Reduction Rules for the Covering Tour Problem

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Abstract

The Covering Tour Problem (CTP) is a generalization of the Traveling Salesman Problem (TSP) which has several actual applications. It is defined on an undirected graph $G = (V \cup W, E)$, where W is a set of vertices that must be covered. The problem consists of determining a minimum length Hamiltonian cycle on a subset of V such that every vertex of W is within a given distance d from, at least, one node in the cycle. This work proposes reduction rules to a generalization of the CTP and also a new Integer Linear Program formulation.

Key words: Covering Tour Problem; mathematical formulation; reduction rules.

1 Introduction

This work presents a filtering technique, composed by some reduction rules that minimize meaningfully CTP original instances'size. This technique can be applied to exact or approximated methods. Being the CTP not very explored, it also proposes a mathematical formulation and a metaheuristic based on GRASP [5] and VNS [6] concepts. The Covering Tour Problem (CTP) was first introduced by Current [1]. This Problem can be defined as follows: let $G = (V \cup W, E)$ be an undirected graph, where $V \cup W = \{1, ..., n\}$ is the vertex set and $E = \{(i, j) \mid i, j \in V \cup W, i < j\}$ is the edge set. Vertex $s = 1$ is the source node, V is a set of vertices that might be visited, $T \subseteq V$ is a set of vertices that must be visited ($s \in T$), and W is a set of vertices that must be covered. A distance matrix $C = (c_{ij})$, defined on E, uses the Euclidean metric. The problem consists of determining a minimum length tour or a Hamiltonian cycle over a subset of $V \cup W$ in such way that the tour contains all vertices of T, and every vertex of W is covered by the tour, i.e., it lies within a distance d from a

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vertex of the tour. The CTP is NP-Hard as it reduces to a TSP when $d = 0$ and $V = W$. Such matter has not received much attention in the literature so far. Two heuristic and two exact methods have been presented for the CTP. The first heuristic was proposed to generate a set of solutions to the exact method [2]. The second one [3] combines a heuristic for the TSP to another one for the Set Covering. Maniezzo et al. [4] presented three Scatter Search metaheuristic to the CTP. The Covering Tour Problem was first formulated by Current and Schilling [2]. Later, Gendreau et al. [3] and Maniezzo et al. [4] presented new formulations for the CTP. Both existing formulations provide traditional constraints to avoid unconnected subtours dealing with an exponential number of associate constraints. This paper proposes a new linear integer programming formulation based on flow variables which satisfies the subtour elimination constraints in another form. The basic formulation idea is to associate a flow variable to each edge $(i, j) \in E$. This approach allows to generate a polynomial number of associate constraints and not an exponential number as the ones already proposed. The presented formulation considers a generalized version of the CTP, where among all vertices of T and a subset of V, the tour may also contain the vertices of W. Let y_k, for $k \in V$, be a $(0-1)$ binary variable equal to 1 if and only if vertex k belongs to the tour. If $k \in T$, then y_k is necessarily equal to 1. Let x_{ij}, $i, j \in V$ and $i \neq j$, another binary variable, equal to 1 if and only if edge (i, j) belongs to the tour. A binary coefficients δ_{lk}, equal to 1 if and only if $l \in W$ can be covered by $k \in V$ (i.e.,$c_{lk} \leq c$), and let $S_l = \{k \in V \mid \delta_{lk} = 1\})$ for every $l \in W$. We also defined a non-negative integer flow variable z_{ij}, associated to each edge $(i, j) \in E$, $i \neq j$. It represents the flow through the arc (i, j). Then the CTP can be stated as:

$$(PRR)\ minimize \sum_{i<j \,|\, i,j \in V} c_{ij}x_{ij}, \qquad st: \tag{1}$$

$$\sum_{k \in S_l} y_k \geq 1 \qquad (\forall l \in W), \tag{2}$$

$$\sum_{i<k} x_{ik} + \sum_{j<k} x_{kj} = 2y_k \qquad (\forall k \in V \cup W), \tag{3}$$

$$\sum_{j \in V \cup W} z_{kj} = \sum_{i \in V \cup W} z_{ik} + y_k \qquad (\forall k \in V \cup W - \{s\}), \tag{4}$$

