objectives
    - Maximize revenue
      - Some games in specific days
      - Transmission rights
    - Minimize costs (traveled distance)
    - Maximize athlete performance (time to rest)
    - Fairness (avoid playing many strong teams in a row)
    - Avoid conflicts (rival fans)

Optimization problems in sports

- Sports competitions organization
  - Different conflicting objectives
    - Maximize revenue
      - Some games in specific days
      - Transmission rights
    - Minimize costs (traveled distance)
    - Maximize athlete performance (time to rest)
    - Fairness (avoid playing many strong teams in a row) — this work
    - Avoid conflicts (rival fans)

Preliminary definitions

- Single round robin tournaments
  - Competition involving n (even) teams
  - Each team plays against every other exactly once
    - Compact tournament: $n - 1$ rounds
  - Each team plays exactly once per round.

Preliminary definitions

- Single round robin tournaments – an example:

Round 1	Round 2	Round 3	Round 4	Round 5
1 - 6	2 - 6	3 - 6	4 - 6	5 - 6
2 - 5	1 - 3	2 - 4	3 - 5	4 - 1
3 - 4	4 - 5	1 - 5	1 - 2	2 - 3

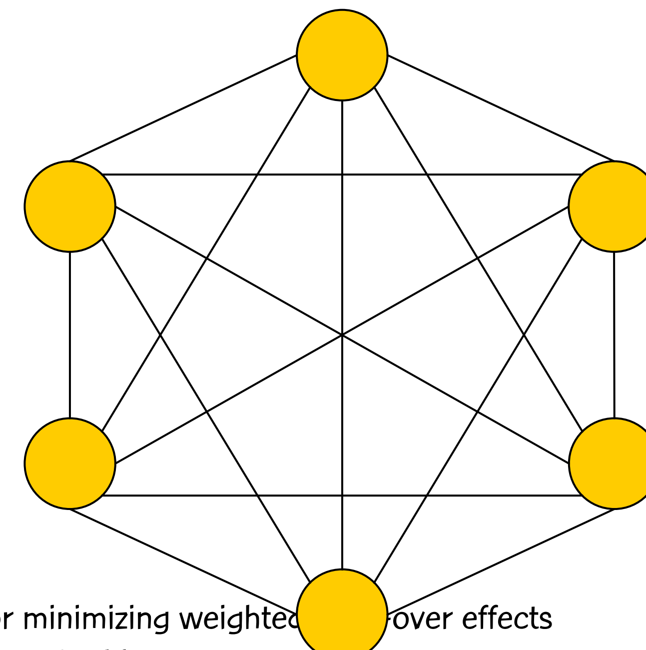
Teams

Rounds

	1	2	3	4	5	6
1	6	5	4	3	2	1
2	3	6	1	5	4	2
3	5	4	6	2	1	3
4	2	1	5	6	3	4
5	4	3	2	1	6	5

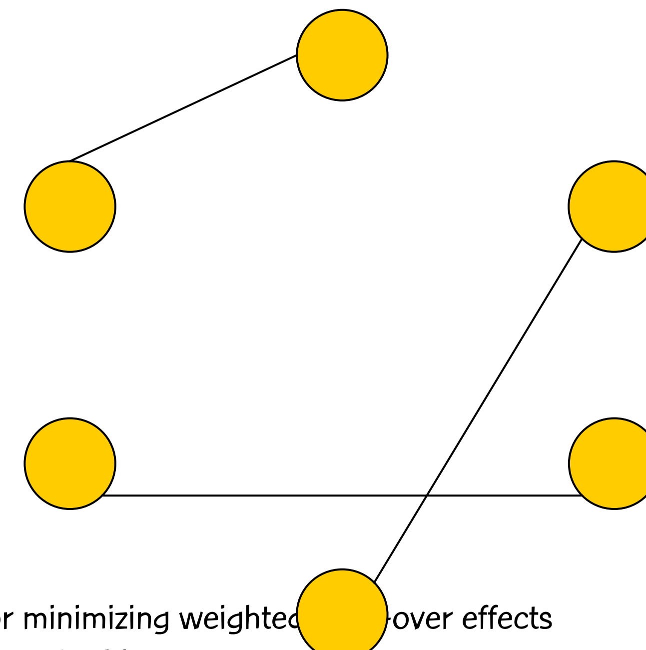
Preliminary definitions

- 1-factors and 1-factorizations
  - Mathematical objects defined over a graph G = (V,E)
    - This work: complete graph



