Scheduling the Brazilian Soccer Championship

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Motivation

- Soccer is widely practiced in Brazil
- Tournament is the most important in the country
- Revenues from sponsors, advertising, merchandising, tickets, and TV
- Major sponsor is TV Globo (largest media network in Brazil), who imposes constraints on games to be broadcast:
  - Most attractive games are those involving a subset of elite teams with more fans and, consequently, with larger broadcast shares
  - They involve elite teams from Rio de Janeiro and São Paulo (richest cities, with larger revenues from advertising)

Motivation

- Tournament organized by CBF (Brazilian Soccer Confederation), who imposes fairness constraints on the schedule
- Tournament lasts 7 months, every team plays at most twice a week
- Revenues and attractiveness of the tournament strongly depend on the schedule of the games
- Elite teams (11): more fans and best records in previous years
  - four from São Paulo
  - four from Rio de Janeiro

Problem statement

- The problem of creating the tournament schedule is faced every year by CBF
- Structure: mirrored double round robin
  - Even number of teams
  - Every team faces each other twice, once at home and once away
  - Every team plays exactly one game per round
  - The schedule is mirrored and divided in two halves:
    - In the first half, every team faces each other once
    - In the second, the teams face each other again in the same order, but with reversed venues

Problem statement

- Constraints:
  1. Every team playing home (resp. away) in the first round plays away (resp. home) in the last round
  2. Every team plays once at home and once away in the two first rounds and in the two last rounds
  3. After any number of rounds, the difference between the number of home and away games played by any team is at most one
  4. Some pairs of teams with the same home city have complementary patterns: when one of them plays at home, the other plays away (and vice-versa) (security and ensuring one game in their city every round)

Problem statement

- Constraints:
  5. Flamengo and Fluminense have complementary patterns in the last four rounds (same stadium)
  6. Regional games between teams from the same city are not to be played in mid-week rounds or in the last six rounds (attractiveness)
  7. Games cannot be broadcast to the same city where they take place (TV rights)
  8. Only one game per round can be broadcast to each city (TV rights)

Problem statement

- Constraints:
  9. There should be at least one team from Rio de Janeiro (resp. São Paulo) playing outside Rio de Janeiro (resp. São Paulo) in every round (TV rights)
  10. If in some round there is only one team from Rio de Janeiro (resp. São Paulo) playing outside Rio de Janeiro (resp. São Paulo), then this game should not be held in Belém (TV equipment accessibility)

Problem statement

- Objectives:
  - Breaks: minimize the number of times the teams play two consecutive home (or away) games
  - Broadcast: maximize the number of rounds in which there is at least one game of an elite team of São Paulo playing away with another elite team plus the number of rounds in which there is at least one game of an elite team of Rio de Janeiro playing away with another elite team
    - Rio de Janeiro and São Paulo are the richest cities, with larger revenues from TV and advertising
    - Only one game per round can be broadcast to each city

