Abstract. A specification language for real-time software systems is presented. Notions from Category Theory are used to specify how the components of a system should interact. The potential role of the proposed language in the search for interoperability of specification formalisms is briefly discussed.

1 Introduction

The aim of this work is to present RT-Community, a specification language for real-time reactive systems. Using the language, one is able to specify the computations of the individual components of a real-time system as well as the way these components interact with each other. This makes RT-Community suitable as an architecture description language (ADL) in the sense of [1].

RT-Community is an extension of the specification language Community ([7]), from which it inherits its characteristics as a coordination language ([4]). Roughly speaking, this means that it supports the separation between computational concerns (what each component does) and coordination concerns (how components are put together and how they interact to present the behavior expected of the system as a whole).

This paper presents the syntax and informal semantics of RT-Community in section 2 (a formal model-theoric semantics, not included here for lack of space, is found in [2]). Section 3 shows how composition is done in RT-Community by way of some basic notions of Category Theory. Finally, section 4 offers some concluding remarks, especially about the potential role of RT-Community in the search for interoperability of specification formalisms.

2 RT-Community

An RT-Community component has the form shown in Figure 1, where

- $V$ is a finite set of variables, partitioned into input variables, output variables and private variables. Input variables are read by the component from its environment; they cannot be modified by the component. Output variables can be modified by the component and read by the environment; they cannot be modified by the environment. Private variables can be modified by the component and cannot be seen (i.e. neither read nor modified) by the environment. We write $\text{inp}(V)$ for $\text{prv}(V) \cup \text{out}(V)$. We assume given but do not make explicit the specification of the data types over which the variables in $V$ range.
component $P$

\[
\begin{align*}
\text{in} & \quad \text{in} \ (V) \\
\text{out} & \quad \text{out} \ (V) \\
\text{prv} & \quad \text{prv} \ (V) \\
\text{clocks} & \quad C \\
\text{init} & \quad F
\end{align*}
\]

\[
\begin{align*}
\text{do} \\
g & \in sh(\Gamma) \quad g : [T(g), B(g) \rightarrow R(g), \ \|_{v \in D(g)} \ v := F(g, v)] \\
g & \in prv(\Gamma) \quad \text{prv} \quad g : [T(g), B(g) \rightarrow R(g), \ \|_{v \in D(g)} \ v := F(g, v)]
\end{align*}
\]

end component

Figure 1: The form of an RT-Community component

- $C$ is a finite set of clocks. Clocks are like private variables in that they cannot be seen by the environment; they can be consulted (i.e., their current values may be read) by the component; however, the only way a component can modify a clock variable is by resetting it to zero.

- $F$ is a formula specifying the initial state of the component (i.e., conditions on the initial values of variables in $\text{loc}(V)$). The initial values of input variables are unconstrained, and clocks are always initialized to zero.

- $\Gamma$ is a finite set of action names. Actions may be either shared or private. Shared actions are available for synchronization with actions of other components, whereas the execution of private actions is entirely under control of the component.

- The body of the component consists of a set of instructions resembling guarded commands. For each action $g \in \Gamma$, we have the time guard $T(g)$, which is a boolean expression over atomic formulae of the form $x \sim n$ and $x - y \sim n$, where $x, y \in C, n \in \mathbb{R}_{\geq 0}$ and $\sim$ is one of $\{<, >, =\}$; the data guard $B(g)$, which is a boolean expression constructed from the operations and predicates present in the specification of the data types of $V$; the reset clocks $R(g) \subseteq C$, containing the clocks to be reset upon execution of $g$; and the parallel assignment of a term $F(g, v)$ to each variable $v$ that can be modified by $g$. We denote by $D(g)$ the set of variables that can be modified by $g$. This set will be called the write frame (or domain) of $g$.

The execution of a component proceeds as follows: at each step, an action whose time and data guards are true (such an action is said to be enabled) may be executed. If more than one enabled action exists, one may be selected non-deterministically. If an action is selected, the corresponding assignments are effected and the specified clocks are reset. If no action is selected, time passes, the component idles and all clocks are updated accordingly (i.e., the semantics is based on global time). However, three conditions must be met: (1) a private action may not be enabled infinitely often without being infinitely often selected (the fairness condition), (2) time may not pass if there is an enabled private action (the urgency condition), and (3) a shared action may not be disabled solely by the passage of time (the persistency condition).