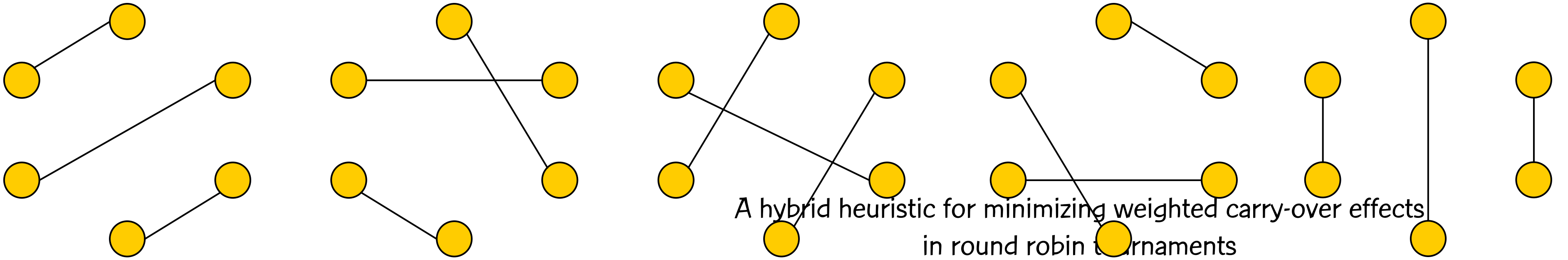
Preliminary definitions

- 1-factors and 1-factorizations
  - Mathematical objects defined over a graph G = (V,E)
    - This work: complete graph
  - Factor
    - Spanning sub-graph of G
  - 1-Factor
    - 1-regular factor



Preliminary definitions

- 1-factors and 1-factorizations
  - Mathematical objects defined over a graph $G = (V,E)$
    - This work: complete graph
  - Factorization
    - Set of edge-disjoint factors $G^1=(V,E^1)$, ..., $G^p=(V,E^p)$, such that $E^1 \cup \ldots \cup E^p = E$
  - 1-Factorization
    - Factorization containing only 1-factors



Preliminary definitions

- 1-factorizations and round robin tournaments
  - Vertex ←→ Team
  - Edge ←→ Game
  - 1-Factor ←→ Round
  - 1-Factorization ←→ Competition fixture

Carry-over effects minimization problem

- Carry-over effects
  - Team B receives a carry-over effect (COE) due to team A if there is a team X that plays against A in round r and against B in round r+1

	1	2	3	4	5	6	7
A	H	C	D	E	F	G	B
B	C	D	E	F	G	H	A
C	B	A	F	H	E	D	G
D	E	B	A	G	H	C	F
E	D	G	B	A	C	F	H
F	G	H	C	B	A	E	D
G	F	E	H	D	B	A	C
H	A	F	G	C	D	B	E

A hybrid heuristic for minimizing weighted carry-over effects in round robin tournaments

Carry-over effects minimization problem

- Carry-over effects
  - Team B receives a carry-over effect (COE) due to team A if there is a team X that plays against A in round r and against B in round r+1

	1	2	3	4	5	6	7
A	H	C	D	E	F	G	B
B	C	D	E	F	G	H	A
C	B	A	F	H	E	D	G
D	E	B	A	G	H	C	F
E	D	G	B	A	C	F	H
F	G	H	C	B	A	E	D
G	F	E	H	D	B	A	C
H	A	F	G	C	D	B	E

Team A receives COE due to B

Team G receives COE due to D

Team A receives COE due to E

A hybrid heuristic for minimizing weighted carry-over effects in round robin tournaments

Carry-over effects minimization problem

- Carry-over effects matrix
  - SRRT and carry-over effects matrix (COEM)

	1	2	3	4	5	6	7
A	H	C	D	E	F	G	B
B	C	D	E	F	G	H	A
C	B	A	F	H	E	D	G
D	E	B	A	G	H	C	F
E	D	G	B	A	C	F	H
F	G	H	C	B	A	E	D
G	F	E	H	D	B	A	C
H	A	F	G	C	D	B	E

