Scheduling the Brazilian Football Tournament: Solution Approach and Practice

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Relevance

- Professional football leagues are big businesses.
- TV networks pay broadcast rights, but want the most attractive games in prime time.
- Gate attendances and tournament attractiveness also depend on the schedule.
- Geographical, technical, fairness, and security constraints impose intricate patterns of game playing.
- Competitions played in parallel require logistics and strong coordination of travel and game schedules.
- Good schedules are of major interest for teams, leagues, sponsors, fans and the media.
- Recent surveys on sports scheduling: Rasmussen & Trick 2008, Kendall et al. 2010. and Ribeiro 2011.

Tournament basics

- Most important sport event in the country.
- Yearly tournament organized by CBF, lasts for seven months.
- TV Globo is the major sponsor: largest media group and television network in Brazil.
- Fair and balanced schedules are major issues for attractiveness and confidence in the outcome.
- TV sponsors condition their support to schedules that make it possible to broadcast the most important games by open channels at prime time.
- Cities hosting two or more teams impose additional security constraints to avoid clashes of fans before or after the games.

Past experience

- Few professional football leagues adopted optimization models and software to date: Nurmi et al. 2010
  - hardness of the problem
  - fuzzy preference restrictions and criteria that can be hardly described
  - resistance of teams and leagues to the use of new tools that introduce modern techniques in sports management
- Austria and Germany, Italy, Chile, Belgium, Denmark, and Honduras (current situation and practical use of most projects is unknown).
- This work describes the formulation, implementation and practical use of the optimization software developed in partnership with CBF to schedule the first (Series A) and second (Series B) divisions of the Brazilian football tournament.

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Definitions 1/3

- Tournament played by an even number $n = 20$ of teams.
- Each team has its own venue: home games vs. away games.
- Compact mirrored double round robin tournament.
- Every team plays exactly once in every round.
- Every game occurs twice.
- Games in the first half are played in the same order as in the second, with exchanged venues.
- Schedule establishes the round and the venue in which each game is played.
- A home-away pattern (HAP) determines in which condition (home or away) each team plays in each round.
- There is a home (away) break whenever a team plays two consecutive home (away) games.

Definitions 2/3

- Weekend rounds are played in Saturdays and Sundays.
- Midweek rounds are played in Wednesdays and Thursdays.
- Double weekend rounds: weekend rounds in both phases.
- Dates available for game playing change from year to year and have to be coordinated with other competitions.
- Last four qualified teams in Series A are downgraded to play Series B next year, while the four best qualified teams in Series B are upgraded to Series A.
- TV rights, marketing revenues, and gate attendances are much larger for teams in Series A.
- Group of Twelve formed by the strongest founding teams of the league: larger broadcast rights.

Definitions 3/3

- Teams are organized in pairs (imposed by CBF).
- Teams in the same pair have complementary home-away patterns of game playing.
- Odd teams (imposed by CBF) play their first game at home.
- Regional games involve two opponents with home cities in the same state.
- Classic games (or classics) involve two opponents with the same home city:
  - Long tradition of rivalry in the history of their games.
  - They attract larger gate attendances and more interest by fans and media.

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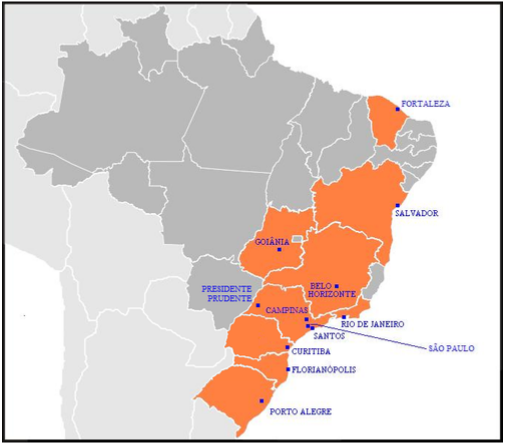
Home cities

- São Paulo is the richest state with a number of strong teams not only in the capital.
- Teams from all other states have their home cities in the capitals.



