

A hybrid heuristic for a broad class of vehicle routing problems with heterogeneous fleet

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Abstract We consider a family of rich vehicle routing problems (RVRP) which have the particularity to combine a heterogeneous fleet with other attributes, such as backhauls, multiple depots, split deliveries, site dependency, open routes, duration limits, and time windows. To efficiently solve these problems, we propose a hybrid metaheuristic which combines an iterated local search with variable neighborhood descent, for solution improvement, and a set partitioning formulation, to exploit the memory of the past search. Moreover, we investigate a class of combined neighborhoods which jointly modify the sequences of visits and perform either heuristic or optimal reassignments of vehicles to routes. To the best of our knowledge, this is the first unified approach for a large class of heterogeneous fleet RVRPs, capable

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of solving more than 12 problem variants. The efficiency of the algorithm is evaluated on 643 well-known benchmark instances, and 71.70% of the best known solutions are either retrieved or improved. Moreover, the proposed metaheuristic, which can be considered as a matheuristic, produces high quality solutions with low standard deviation in comparison with previous methods. Finally, we observe that the use of combined neighborhoods does not lead to significant quality gains. Contrary to intuition, the computational effort seems better spent on more intensive route optimization rather than on more intelligent and frequent fleet re-assignments.

Keywords Rich vehicle routing · Heterogeneous fleet · Matheuristics · Iterated local search · Set partitioning

1 Introduction

The capacitated Vehicle Routing Problem (VRP) is one of the most studied problems in the field of combinatorial optimization. Since the seminal work of [Dantzig and Ramser \(1959\)](#), many additional constraints, objectives and decision subsets, called problem attributes, have been combined with the classical version of the problem. Such attributes include multiple depots, pickup and delivery, backhauls, heterogeneous fleets, time windows, among others. The reader is referred to [Vidal et al. \(2013a\)](#) for a recent survey and classification on the most common attributes adopted in the VRP literature. Several variants that consider each of these attributes individually received a lot of attention over the past few years. However, in practical applications, many attributes tend to appear together, thus increasing the resolution challenges. Several recent articles have attempted to cope with this increasing variety of problems. The term *rich*, in particular, has being widely adopted to describe VRP versions composed of multiple attributes. Given the importance of solving real-world problems, unified solution methods capable of tackling many VRP variants are of high importance. This explains the recent trend in the development of this type of approach ([Røpke and Pisinger 2006](#); [Subramanian et al. 2013](#); [Derigs and Vogel 2014](#); [Vidal et al. 2014](#)).

One important aspect of practical VRP applications is the frequent use of a *heterogeneous* fleet of vehicles ([Hoff et al. 2010](#)), with different capacities and operational costs. This type of VRPs was excluded from most unified frameworks available in the literature. For example, the framework of [Vidal et al. \(2014\)](#) considered VRPs with heterogeneous fleet, but only in the case where the fleet is unlimited (fleet size and mix VRP—FSMVRP).

The contributions of this work are as follows.

- We propose the first unified algorithm designed to solve a broad class of Heterogeneous Fleet RVRPs (HFRVRPs), thus filling the methodological gap of previous works and extending the range of applications to richer and more challenging variants.
- Our algorithm is capable of dealing with at least 10 distinct and well-known attributes, such as multiple depots, time windows, (mixed) backhauls, site dependency, and split deliveries. The attributes may be considered one at a time (classical variants) or simultaneously (rich variants), leading to a wide gamut of problems. In practice, hundreds of variants could be formed with those attributes. Obviously, for comparison purposes, we have decided to test our algorithm only on those variants where there has been substantial research and publicly available instances.
- The algorithm generalizes the ILS-RVND-SP matheuristic of [Subramanian et al. \(2013\)](#), originally designed for vehicle routing problems with homogeneous fleet, and the alter-

native version of this matheuristic (Subramanian et al. 2012) that was developed for classical Heterogeneous Fleet VRPs (HFVRPs). ILS-RVND-SP combines an Iterated Local Search (ILS—Lourenço et al. 2010) with Randomized Variable Neighborhood Decent (RVND) and an integer programming-based optimization over a Set Partitioning (SP) formulation. The routes generated by the ILS-RVND heuristic (Penna et al. 2013) are used to create a pool of promising routes for the SP. The SP problem is then solved by a Mixed Integer Programming (MIP) solver, which interacts with the ILS-RVND during its execution.