RRT

	A	B	C	D	E	F	G	H
A	0	0	3	0	1	2	1	0
B	5	0	0	0	1	0	0	1
C	0	1	0	3	0	3	0	0
D	0	2	0	0	2	0	3	0
E	1	1	0	2	0	2	0	1
F	0	0	0	0	2	0	3	2
G	0	3	1	0	0	0	0	3
H	1	0	3	2	1	0	0	0

COE Matrix

A hybrid heuristic for minimizing weighted carry-over effects in round robin tournaments

Carry-over effects minimization problem

- RRT and carry-over effects matrix (COEM)

	1	2	3	4	5	6	7
A	H	C	D	E	F	G	B
B	C	D	E	F	G	H	A
C	B	A	F	H	E	D	G
D	E	B	A	G	H	C	F
E	D	G	B	A	C	F	H
F	G	H	C	B	A	E	D
G	F	E	H	D	B	A	C
H	A	F	G	C	D	B	E

RRT

	A	B	C	D	E	F	G	H
A	0	0	3	0	1	2	1	0
B	5	0	0	0	1	0	0	1
C	0	1	0	3	0	3	0	0
D	0	2	0	0	2	0	3	0
E	1	1	0	2	0	2	0	1
F	0	0	0	0	2	0	3	2
G	0	3	1	0	0	0	0	3
H	1	0	3	2	1	0	0	0

COE Matrix

Carry-over effects minimization problem

	A	B	C	D	E	F	G	H
A	0	0	3	0	1	2	1	0
B	5	0	0	0	1	0	0	1
C	0	1	0	3	0	3	0	0
D	0	2	0	0	2	0	3	0
E	1	1	0	2	0	2	0	1
F	0	0	0	0	2	0	3	2
G	0	3	1	0	0	0	0	3
H	1	0	3	2	1	0	0	0

$COEM_{DG} = 3$

$COEM_{FH} = 2$

COE matrix

$$COEV = \sum_{i=A}^{H} \sum_{j=A}^{H} (COEM_{ij})^2$$

A hybrid heuristic for minimizing weighted carry-over effects
in round robin tournaments

Carry-over effects minimization problem

	A	B	C	D	E	F	G	H
A	0	0	3	0	1	2	1	0
B	5	0	0	0	1	0	0	1
C	0	1	0	3	0	3	0	0
D	0	2	0	0	2	0	3	0
E	1	1	0	2	0	2	0	1
F	0	0	0	0	2	0	3	2
G	0	3	1	0	0	0	0	3
H	1	0	3	2	1	0	0	0

$COEM_{DG} = 3$

$COEM_{FH} = 2$

COE Matrix

minimize!!!

$$COEV = \sum_{i=A}^{H} \sum_{j=A}^{H} (COEM_{ij})^2$$

A hybrid heuristic for minimizing weighted carry-over effects
in round robin tournaments

Carry-over effects minimization problem

- Example: Karate-Do competitions
- Groups playing round-robin tournaments
  - Pan-american Karate-Do championship
  - Brazilian classification for World Karate-Do championship

Carry-over effects minimization problem

- Example: Karate-Do competitions
- Open-weight categories
  - Phisically strong contestants may fight weak ones.
  - A competitor could consecutively fight (phisically) weak opponents coming from matches against strong athletes.
    - This situation should be avoide

Carry-over effects minimization problem

- Problem statement:
  - Find a schedule with minimum COEV
    - RRT distributing the carry-over effects as evenly as possible among the teams.
- Best solution approaches to date in literature:
  - Random generation of permutations of 1-factors
  - Constraint programming
  - Combinatorial designs
    - Starters

Carry-over effects minimization problem – weighted version

- A weight is assigned to each carry-over effect.
- Generalization of the original problem, in which weights set to one.