Teams

Index	Team	City	State	G12?
1	Atlético Goianiense	Goiânia	Goiás	no
2	Goiás	Goiânia	Goiás	no
3	Atlético Mineiro	Belo Horizonte	Minas Gerais	yes
4	Cruzeiro	Belo Horizonte	Minas Gerais	yes
5	Avaí	Florianópolis	Santa Catarina	no
6	Atlético Paranaense	Curitiba	Paraná	no
7	Botafogo	Rio de Janeiro	Rio de Janeiro	yes
8	Fluminense	Rio de Janeiro	Rio de Janeiro	yes
9	Ceará	Fortaleza	Ceará	no
10	Vitória	Salvador	Bahia	no
11	Corinthians	São Paulo	São Paulo	yes
12	São Paulo	São Paulo	São Paulo	yes
13	Flamengo	Rio de Janeiro	Rio de Janeiro	yes
14	Vasco da Gama	Rio de Janeiro	Rio de Janeiro	yes
15	Internacional	Porto Alegre	Rio Grande do Sul	yes
16	Grêmio	Porto Alegre	Rio Grande do Sul	yes
17	Guarani	Campinas	São Paulo	no
18	Grêmio Prudente	Presidente Prudente	São Paulo	no
19	Palmeiras	São Paulo	São Paulo	yes
20	Santos	Santos	São Paulo	yes

Last champions 1990-2011

Year	Team	State	G12?
2011	Corinthians	São Paulo	yes
2010	Fluminense	Rio de Janeiro	yes
2009	Flamengo	Rio de Janeiro	yes
2008	São Paulo	São Paulo	yes
2007	São Paulo	São Paulo	yes
2006	São Paulo	São Paulo	yes
2005	Corinthians	São Paulo	yes
2004	Santos	São Paulo	yes
2003	Cruzeiro	Minas Gerais	yes
2002	Santos	São Paulo	yes
2001	Atlético Paranaense	Paraná	no
2000	Vasco da Gama	Rio de Janeiro	yes
1999	Corinthians	São Paulo	yes
1998	Corinthians	São Paulo	yes
1997	Vasco da Gama	Rio de Janeiro	yes
1996	Grêmio	Rio Grande do Sul	yes
1995	Botafogo	Rio de Janeiro	yes
1994	Palmeiras	São Paulo	yes
1993	Palmeiras	São Paulo	yes
1992	Flamengo	Rio de Janeiro	yes
1991	São Paulo	São Paulo	yes
1990	Corinthians	São Paulo	yes

Contents

Basics

- Schedule should satisfy constraints ranging from fairness to security issues, and from technical to broadcasting criteria.
- Some constraints reflect strategies for maximizing revenues and tournament attractiveness.
- Other constraints attempt to avoid unfair situations that could benefit one team or another with a more convenient sequence of games.
- Requirements have been discussed and established over the years by teams, federations, city administrators, security agencies, and sponsors.

Basics

- Schedule should satisfy constraints ranging from fairness to security issues, and from technical to broadcasting criteria.
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- Requirements have been discussed and established over the years by teams, federations, city administrators, security agencies, and sponsors.

Round robin constraints

(A.1) Each team must play every other twice, once at home and once away (double round robin).

(A.2) Each team must play exactly once in each round, either home or away (compact schedule).

(A.3) Each team must play every other exactly once in the first (resp. second) phase, along the $n-1$ initial (resp. last) rounds. Games in the second phase are played exactly in the same order as in the first, but with interchanged venues. (mirrored schedule).

- In consequence, schedule of the second phase is directly determined by that of the first.

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Home-away patterns of game playing

(B.1) Teams in the same pair have complementary home-away patterns (one of them has home rights in their games).

(B.2) Teams playing at home in the first round are chosen by CBF (teams that played away in the first round of the previous year's tournament and teams upgraded from lower division).

(B.3) Teams alternate home and away games along the first four and the last two rounds (no breaks).

(B.4) Teams playing at home (away) in the first round will play away (home) in the last.