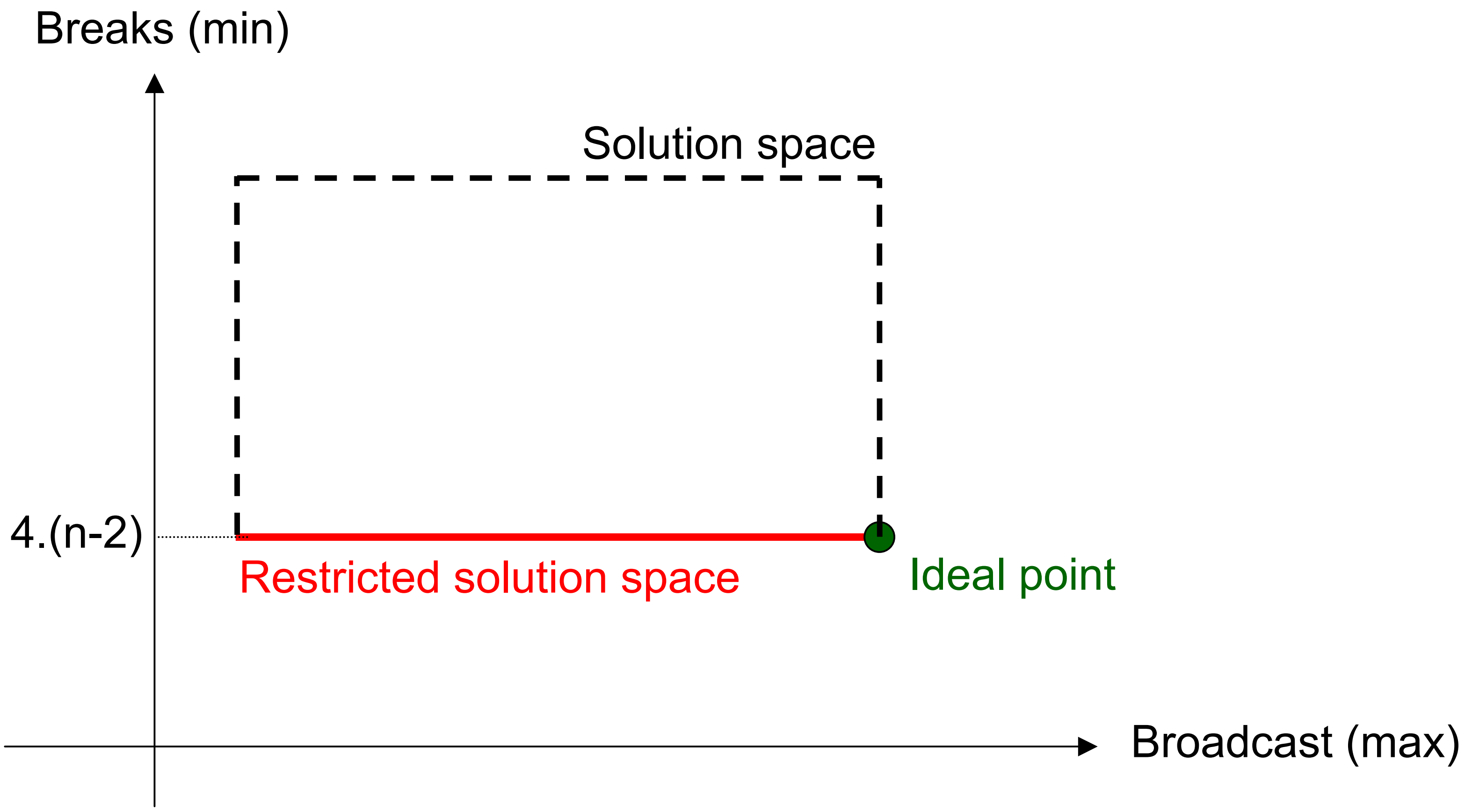
Solution approach

- Straightforward integer programming formulation cannot be solved by commercial solvers in less than a day of execution
- Distances between venues are not relevant
- Attempt to tackle the problem by an exact method
- Similar approach to that used by Nemhauser & Trick (OR, 1998) to exactly solve the problem of scheduling a basketball league

Solution approach

- Bi-objective problem: (min breaks, max broadcast)
  - Ideal point: both objectives optimized by same solution
- Approach:
  - Maximize the broadcast objective, with the number of breaks fixed at the minimum (constraint)
  - If the maximum of the broadcast objective is equal to the unrestricted maximum, then solution is an ideal point (optimizing both objectives)
- In the following, we show that constraints (1) and the mirrored structure impose that the number of breaks is bounded by 4.(n-2), where n denotes the number of teams

Solution approach



Solution approach

- Only two teams may have no breaks (HAHAHA and AHAHAH) in a round robin tournament, the other have at least one break: breaks $\geq$ n-2
- Mirrored tournament implies that a break in the first half is repeated in the second: breaks $\geq$ 2.(n-2)
- Mirrored tournament implies that every team with an odd number of breaks in the first half has an extra break in the first round of the second half: breaks $\geq$ 3.(n-2)
- A team with an odd number of breaks plays its last game in the same playing condition as in the first game, violating constraints (1): breaks $\geq$ 4.(n-2)

Solution approach

- Broadcast objective can also be bound:
  - Not greater than twice the number of rounds (one game broadcast to São Paulo and another to Rio de Janeiro in every round)
  - Not greater than the number of candidate games to be broadcast:

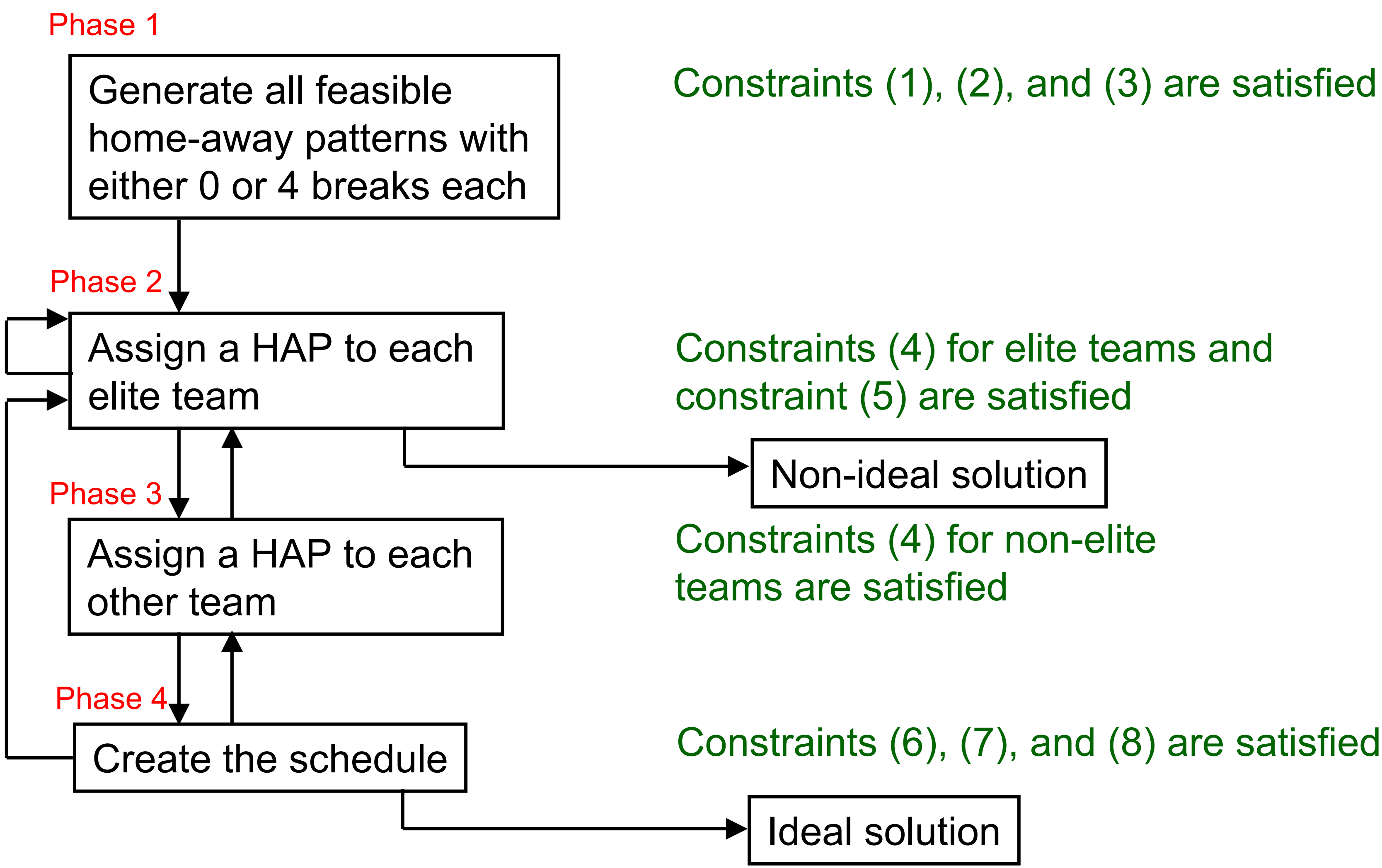
$$| \text{elite teams from SP} | \times | \text{elite teams outside SP} | +$$
$$| \text{elite teams from RJ} | \times | \text{elite teams outside RJ} |$$