Carry-over effects minimization problem – weighted version

- Motivation:
  - Original problem assumes equally strong teams and athletes
  - Real competitions: there are no stronger or weaker teams or athletes
    - Estimates of teams or athletes strengths are usually known beforehand.

$$COEV = \sum_{i=A}^{H} \sum_{j=A}^{H} w_{ij} . (COEM_{ij})^2$$

A hybrid heuristic for minimizing weighted carry-over effects in round robin tournaments

Carry-over effects minimization problem – weighted version

- Mathematical formulation by integer programming

- $y_{kij} = \begin{cases} 1, \text{ if team i plays against team j in round k} \\ 0, \text{ otherwise} \end{cases}$

Minimize $$\sum_{i-1}^{n} \sum_{j=1}^{n} w_{ij} . (z_{ij})^2$$

Entries from the COE Matrix

with $$z_{ij} = \sum_{l=1}^{n} \sum_{k=1}^{n-1} \left(y_{kli} * y_{(k+1)lj} \right), \quad \text{for } i \neq j$$

Carry-over effects minimization problem – weighted version

- subject to:

(1) $y_{kij} = y_{kji}$, for $i, j, k = 1,\ldots,n$

(2) $\sum_{i=1}^{n} y_{kij} = 1$, for $j, k = 1,\ldots,n$

...

	A	B	C	D	E	F	G	H
1	H	C	B	E	D	G	F	A
2	C	D	A	B	G	H	E	F
3	D	E	F	A	B	C	H	G
4	E	F	H	G	A	B	D	C
5	F	G	E	H	C	A	B	D
6	G	H	D	C	F	E	A	B
7	B	A	G	F	H	D	C	E

Carry-over effects minimization problem – weighted version

- subject to:

(3) $$\sum_{k=1}^{n-1} y_{kij} = 1 \text{ , for } i, j = 1,\dots,n : i \neq j$$

(4) $$y_{kii} = 0 \text{ , for } i, k = 1,\dots,n$$

...

	A	B	C	D	E	F	G	H
1	H	C	B	E	D	G	F	A
2	C	D	A	B	G	H	E	F
3	D	E	F	A	B	C	H	G
4	E	F	H	G	A	B	D	C
5	F	G	E	H	C	A	B	D
6	G	H	D	C	F	E	A	B
7	B	A	G	F	H	D	C	E

Carry-over effects minimization problem – weighted version

- subject to:

(5) $y_{nij} = y_{1ij}$, for $i, j = 1,\ldots,n$

(6) $y_{kij} \in \{0,1\}$, for $i, j, k = 1,\ldots,n$

	A	B	C	D	E	F	G	H
1	H	C	B	E	D	G	F	A
2	C	D	A	B	G	H	E	F
3	D	E	F	A	B	C	H	G
4	E	F	H	G	A	B	D	C
5	F	G	E	H	C	A	B	D
6	G	H	D	C	F	E	A	B
7	B	A	G	F	H	D	C	E

Previous approaches

- Russel (1980)
  - First proposed method.
  - Provides optimal solutions when $n = 2^p$.
- Anderson (1997)
  - Makes use of pre-enumerated starters.
  - Work not well-known.

Previous approaches

- Trick (2000), Henz et al. (2001)
  - Methods not tailored to the problem in hand
- Miyashiro and Matsui (2006)
  - Random generation os solutions
  - Constraint programming

Solution approach

- Multi-start + ILS heuristic
- Solutions represented by 1-factorizations
  - Canonical factorizations
  - Binary 1-factorizations
- Constructive algorithms
  - Rearragment of the 1-factors of a solution (TSP-like greedy algorithms)
    - Nearest neighbor
    - Arbitrary insertion

Solution approach

- Local search
  - Round Swap (RS)
  - Team Swap (TS)
  - Partial Round Swap (PRS)
  - Partial Team Swap (PTS)
- Pertubations
  - Ejection chain: Game Rotation (GR)

Multi-start + ILS heuristic

- Multi-start phase: generation of 100 solutions
  - 50% based on canonical 1-factorizations
  - 50% based on binary 1-factorizations (whenever possible)
  - Constructive methods applied to the 1-factors of the 1-factorizations
  - Local search
- Best solution of the multi-start phase is the input for the ILS algorithm

Multi-start + ILS heuristic

```
For iteration = 1 to NUMBER_OF_ITERATIONS Do
    For try = 1 to 100 Do
        S ← Initial_Solution();
        S ← Local_Search(S);
        S* ← Update_Best_Solution(S, S*);
    End-For;
    S ← S*;
    While Not Stopping-Criterion Do
        S' ← Pertubation(S);
        S' ← Local_Search(S');
        S ← Acceptance_Criterion(S, S');
        S* ← Update_Best_Solution(S, S*);
    End-While;
End-For;
```