(B.5) All teams have the same number of home breaks and away breaks in each of the two phases.

(B.6) The number of breaks should be minimum (each team has one home break and one away break in each phase).

(B.7) Each team plays nine games at home (away) in each phase, regardless of the game with the other team in the same pair.

Classic and regional games

(C.1) As many classics (derbies) as possible should be played in double weekend rounds (larger gate attendances and audiences).

(C.2) No regional games in the first three (fans are less motivated) or in the last four (to avoid local arrangements) rounds.

(C.3) Whenever a classic is played at São Paulo, there are no games between G12 teams of Rio de Janeiro and São Paulo (to avoid competition that could divide the interest of the public).

(C.4) Whenever a classic is played at Rio de Janeiro, there are no games between G12 teams of Rio de Janeiro and São Paulo.

(C.5) There can be at most one classic game played in each city in any round (to avoid security and circulation problems).

(C.6) No team can play two classics in consecutive rounds (negative impact on team motivation and on the interest of its fans).

(C.7) Some games have to be scheduled at specific rounds.

Geographical and G12 constraints

- (D.1) No team from São Paulo can play five or more consecutive games in that state (this would be an advantage for this team, because these cities are very close to each other).
- (D.2) No team from a state other than São Paulo can play four or more consecutive games in that state (to avoid the privilege of staying very long at home).
- (D.3) Every team must play a game outside the state where its home city is located in either one of the first two rounds.
- (D.4) There are at least two and at most four games involving only G12 teams in every round (even distribution of games between strong teams along the tournament).
  - Multiple games between strong teams in any round, offering more choices for broadcasting.
- (D.5) No team can play more than five consecutive games against G12 teams (to avoid a long series of hard games).

Perfect matching of paired teams

(E.1) In the quest for a fair and balanced schedule, CBF imposes additional constraints to enforce a tight equilibrium to any two teams belonging to the same pair.

- Let (A,B) and (C,D) be any two pairs of paired teams.
- If A plays with C at home (away) in the first phase, then it plays away (at home) with D. Consequently, B plays away (at home) with C and at home (away) with D in this phase.
- Same constraints automatically implied for the second phase.
- Such constraints lead to a strong equilibrium between teams in the same cities and regions and are considered by CBF officials as one of the most important to be enforced.

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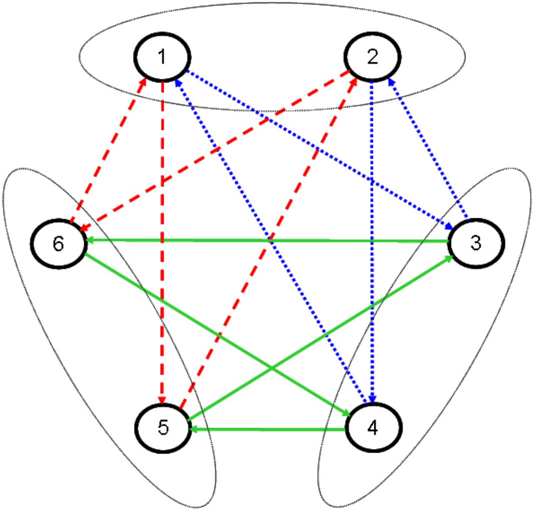
Perfect matching of six paired teams

- Oriented arc (i, j) means that team i plays away with team j.



Discussion

- Maximization of gate attendances and TV audiences is a major issue, since major revenues come from broadcast and merchandising rights: sponsors request good schedules drawing large audiences.
- Good schedules maximize gate attendances and TV audiences: search for a schedule with a maximum number of classic games played in double weekend rounds.
- Fair and balanced schedules are a major issue for the attractiveness of the tournament and for the confidence in its outcome, playing a major role in the success of the competition: most constraints handle fairness and equilibrium requirements.

Elsewhere. . .

Elsewhere. . . witchcraft?