A model-theoretic semantics of RT-Community based on Time-Labelled Transition Systems can be found in [2].
component snooze
   in  float initialInterval, float minimum
   out  bool ringing
   prv  float interval
   clocks  c
   init  ~ringing ∧ interval = −1
   do
      [0] firstRing:  → ringing := true || interval := initialInterval
      [0] snooze:  ringing ∧ interval > minimum →
         reset(c) || ringing := false || interval := interval/2
      [0] off:  ringing → ringing := false || interval := −1
      [0] prv timeout:  c == interval → ringing := true
   end component

Figure 2: The snooze component

2.1 A Simple Example

We present a component for the “snooze” feature of an alarm clock. Component snooze is activated when the timekeeping component of the alarm clock (not shown) reaches the preset time, as indicated by action firstRing. This action sets the output variable ringing to true, a change that may be detected by a “bell” component (not shown either). If the user presses the “off” button at this point, the alarm and the snooze component are turned off, as indicated by the off action. However, if the user presses the “snooze” button (action snooze), the alarm stops ringing, only to ring again after a preset time interval. This second ringing of the alarm is activated by the snooze component upon detecting the timeout (private action timeout). Now, if the user presses the “snooze” button this time, he will be allowed to sleep for an additional period with half the duration of the initial interval. This pattern repeats, with the interval being halved each time the alarm rings and the user presses the “snooze” button, until either the user presses the “off” button or the interval reaches a certain minimum duration (in this last case, the alarm will go on ringing until the user presses the “off” button).

The duration of the initial interval and the minimum duration are provided by the environment of the snooze component, as indicated by input variables initialInterval and minimum. The specification of the snooze component is given in Figure 2. There, time guards and data guards that have the constant truth value true are omitted, and the resetting of a clock c is indicated by the reset(c) instruction.

3 Composing RT-Community Components

We use basic concepts from Category Theory ([6, 10]) to define how composition is done in RT-Community. The use of Category Theory allows us to describe the interaction of components in an abstract fashion, thus establishing the essentials of their coordination in such a way that RT-Community components may be replaced by specifications in other formalisms (e.g. a temporal logic with time-bounded operators such as MTL – see [3]), a desirable feature when our ultimate goal is to promote the interoperability of formalisms.
3.1 The Categories of Signatures and Components

Formally, an RT-Community component is a pair \((\Sigma, \Delta)\) with \(\Sigma\) the signature and \(\Delta\) the body of the component.

**Definition 3.1 (Signature of a component).** An RT-Community signature is a tuple \(\Sigma = < V, C, \Gamma, tv, ta, D, R >\), where \(V\) is a finite set of variables, \(C\) is a finite set of clocks, \(\Gamma\) is a finite set of action names, \(tv : V \rightarrow \{\text{in, out, prv}\}\) is the typing function for variables, \(ta : \Gamma \rightarrow \{\text{shr, prv}\}\) is the typing function for action names, \(D : \Gamma \rightarrow 2^{\text{loc}(V)}\) is the write frame function for action names, and \(R : \Gamma \rightarrow 2^C\) is the reset clock function for action names.

**Definition 3.2 (Body of a component).** The body of an RT-Community component with signature \(\Sigma\) is a tuple \(\Delta = < T, B, F, I >\), with \(T : \Gamma \rightarrow \mathcal{PROP}(C)\) the time guard function for action names, \(B : \Gamma \rightarrow \mathcal{PROP}(V)\) the data guard function for action names, \(F : \Gamma \rightarrow (\text{loc}(V) \rightarrow \mathcal{TERM}(\mathcal{M}(V)))\) the assignment function for action names, and \(I\) a formula over \(\text{loc}(V)\) specifying the initial state(s) of the component. Here, \(\mathcal{PROP}(C)\) denotes the set of boolean propositions over action names, \(\mathcal{PROP}(V)\) denotes the set of boolean propositions over variables and \(\mathcal{TERM}(\mathcal{M}(V))\) is the set of terms of the term algebra of the data types involved. Function \(F\) must respect sorts when assigning terms to variables.