Computational results

- Weighted instances
  - Random instances
  - Linear instances
  - Perturbed linear instances
  - Real-life inspired instances
- Link:
  - http://www.esportemax.org/index.php/Instances_for_the_Problem_of_Minimizing_the_Weighted_Carry-Over_Effects_Value

Computational results

- Weighted instances
  - Random instances
    - Weight w_{ij} is a random number in the interval $[1..2n]$
  - Linear instances
    - Each team is assigned a strength $s(i) = i$
    - Weight w_{ij} is set to $|s(i) - s(j)|$
  - Perturbed linear instances
    - Each team is assigned a strength $s(i) = i$
    - Weight w_{ij} is set to $|s(i) - s(j) + a|$
      - *a* is a number in the interval [-n/2..n/2]

Computational results

- Weighted instances
  - Real-life inspired instances
  - Based on the Brazillian soccer championship
    - Double round robin tournament
    - Editions of 2003, 2004, 2005, 2006, 2007, 2008
    - Each team is assigned a strength s(i)
      - s(i) = number of points obtained
    - Weight w_{ij} is set to $|s(i) - s(j)|$
- Non-weighted instances
  - Unitary weights
  - Equivalent to original problem

Computational results

- Real-life inspired instances

Instances	Average COEV	Best COEV	Average time (s)	Largest time (s)
Inst24brazil2003	7730,4	7542,0	13897,8	15755,0
Inst24brazil2004	7179,6	7088,0	12992,8	13468,0
Inst22brazil2005	5228,8	5158,0	10599,0	13959,0
Inst20brazil2006	5310,0	5236,0	5705,4	6358,0
Inst20brazil2007	4876,0	4834,0	2715,8	3655,0
Inst20brazil2008	4045,6	3944,0	6805,6	8308,0

Computational results

- Non-weighted instances

n	Average COEV	Best COEV	Average time (s)	Largest time (s)
4	12,0	12,0	0,2	1,0
6	60,0	60,0	0,4	1,0
8	56,0	56,0	5,4	6,0
10	111,6	108,0	12,0	13,0
12	164,8	160,0	24,6	26,0
14	254,0	254,0	8,8	9,0
16	259,2	240,0	129,8	140,0
18	401,6	400,0	28,8	31,0
20	491,2	486,0	65,2	72,0

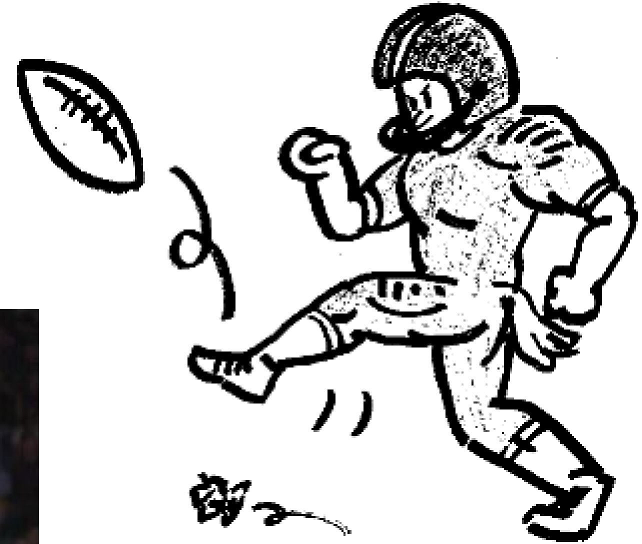
Computational results

- Non-weighted instances

n	Best known COEV	Proposed method
4	12	12
6	60	60
8	56	56
10	108	108
12	176	**160**
14	**234**	254
16	240	240
18	**340**	400
20	**380**	486

Concluding remarks

- Minimizing carry-over effects in round robin tournaments
  - Weighted version
  - Mathematical formulation
  - ILS metaheuristic:
    - Reveals itself as a very appropriate approach for similar problems.
  - Benchmark instances:
    - Online



stic for minimi
in round rob