Contents

Preliminaries and notation 1/2

- Complete schedule of a mirrored double round robin tournament is determined by the first phase.
- Home-away patterns are formed by two symmetric halves.
- Computation of optimal schedules based on first phase HAPs.
- $T = \{1, \ldots, n\}$ is the set of participating teams:
  - Odd indexed teams play at home in the first round.
  - Teams indexed by $2 \cdot e - 1$ and $2 \cdot e$ belong to the same pair.
- $H = \{1, \ldots, N\}$ is the set of feasible first phase HAPs starting with a home game:

$$h(\ell, k) = \begin{cases} 1, & \text{if HAP } \ell \text{ has a home game in round } k, \\ 0, & \text{otherwise.} \end{cases}$$

- If HAP ℓ is assigned to odd indexed team $2 \cdot e - 1$, then its complementary HAP is assigned to team $2 \cdot e$ in the same pair.

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Preliminaries and notation 2/2

- C: set of all cities hosting two or more teams.
- $T(c)$: subset of teams whose home city is c.
- $c = 1$ corresponds to São Paulo and $c = 2$ to Rio de Janeiro.
- SP: subset of teams in the state of São Paulo.
- $\bar{R}$: subset of rounds where regional games cannot be played.
- D: subset of double weekend rounds.
- $G12$: subset of teams forming the Group of Twelve.

Decision variables

Variables x: games, rounds and venues

$$x_{ijk} = \begin{cases} 1, \text{if team } i \text{ plays at home with } j \text{ in round } k, \\ 0, \text{otherwise.} \end{cases}$$

Variables y: teams and home-away assignments

$$y_{e\ell} = \begin{cases} 1, \text{if team } 2 \cdot e - 1 \text{ follows first phase HAP } \ell, \\ 0, \text{otherwise.} \end{cases}$$

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Model

Objective function maximizes the number of classic games that can be played in double weekend rounds (larger gate attendances and TV audiences):

$$\max \sum_{k \in D} \sum_{c \in C} \sum_{i \in T(c)} \sum_{\substack{j \in T(c) \\ j \neq i}} (x_{ijk} + x_{jik}) \tag{1}$$

Each phase is a SRR tournament (every team play against every other exactly once in each phase):

$$\sum_{k=1}^{n-1} (x_{ijk} + x_{jik}) = 1, \qquad \forall i, j \in T : i < j \tag{2}$$

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Model

Each team must have one home break and one away break in each phase. Teams that play at home in the first round will play ten home games and nine away games in the first phase. Therefore, the game between the two teams in each pair must be played at the home of the odd indexed team in the first phase:

$$\sum_{k=1}^{n-1} x_{2e-1,2e,k} = 1, \qquad e = 1, \ldots, n/2 \tag{3}$$

Model

No regional games in the first three rounds:

$$\sum_{\substack{c \in C \\ c \neq 1}} \sum_{i \in T(c)} \sum_{\substack{j \in T(c) \\ j \neq i}} (x_{ijk} + x_{jik}) + \sum_{i \in SP} \sum_{\substack{j \in SP \\ j \neq i}} (x_{ijk} + x_{jik}) = 0, \qquad \forall k \in \bar{R} \tag{4}$$

Model

At most one classic in Rio de Janeiro and one classic in São Paulo, but not in concurrency with a G12 game:

$$\sum_{i \in T(1)} \sum_{j \in T(2)} (x_{ijk} + x_{jik}) + 4 \cdot \sum_{i \in T(1)} \sum_{\substack{j \in T(1) \\ j \neq i}} (x_{ijk} + x_{jik}) \leq 4, \qquad k = 1, \ldots, n-1 \quad (5)$$

$$\sum_{i \in T(2)} \sum_{j \in T(1)} (x_{ijk} + x_{jik}) + 4 \cdot \sum_{i \in T(2)} \sum_{\substack{j \in T(2) \\ j \neq i}} (x_{ijk} + x_{jik}) \leq 4, \qquad k = 1, \ldots, n-1 \quad (6)$$

Model

At least one round in between two consecutive classics of the same team (at most one classic game in every two consecutive rounds):

$$\sum_{i \in T(c)} (x_{ijk} + x_{jik} + x_{ij,k+1} + x_{ji,k+1}) \leq 1,$$
$$\forall c \in C : |T(c)| \geq 3, \quad \forall j \in T(c), \quad k = 1, \ldots, n-2 \quad (7)$$