We define the category of RT-Community signatures. In what follows, it is helpful to keep in mind that \(\Sigma_1\) is the signature of a component (referred to as “the component”) that is embedded in a system (referred to as “the system”) whose signature is \(\Sigma_2\).

**Definition 3.3 (Category of signatures).** \(\text{Sign}\) is the category that has signatures of RT-Community as objects; a morphism \(\sigma : \Sigma_1 \rightarrow \Sigma_2\) in \(\text{Sign}\) is a triple \(< \sigma_v, \sigma_c, \sigma_a >\) defined as follows:

\[\sigma_v : V_1 \rightarrow V_2\]

- For all \(v \in V_1\), \(\text{sort}_2(\sigma_v(v)) = \text{sort}_1(v)\). Variables of the component are mapped to variables of the system in such a way that sorts are preserved;

- For all \(o, i, p \in V_1\), \(o \in \text{out}(V_1) \Rightarrow \sigma_v(o) \in \text{out}(V_2)\), \(i \in \text{in}(V_1) \Rightarrow \sigma_v(i) \in \text{in}(V_2) \cup \text{out}(V_2)\), \(p \in \text{prv}(V_1) \Rightarrow \sigma_v(p) \in \text{out}(V_2)\). The nature of each variable is preserved, with the exception that input variables of the component may become output variables of the system. This is because, as will be seen below, an input variable of a component may be “connected” to an output variable of another component; when this happens, the resulting variable must be considered an output variable of the system: its value can be modified by the system, and it remains visible to the environment (which, however, cannot modify it).

\(\sigma_c : C_1 \rightarrow C_2\) is a total, injective function from the clocks of the component to the clocks of the system. In other words, all clocks of the component must retain their identity in the system.

\(\sigma_a : \Gamma_2 \rightarrow \Gamma_1\) is a partial function from the actions of the system to the actions of the component. \(\sigma_a\) is partial because an action of the system may or may not correspond to an action of the component; i.e., if the component does not participate in a given action
$g$ of the system, then $\sigma_a(g)$ is undefined. Furthermore, note the contravariant nature of $\sigma_a$ compared to $\sigma_v$ and $\sigma_c$: because each action of the system can involve at most one action of the component, and each action of the component may participate in more than one action of the system, the relation must be functional from the system to the component. Besides, $\sigma_a$ must satisfy the following conditions ($D(v)$ for a variable $v$ denotes the set of actions having $v$ in their domain):

- For all $g \in \Gamma_2$ for which $\sigma_a(g)$ is defined, $g \in shr(\Gamma_2) \Rightarrow \sigma_a(g) \in shr(\Gamma_1)$ and $g \in prv(\Gamma_2) \Rightarrow \sigma_a(g) \in prv(\Gamma_1)$. An action of the system is of the same nature (shared or private) as the component action involved in it.

- For all $g \in \Gamma_2$ for which $\sigma_a(g)$ is defined, and for all $v \in loc(V_1)$, $v \in D_1(\sigma_a(g)) \Rightarrow \sigma_v(v) \in D_2(g)$ and $g \in D_2(\sigma_v(v)) \Rightarrow \sigma_a(g) \in D_1(v)$. If a component variable is modified by a component action, then the corresponding system variable is modified by the corresponding system action. Besides, system actions where the component does not participate cannot modify local variables of the component.

The following item states analogous conditions for clocks of the component:

- For all $g \in \Gamma_2$ for which $\sigma_a(g)$ is defined, and for all $c \in C_1$, $c \in R_1(\sigma_a(g)) \Rightarrow \sigma_c(c) \in R_2(g)$ and $g \in R_2(\sigma_c(c)) \Rightarrow \sigma_a(g) \in R_1(c)$.

We define the category of RT-Community components, with whole components as objects and special signature morphisms between them.