  - Second bound is better (smaller) in the instances solved

Solution approach

- Nemhauser & Trick (OR, 1998):
  1. Create feasible home-away patterns (HAP)
  2. Assign a HAP to every team
  3. Create the schedule regarding the previously determined HAP assignments
- Our approach: four phases

Solution approach



Phase 1: create all feasible HAPs

- Mirrored structure, constraints (1), and breaks objective:
  - Feasible patterns are those with either 0 (only two patterns) or 2 breaks in the first half
- Constraints (2):
  - Feasible patterns are those without breaks in the second and last rounds of the first half
- Constraints (3):
  - Feasible patterns are those without breaks in even rounds of the first half
- There are 58 feasible patterns for n=20 and 74 feasible patterns for n=22

Phase 1: create all feasible HAPs

- Small number of feasible patterns allows their complete enumeration
- After this phase, we have the complete set of feasible patterns:

H	A	H	A	H
A	H	A	A	H
		:		
H	A	H	H	A

Phase 2: assign HAPs to elite teams

- Assign (by explicit exhaustive enumeration) a HAP to each elite team satisfying constraints (4) and (5).
- Create and solve a linear programming model considering:
  - Partial assignment (HAPs assigned to elite teams)
  - Constraints (6), (7), and (8)
  - Maximization of the broadcast objective
- LP optimal value is an upper bound to the broadcast objective of this partial assignment

Phase 2: assign HAPs to elite teams

- If LP bound is not better (smaller or equal) than the value of the broadcast objective for the best known solution, then the partial assignment can be discarded and a new partial assignment is enumerated.
- Otherwise, proceed to Phase 3
- If all partial assignments have been tested, the algorithm stops and returns the best known (non-ideal) solution

Phase 3: assign HAPs to other teams

- Complete the partial HAP assignment: assign a HAP to each non-elite team satisfying constraints (4) and proceed to the last phase
- If all possible ways to complete the partial assignment have been tested, return to phase 2 to consider a new partial assignment

H	A	H	A	H	Fluminense
A	H	A	A	H	Botafogo
		:			
H	A	H	H	A	Santos

Phase 4: create the schedule

- Create and solve an integer programming model:
  - Complete HAP assignment
  - Constraints (6), (7), and (8)
  - Maximization of the broadcast objective
- HAP-based model can be quickly solved by a commercial solver

Phase 4: create the schedule

- Outcome of the integer programming model:
  - If the optimal value is equal to the broadcast bound, then the algorithm terminates with an ideal solution. Otherwise, possibly update the best known solution
  - If problem is infeasible or if the optimal value is smaller than the LP bound obtained in phase 2, then return to phase 3 to enumerate a new way to complete the partial assignment
  - If the optimal value is equal to the LP bound obtained in phase 2, then return to phase 2 to enumerate a new partial assignment

Numerical results

- Algorithm tested to schedule the 2005 and 2006 tournaments:
  - 2005: 22 teams
  - 2006: 20 teams
- 11 elite teams, four from São Paulo and four from Rio de Janeiro
- Optimal solutions obtained in less than 10 minutes on a standard Pentium IV processor with 256 Mbytes

Comparison table: 2005

	Official schedule	HAP-ILP schedule
Constraints (1)	yes	yes
Constraints (2)	yes	yes
Constraints (3)	no	yes
Constraints (4)	yes	yes
Constraints (5)	no	yes
Constraints (6)	no	yes
Constraints (7)	yes	yes
Constraints (8)	yes	yes
Breaks	156	80 (optimal)
Broadcast	43	56 (optimal)

Comparison table: 2006

	Official schedule	HAP-ILP schedule
Constraints (1)	yes	yes
Constraints (2)	yes	yes
Constraints (3)	no	yes
Constraints (4)	yes	yes
Constraints (5)	no	yes
Constraints (6)	no	yes
Constraints (7)	yes	yes
Constraints (8)	yes	yes
Breaks	172	72 (optimal)
Broadcast	47	56 (optimal)

Conclusions

- Optimal solutions found in a few minutes
- Manual schedule currently used does not satisfy all constraints
- HAP-based ILP schedule satisfies all constraints and finds optimal solutions for both objectives (ideal solutions)
- TV Globo is interested by the HAP-based ILP schedules
- CBF is evaluating the use of the HAP-based ILP schedules