Model

Any team whose home city is located at the state of São Paulo plays away at least once outside the state in any sequence of five consecutive games (for both the first and second phases):

$$\sum_{i \notin SP} (x_{ijk} + x_{ij,k+1} + x_{ij,k+2} + x_{ij,k+3} + x_{ij,k+4}) \geq 1,$$
$$\forall j \in SP, \quad k = 1, \ldots, n-5 \quad (8)$$

$$\sum_{i \notin SP} (x_{jik} + x_{ji,k+1} + x_{ji,k+2} + x_{ji,k+3} + x_{ji,k+4}) \geq 1,$$
$$\forall j \in SP, \quad k = 1, \ldots, n-5 \quad (9)$$

Model

Any team whose home city is not located at the state of São Paulo plays away at least once outside the state in any sequence of four consecutive games (for both the first and second phases):

$$\sum_{i \notin T(c)} (x_{ijk} + x_{ij,k+1} + x_{ij,k+2} + x_{ij,k+3}) \geq 1, \qquad \forall c \in C \setminus \{1\}, \quad \forall j \in T(c), \quad k = 1, \ldots, n-4 \tag{10}$$

$$\sum_{i \notin T(c)} (x_{jik} + x_{ji,k+1} + x_{ji,k+2} + x_{ji,k+3}) \geq 1, \qquad \forall c \in C \setminus \{1\}, \quad \forall j \in T(c), \quad k = 1, \ldots, n-4 \tag{11}$$

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$$\forall j \in T(c), \quad k = 1, \ldots, n-4 \quad (12)$$

$$\sum_{i \notin T(c)} (x_{jik} + x_{ji,k+1} + x_{ji,k+2} + x_{ji,k+3}) \geq 1, \qquad \forall c \in C \setminus \{1\},$$
$$\forall j \in T(c), \quad k = 1, \ldots, n-4 \quad (13)$$

Model

There are at least two and at most four games involving only G12 teams in every round:

$$2 \leq \sum_{i \in G12} \sum_{\substack{j \in G12 \\ j \neq i}} x_{ijk} \leq 4, \qquad k = 1, \ldots, n-1 \tag{14}$$

Model

No team can play more than five consecutive games against G12 teams:

$$\sum_{i \notin G12 : i \neq j} \left(x_{ij,k} + x_{ij,f(k+1)} + x_{ij,f(k+2)} + x_{ij,f(k+3)} + \right.$$
$$+ x_{ij,f(k+4)} + x_{ij,f(k+5)} + x_{ji,k} + x_{ji,f(k+1)} +$$
$$\left. + x_{ji,f(k+2)} + x_{ji,f(k+3)} + x_{ji,f(k+4)} + x_{ji,f(k+5)} \right) \geq 1,$$
$$\forall i \in T, \quad k = 1, \ldots, n-1 \quad (15)$$

with the function

$$f(a) = \begin{cases} a, & \text{if } a \leq n-1, \\ a-(n-1), & \text{otherwise,} \end{cases}$$

being used to handle sequences of consecutive games that start in the first phase and finish in the second.

Model

Each odd indexed team follows a different home-away pattern starting with a home game:

$$\sum_{\ell \in H} y_{e\ell} = 1, \qquad e = 1, \dots, n/2 \tag{16}$$

$$\sum_{e=1}^{n/2} y_{e\ell} \leq 1, \qquad \forall \ell \in H \tag{17}$$

Model

Assigned home-away patterns determine whether each odd indexed team plays at home or away in each round:

$$\sum_{\substack{i \in T \\ i \neq 2e-1}} x_{2e-1,ik} = \sum_{\ell \in H} h(\ell, k) \cdot y_{e\ell}, \qquad \forall e = 1, \dots, n/2, \quad k = 1, \dots, n-1 \tag{18}$$

$$\sum_{\substack{i \in T \\ i \neq 2e-1}} x_{i,2e-1,k} = 1 - \sum_{\ell \in H} h(\ell, k) \cdot y_{e\ell}, \qquad \forall e = 1, \dots, n/2, \quad k = 1, \dots, n-1 \tag{19}$$