**Definition 3.4 (Category of components).** Comp is the category that has components of RT-Community as objects; a morphism $\sigma : (\Sigma_1, \Delta_1) \rightarrow (\Sigma_2, \Delta_2)$ in Comp is a signature morphism $\sigma : \Sigma_1 \rightarrow \Sigma_2$ satisfying the following conditions:

- For all actions $g$ in $\Gamma_2$ with $\sigma_a(g)$ defined, we have $\Phi \models B_2(g) \rightarrow \bar{\sigma}(B_1(\sigma_a(g)))$ and $\Phi \models T_2(g) \rightarrow \bar{\sigma}(T_1(\sigma_a(g)))$, where $\Phi$ is a suitable axiomatization of the specification of the data types involved, and $\bar{\sigma}$ is the extension of $\sigma_v$ to the language of terms and propositions. The behavior of the system $(\Sigma_2, \Delta_2)$ is such that an action $g$ of the system cannot disrespect the data and time guards of the corresponding action $\sigma_a(g)$ of the component; i.e., the system can only strengthen said guards.

- $\Phi \models I_2 \rightarrow \bar{\sigma}(I_1)$, with $\Phi$ and $\bar{\sigma}$ as above. The initial state of the component must be implied by the initial state of the system.

- For all actions $g$ in $\Gamma_2$ with $\sigma_a(g)$ defined, and for all local variables $v$ in $D_1(\sigma_a(g))$, we have $F_2(g)(\sigma_v(v)) = \sigma(F_1(\sigma_a(g))(v))$, where, as before, $\sigma$ is the extension of $\sigma_v$ to the language of terms and propositions. Recall that $F$ is the function that assigns to each action $g$ a mapping from the variables in the action’s domain $D(g)$ to terms of the term algebra of the data types involved. This means that an action $g$ of the system can only assign to a local variable of the component the “translation” of the value that the corresponding action $\sigma_a(g)$ of the component does.
3.2 Channels and Configurations

The interaction of two components $P_1$ and $P_2$ may be either in the form of the connection of variables or in the form of the synchronization of actions (or both). This interaction is specified using a third component, called a channel. Given components $P_1$ and $P_2$ and a channel $P_c$, the composition of $P_1$ and $P_2$ via $P_c$ is represented by the diagram

$$P_1 \xrightarrow{\sigma_1} P_c \xrightarrow{\sigma_2} P_2$$

When certain conditions are met, such a diagram is called a configuration. In a configuration, morphisms $\sigma_1$ and $\sigma_2$ specify how $P_1$ and $P_2$ should interact, as discussed below:

**Variables:** The only variables that the channel $P_c$ can contain are of the input kind, so they can be mapped to input or output variables of $P_1$ and $P_2$. Given an input variable $v$ of $P_c$, we say that variables $\sigma_1(v)$ and $\sigma_2(v)$ of $P_1$ and $P_2$, respectively, are connected. If two input variables are connected, the result will be an input variable of the composite system. If an input variable and an output variable are connected, the result will be an output variable of the composite system. We do not allow two output variables to be connected (a diagram where this happens is not considered a well-formed configuration). Furthermore, private variables and clocks cannot be connected, so we do not allow the channel $P_c$ to have private variables or clocks.

**Actions:** The only actions that the channel $P_c$ can contain are of the shared kind. Furthermore, these actions must be “empty” in the sense that their time and data guards are the constant values $\text{true}$, they do not reset any clocks and do not modify any variables (i.e. their write frame is empty). This means that the channel $P_c$ is a “neutral” component with no behavior of its own, serving only as a kind of “wire” for joining the actions of $P_1$ and $P_2$.

Given an action $g_1$ of $P_1$ and an action $g_2$ of $P_2$ having $\sigma_1(g_1) = \sigma_2(g_2) = g$ for $g$ an action of $P_c$, we say that $g_1$ and $g_2$ are synchronized. When two actions are synchronized, the result will be a joint action of the composite system. This joint action will be of the shared kind, its data and time guards being the conjunction of the corresponding guards of $g_1$ and $g_2$, its effect being to reset the union of the sets of clocks reset by $g_1$ and $g_2$ and to perform the assignments in the union of the sets of assignments dictated by $g_1$ and $g_2$.