$$\sum_{j \in V \cup W} z_{sj} = 1 \quad and \quad \sum_{j \in V \cup W} z_{js} = \sum_{j \in V \cup W} y_j \tag{5}$$

$$x_{ij} \leq z_{ij} \quad (\forall i, \forall j \in V \cup W); \tag{6}$$

$$x_{ij} \geq (z_{ij})/(|V| + |W| + 1) \qquad (\forall i, \forall j \in V \cup W), \tag{7}$$

$$y_k = 1 \qquad (\forall k \in T), \tag{8}$$

$$y_k \in \{0, 1\} \quad (\forall k \in V \backslash T) \quad and \quad x_{ij} \in \{0, 1\} \quad (\forall i, \forall j \in V \cup W) \tag{9}$$

$$z_{ij} \in Z^{+} \quad (\forall i, \forall j \in V \cup W). \tag{10}$$

In the formulation, defined by constraints (1)-(10), the constraints (2) ensure that every vertex of W is covered by the tour. The constraints (3) represent the flow conservation equations . Constraints (4) and (5), unable disconnect subtours. The constraints (6) and (7) ensure that the generated tours from the flow variable (z_{ij}) and from decision variable (x_{ij}) match. Constraints (8) ensure that every vertex of T belongs to the tour. Finally, constraints (9) and (10) represent the integrality requirements.

2 Reduction Rules for the Generalized version of the CTP

This work considers a generalized version of the CTP (GCTP). It consists of finding a minimum length tour also through a subset of W, instead of only a subset of V. In this circumstances, the existing reduction rules does not work. Being the CTP a very large instance problem, the use of filtering techniques give effectiveness to exact methods and to high complexity heuristics. Below, some rules for the GCTP are proposed. Let $Cob = (cob_{ij})$ be a $|W| \times |V \cup W|$ $(0-1)$ matrix where $cob_{ij} = 1$, $i \in W, j \in V \cup W$ if and only if $c_{ij} \leq d$. Sets W and V can be reduced by applying the following reduction rules only once:

- Transform every vertex $i \in W$ for which $cob_{ik} = 1$, with $k \in T$, in a vertex $l \in V \backslash T$;
- Remove every vertex i for which $cob_{ji} = 0$, $\forall j \in W$, from $V \backslash T$.

In addition, a metaheuristic algorithm was developed to approximately solve the CTP, following the guidelines of the GRASP (*Greedy Randomized Adaptive Search Procedures*, proposed by Feo and Resende [5]. In the first phase, the metaheuristic generates an initial feasible solution for the problem. In the local search phase, concepts of VNS (Variable Neighborhood Search)[6] are used.

$\lvert(V \bigcup W)\rvert$	EM	EM+RR	GRASP	GRASP+RR
10	5,6	0,2	3,1	0
15	1239	7,8	16,4	4,3
20	**	203,4	54	29,7
25	**	4759,9	145	93,4

Table 1

Required time (in sec) to find the optimal solution. The (**) denotes that exact solutions were not found in 4 hours of processing time. EM: exact method using original graph, EM+RR: exact method using reduction rules, GRASP: GRASP with original graph, GRASP+RR: GRASP using reduction rules

This work presents a generalized version of the CTP called *Generalized Covering Tour Problem* (GCTP). The contributions of this work include a new mathematical formulation and some reduction rules for the GTSP. Partial computational testes show a significant reduction in the instances' size, something about 40% to 60% with instances generated randomly, varying the percentage of $|T|$ and $|W|$. In addition to the results described in Table 1, a set of instances with 200 and 300 nodes were also considered . The execution time in both cases was fixed at 1200 seconds. The final solutions obtained by our heuristic GRASP using the proposed reduction rules were (in average) 63,6% and 64% respectively better than the results produced by GRASP when applied to the original graphs. These results show the reduction rule impact to approximately or optimally solve the GTSP.

Related to exact methods, we verify that the already existing formulations for the CTP are inappropriate when implemented to softwares like XPRESS because of the high number of constraints to avoid disconnected subtours. Hence, a new mathematical formulation that decreases the number of lines in the input matrix is presented. The reduction tests here proposed enable a more intensive search even with metaheuristic under more sophisticated mechanisms like the one proposed in this paper which is based on GRASP/VNS concept.

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