Model

Assigned home-away patterns determine whether each even indexed team plays at home or away in each round:

$$\sum_{\substack{i \in T \\ i \neq 2e}} x_{2e,ik} = 1 - \sum_{\ell \in H} h(\ell, k) \cdot y_{e\ell}, \qquad \forall e = 1, \ldots, n/2, \quad k = 1, \ldots, n-1 \quad (20)$$

$$\sum_{\substack{i \in T \\ i \neq 2e}} x_{i,2e,k} = \sum_{\ell \in H} h(\ell, k) \cdot y_{e\ell}, \qquad \forall e = 1, \ldots, n/2, \quad k = 1, \ldots, n-1 \quad (21)$$

Model

Perfect matching of paired teams (any team i will play at home against exactly one of the two teams indexed by $2 \cdot e - 1$ and $2 \cdot e$ in each phase):

$$\sum_{k=1}^{n-1} (x_{i,2e-1,k} + x_{i,2e,k}) = 1,$$

$$e = 1, \ldots, n/2, \quad \forall i \in T : i \neq 2e - 1, i \neq 2e \quad (22)$$

Integrality constraints:

$$x_{ijk} \in \{0, 1\}, \qquad \forall i \in T, \quad \forall j \in T : j \neq i, \quad k = 1, \ldots, n-1 \quad (23)$$

$$y_{e\ell} \in \{0, 1\}, \qquad e = 1, \ldots, n/2, \quad \ell = 1, \ldots, N \quad (24)$$

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$$\sum_{k=1}^{n-1} (x_{i,2e-1,k} + x_{i,2e,k}) = 1,$$

$$e = 1, \ldots, n/2, \quad \forall i \in T : i \neq 2e-1, i \neq 2e \tag{22}$$

Integrality constraints:

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Contents

Decomposition strategy

- Complete formulation enumerating all home-away patterns starting with a home game could not be solved by a commercial solver within an entire day of computations.
- Three-phase solution approach:
  - "First-break, then-schedule" decomposition scheme
  - Nemhauser & Trick 1998: scheduling a basketball league
- First phase: creation of feasible home-away patterns
- Second phase: assignment of a different HAP to each team
- Third phase: construction of an optimal schedule by solving a simpler integer program with HAPs already assigned

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Decomposition strategy

- Complete formulation enumerating all home-away patterns starting with a home game could not be solved by a commercial solver within an entire day of computations.
- Three-phase solution approach:
  - "First-break, then-schedule" decomposition scheme
  - Nemhauser & Trick 1998: scheduling a basketball league
- First phase: creation of feasible home-away patterns
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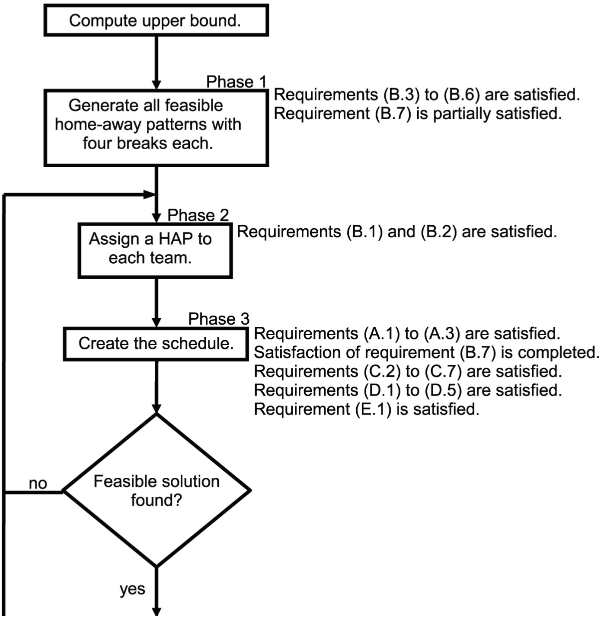
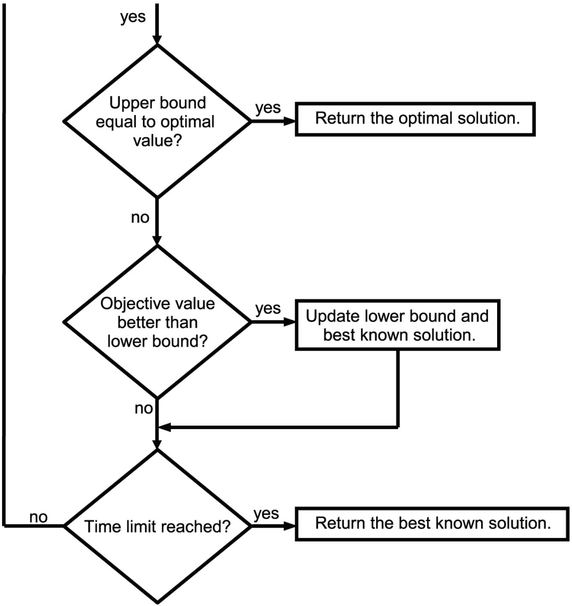