Formally, the system that results from the composition of $P_1$ and $P_2$ through channel $P_c$ is the pushout (in the category $\text{Comp}$) of the configuration above, written $P_1 +_{P_c} P_2$. In general, for a configuration diagram involving any (finite) number of channels and components, the resulting system is given by the colimit of the diagram. A configuration is said to be well-formed when it satisfies the conditions described above. For well-formed configurations, the colimit will always exist.

3.3 A Simple Example

In order to illustrate composition in RT-Community, we present the timekeeping component of the alarm clock discussed in Section 2.1 and show how the timekeeping and snooze components can be put together. The snooze component was shown in Figure 2. The timekeeping component is shown in Figure 3. There, it is assumed that a data type $\text{Time}$ is available, and that adding 1 to a variable of this data type corresponds to adding one second to the time value it contains. Besides, the fact that an action performs no assignments and resets no clocks (like the ring action) is indicated by the abbreviation skip.
component timekeeping
    in Time alarmTime, currentTime
    out float snoozeInterval, float minimum
    prv int ticksPerSec, Time now, boolean alarmOn
    clock c
    init snoozeInterval = 10 ∧ minimum = 1 ∧ ticksPerSec = ... ∧ ¬ alarmOn
    do
        [] setTime: → now := currentTime || reset(c)
        [] setAlarm: → alarmOn := true
        [] ring: alarmOn ∧ now == alarmTime → skip
        [] alarmOff: → alarmOn := false
        [] prv keepTime: c == ticksPerSec → now := now + 1 || reset(c)
    end component

Figure 3: The timekeeping component

When composing the timekeeping and snooze components, we want to identify variables snoozeInterval and initialInterval, as well as variables minimum (in timekeeping) and minimum (in snooze), so that the input variables in snooze will contain the constant values provided by timekeeping.

Furthermore, we want to synchronize actions ring and firstRing so that snooze will be activated exactly when timekeeping detects that the current time equals the time the alarm has been set to ring. We also want to synchronize the alarmOff and off actions, meaning that when the user presses the “off” button, both the snooze and the alarm mechanisms are turned off.

Notice that the resulting composite system is still open, in the sense that there are still unconnected input variables: currentTime and alarmTime receive values given by the user when he or she wants to set the time or the alarm, operations that are made available by the shared actions setTime and setAlarm, respectively.

Further interaction with the environment is given by the following features:

- The snooze action of the snooze component remains as a shared action of the system; it must be synchronized with an action of the environment representing the pressing of the “snooze” button while the bell is ringing.

- The joint action (alarmOff | off) is a shared action of the system that must be synchronized with an action of the environment representing the pressing of the “off” button while the bell is ringing.

- The output variable ringing in the snooze component must be connected to an input variable of a component representing the actual bell mechanism.

4 Concluding Remarks

RT-Community, like untimed Community, lends itself to the specification of architectural connectors that express interaction of a more complex nature than in the examples presented here. Other features of the language include the capacity for underspecification, such as lower
and upper bounds for action data guards (i.e., safety conditions and progress conditions, respectively), and the partial specification of the effect of an action \( g \) on its write frame \( D(g) \), by means of a boolean expression (involving primed variables, as is customary in other formalisms) instead of parallel assignment.

When specifying the components of a real-time system, it may be appropriate to use formalisms of a higher level of abstraction than RT-Community. One example would be the use of a real-time temporal logic (e.g. MTL – [3]) to specify the behavior of a component. In fact, we expect that by using mappings between logics such as those described in [5], a wider range of formalisms may be employed in the specification of a single system, allowing for a situation of interoperability among logics and specification languages.

The adaptation of RT-Community to systems involving mobility and dynamic reconfiguration is the subject of current study. A similar goal is being pursued in relation to untimed Community ([9]), where graph rewriting is used to reflect runtime changes in the system. We are investigating an alternative approach, where channels may be passed between components to allow them to engage in new connections at execution time – a strategy similar to the one used in \( \pi \)-calculus ([8]).

RT-Community is currently being contemplated for the specification of hypermedia presentations, a domain where real-time constraints occur naturally.

References