Compute upper bound.
Phase 1
Generate all feasible home-away patterns with four breaks each.
Requirements (B.3) to (B.6) are satisfied.
Requirement (B.7) is partially satisfied.
Phase 2
Assign a HAP to each team.
Requirements (B.1) and (B.2) are satisfied.
Phase 3
Create the schedule.
Requirements (A.1) to (A.3) are satisfied.
Satisfaction of requirement (B.7) is completed.
Requirements (C.2) to (C.7) are satisfied.
Requirements (D.1) to (D.5) are satisfied.
Requirement (E.1) is satisfied.
Feasible solution found?
no
yes


yes
Upper bound equal to optimal value?
yes
Return the optimal solution.
no
Objective value better than lower bound?
yes
Update lower bound and best known solution.
no
no
Time limit reached?
yes
Return the best known solution.

Preprocessing

- Algorithm computes an upper bound to the maximum number of classic games that can be played at double weekend rounds.
- Upper bound depends on participating teams and calendar.
- Example: 2010 edition
  - Three classic games can be played in any double weekend round: Atlético Mineiro vs. Cruzeiro, Grêmio vs. Internacional, and Goiás vs. Atlético Goianiense.
  - Four teams in São Paulo: six classic games in each phase.
  - There are only four double weekend rounds: 11, 12, 13, and 15.
  - If classics are played in three consecutive rounds, at least one team would play two classics in consecutive rounds.
  - Therefore, only three classics can be played in São Paulo in double weekend rounds.
  - Same reasoning applies to Rio de Janeiro.
  - Upper bound: no more than nine out of 15 classic games can be played in double weekend rounds.

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Phase 1: HAP generation

- Home-away patterns of mirrored schedules may be seen as divided into two symmetric halves.
- Second half is completely determined by the first.
- Only HAPs with an even number of breaks in the first half are feasible.
- Number of feasible HAPs with exactly one home break and one away break is equal to $2 \cdot ((n-6)/2 - 1) \cdot (n-6)/2 = 84$ for $n = 20$ teams.
- Relatively small number of feasible HAPs with exactly two breaks each makes it possible to complete enumerate all of them.

Phase 2: HAP assignment

- Random assignment of two complementary HAPs to each pair of teams.
- Requirements (B.1) to (B.7) are all satisfied.

Phase 3: Schedule creation

- Build a simpler integer program by fixing the HAPs assigned to each team.
- Variables $y_{e\ell}$ are no longer necessary.
- Constraints (16) and (17) automatically satisfied.
- RHS of constraints (18) to (21) set to 0 or 1: fixations
- Reduced problem is solvable by a commercial solver.
- Optimal value equal to upper bound: solution is optimal.
- Optimal value smaller than the upper bound but better than lower bound: update the best known solution.
- Maximum time limit or another stopping criterion: return the best known solution.
- Otherwise, if the integer program is infeasible or its optimal value is smaller than the upper bound, then return to the second phase and perform new HAP assignments.

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Contents

Input data

- Participating teams change every year: the last four teams in Series A in the precedent year are replaced by the best four in Series B.
- Paired teams: pairs are usually formed by teams with the same home city. Cities hosting three of more teams offer different possibilities.
- Teams playing their first games at home are chosen by CBF. Teams upgraded from Series B usually play at home, to give a chance to their fans to celebrate the ascension.
- Games with predefined dates: for instance, one may be interested in scheduling the game between two teams with the same home city to a major local holiday.
- Calendar dates also change every year: they define midweek, weekend, and double-weekend rounds. Very tight calendar imposed by CBF for 2010, due to the interruption of the competition during the World Cup in June and July.

Contents

Development and validation

- Optimization software coded in C++ developed and validated over the last three years takes a few minutes of running time.
- Staff of CBF and TV Globo participated actively in the formulation and validation.
- Database with historical data supports the user to avoid repetitions of schedules and situations observed in previous years, such as playing at home or away in the first round.
- System validated with data from the 2005 and 2006 editions of the tournament, played by respectively 22 and 20 teams.
- Official schedules violated several requirements, while the optimized schedules met all constraints.
- 156 (172) breaks in the schedule played in 2005 (2006): far greater than the minimum number of breaks.
- Optimized schedules made it possible to broadcast all 56 of the most attractive games without conflicts, while only 43 (47) games could be broadcast in 2005 (2006).

Practice: 2009

- Software first used in 2009 as the official scheduler.
- Alternative schedules provided to the users: new requirements proposed and implemented along the decision process.
- Most attractive tournament in recent times: four teams still in contention for the title when the last round started.
- Title changed hands several times, due to changes in the scores of the ten simultaneous games underway.
- Champion was known only when the last game ended, contrarily to previous years in which the winners were known up to four rounds before the end of the tournament, making the games of the last rounds very uninteresting:
  - Many attractive games played in the last rounds, keeping high the interest of the public and the gate attendances.
  - Attractive scenario partly due to a fair and equilibrated schedule of games, in which no team had specific advantages or disadvantages over the others.

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Practice: 2010

- Optimization software used for the second time in 2010.
- Once again, the decision makers were happy with the alternative schedules computed by the system and with the choices they had.
- Tournament was hard to schedule:
  - Very tight calendar, due to its interruption in June and July for the World Cup.
  - More midweek rounds and fewer weekend rounds than usual.
  - Impossible to schedule all classics in double weekend rounds.
  - System found a schedule with the maximum of classic games played at double weekend rounds.
  - More constrained problem, since for the first time in many years all G12 teams participate in the tournament.
  - Increase in the number of teams from the state of São Paulo also made it harder to find balanced schedules.

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- Optimization software used for the third time in 2011.
- Some teams have been accused in 2009 and 2010 of facilitating their final games to avoid that their strong local rivals win the championship:
  - Gremio (RS) did not put its best players to play in the final game against Flamengo (RJ) in 2009, to avoid "giving" the championship to Internacional (RS).
  - Player of Palmeiras (SP) was called by bad names when he scored first against Fluminense (RJ) in 2012, since this could lead to Corinthians (SP) winning the title.
- Solution: schedule all local classic games (derbies) in the final rounds.
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Concluding remarks

- Fair and balanced schedules for all teams are a major issue for attractiveness and confidence in the outcome of a tournament.
- Few professional leagues adopted optimization software: this seems to be due not only to the difficulty of the problem and to some fuzzy requirements that can hardly be described and formulated, but also to the resistance of teams and leagues that are often afraid of using new tools that break with the past and introduce modern techniques in sports management.
- Operations Research has certainly proved its usefulness in sports management: besides the quality of the schedules found, the main advantages of the integer programming optimization system are its ease of use and the construction of alternative schedules, making it possible for the organizers to compare and select the most attractive schedule from among different alternatives, contemplating distinct goals.

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