- The generalization includes, among other features, the addition of alternative constructive procedures, neighborhood structures and perturbation mechanisms in order to cope with attributes that were not considered in Subramanian et al. (2013). We also introduce a novel perturbation scheme that exploits the heterogeneous characteristic of the fleet. Moreover, in contrast to Subramanian et al. (2012, 2013), the proposed generalized algorithm now accepts infeasible solutions, which seem to be crucial for obtaining high quality solutions when dealing variants with time windows (Vidal et al. 2013a,b). In addition, move evaluations for this class of variants are now performed via the efficient approach presented in Vidal et al. (2014). Overall, this led to a successful algorithm, which produces solutions of high quality, and sometimes new best solutions, for 12 difficult VRP variants with heterogeneous fleet, 20 sets of benchmark instances and overall 643 test problems.
- Finally, we followed the reasonable belief that optimality gaps could be related to errors in fleet assignment rather than errors in routing. For this reason, we tested more advanced neighborhoods with combine moves on the sequence of visits with a re-optimization (either heuristic or exact) of the fleet assignment decisions. From our experiments, this approach did not lead to significant improvements. As this (negative) result seems very counter-intuitive at first glance, we believe that it deserves some discussion in the present paper.

The remainder of this paper is structured as follows. Section 2 describes the problems under study and Sect. 3 reviews the main works related to RVRP and heterogeneous fleet variants. Section 4 describes the proposed matheuristic as well as the combined neighborhoods with joint routing and assignment optimization. Section 5 reports the computational results and establishes a comparison with the current literature. Section 6 finally concludes.

2 Class of problems considered

In what follows, we provide a formal description of the classical HFVRPs as well as of the main attributes considered in this work.

The HFVRP can be defined as follows: let $G = (V, A)$ be a directed graph where $V = \{0, 1, \dots, n\}$ is a set composed of $n + 1$ vertices, and $A = \{(i, j) : i, j \in V, i \neq j\}$ is the set of arcs. Vertex 0 denotes the depot, where the vehicle fleet is located, while the set $V' = V \setminus \{0\}$ includes the remaining vertices which represent the n customers. Each customer $i \in V'$ has a non-negative demand q_i . The fleet is composed by K different types of vehicles, with $M = \{1, \dots, K\}$. For each $k \in M$, there are m_k available vehicles, each with a capacity Q_k . Every vehicle type is also associated with a fixed cost denoted by f_k . Finally, each arc $(i, j) \in A$ has a length d_{ij} and its traversal cost for vehicle type k is $c_{ij}^k = d_{ij} \times r_k$, where r_k is a cost per distance unit, also called dependent cost or variable cost in the literature. The objective is to determine a fleet composition as well as a set of routes,

$R^k = (i_1, i_2, \dots, i_{|R^k|})$, that minimize the sum of fixed and travel costs in such a way that: (a) every route R^k starts and ends at the depot ($i_1 = i_{|R^k|} = 0$ and $\{i_2, \dots, i_{|R^k|-1} \subseteq V'\}$) and is associated with a vehicle type $k \in M$; (b) each customer belongs to exactly one route; (c) vehicle capacity is not exceeded; (d) for each vehicle type k , the number of vehicles actually used does not exceed m_k .

The HFVRP is \mathcal{NP} -hard since it includes the classical VRP as a special case when all vehicles are identical. The problem was introduced by Golden et al. (1984) under the name Fleet Size and Mix (FSM), which is a variant that assumes an unlimited number of vehicles of each type, i.e., $m_k = +\infty, \forall k \in M$. Fifteen years later, Taillard (1999) proposed the Heterogeneous Fixed Fleet VRP (HFFVRP), a variant in which the number of vehicles of each type is limited.

Since the seminal works of Golden et al. (1984) and Taillard (1999), several HFVRP extensions considering well-known real-life VRP attributes were presented in the literature. We were able to deal with the following ones:

- **Asymmetry (A)**: the costs of two opposite arcs may differ, i.e., c_{ij} is not necessarily equal to c_{ji} , for $i \in V$ and $j \in V$. Although this is the case in cities with one-way streets, a vast majority of VRP articles consider undirected networks which lead to simpler local search procedures.
- **Open (O) routes**: in open VRPs, the vehicle does not need not return to the depot after visiting the last customer, i.e., $c_{i0} = 0$ for all $i \in V$.
- **Multiple depots (MD)**: more than one depot is available, but each vehicle must start and end at the same depot in a route. The number of vehicles per depot is usually limited.
- **Multiple trips (MT)**: each vehicle may perform a sequence of successive trips called multitrip, often limited by a maximum length or duration.
- **Backhauls (B)**: two different types of customers are considered, more precisely, linehaul and backhaul. The first type includes the customers with delivery demands, while the second one includes those with pickup demands. In this case, backhaul customers can only be visited after the last linehaul customer, and a route cannot be only composed of backhaul customers. The vehicle leaves the depot with a load that is equal to the sum of the delivery demands (linehauls) in the route, and returns to the depot with a load that is equal to the sum of the pickup demands (backhauls).
- **Mixed Backhauls (MB)**: similar to the previous case, but there are no constraints on the order in which linehaul and backhaul customers should be visited. The load of a vehicle may increase or decrease along its route.
- **Site dependency (SDep)**: some customers can only be visited by a subset of the existing vehicles. This usually happens when a customer site limits its access to vehicles with some particular characteristics.
- **Split Deliveries (SD)**: customers can be visited more than once and their demands can be split among different vehicles. In this case, it is necessary to decide on the amount of goods to be delivered to each customer by each vehicle.
- **Time Windows (TW)**: a time window $[a_i, b_i]$ and a service time s_i are defined for each vertex $i \in V$. The service of a customer should start within its time window and a vehicle is allowed to arrive at customer i before a_i , but not after b_i . Note that waiting times are allowed.
- **Route Duration (RD) constraints**: there exists a limit D on the duration of a route in terms of distance or time.

3 Related works

The classical HFVRP has been widely studied in the literature, as can be observed in the surveys of Baldacci et al. (2008), Hoff et al. (2010), Irnich et al. (2014) and Koç et al. (2016). RVRPs have also been an object of active research interest. The reader is referred to Cáceres-Cruz et al. (2014a), Derigs and Vogel (2014) and Lahyani et al. (2015) for a comprehensive literature review on this topic. In this section we review some of the main works and milestones for the HFRVRPs.

As in most vehicle routing problems, the progress on solution methods went through several phases: from early constructive methods, towards local search-based heuristics, metaheuristics, and hybrid methods. First, Golden et al. (1984) developed several heuristics for the FSMVRP, while Taillard (1999) presented a column generation heuristic for the HFFVRP. In the following years, nearly all classical metaheuristic frameworks have been considered:

- *Tabu Search (TS)* in Gendreau et al. (1999), Wassan and Osman (2002), Lee et al. (2008), Brandão (2009) and Brandão (2011);
- *Variable Neighborhood Search (VNS)* in Imran et al. (2009);
- *Iterated Local Search (ILS)* in Penna et al. (2013) and a hybrid ILS in Subramanian et al. (2013);
- *Threshold Accepting (TA)* in Tarantilis et al. (2003, 2004);
- *Record-to-Record (RTR)* in Li et al. (2007);
- *Adaptive Memory Programming (AMP)* in Rochat and Taillard (1995), Li et al. (2010);
- *Multi-Start (MS)/Evolutionary Local Search (ELS)* in Duhamel et al. (2011, 2013);
- *Hybrid Genetic Algorithms (HGA)* in Ochi et al. (1998a, b), Lima et al. (2004), Prins (2009), Liu et al. (2009) and Vidal et al. (2014).

Not all these methods were equally successful, and their relative performance can even vary between different benchmark sets. After years of research on metaheuristics for the HFVRP, it was impossible to conclude on a *more suitable* metaheuristic framework. The common viewpoint remains that success cannot be attributed to one method's name, but rather to its specific components, efficient neighborhood structures, a proper balance between intensification and diversification strategies (Blum and Roli 2003; Vidal et al. 2013a), as well as a good distributions of the search effort dedicated to the sequencing and customer-to-vehicle assignment decision sets. Finally, the research on exact methods has also progressed over the last few years. The current state-of-the-art approaches (Choi and Tcha 2007; Pessoa et al. 2009; Baldacci et al. 2009, 2010a, b) can consistently solve instances with 75 customers in a few minutes, as well as some instances of a few 100 customers. However, this performance is still insufficient for many practical applications.

Rich variants of the HFVRP can include a large variety of attributes. From early works on separate problems, which tends to reduce our ability to compare method performances, the literature has evolved towards unified solution frameworks with the potential to be applied and compared on a wide gamut of problems. With this aim, some of the most successful algorithms include the Unified Tabu Search (UTS) of Cordeau et al. (2001), the Adaptive Large Neighborhood Search (ALNS) of Pisinger and Røpke (2007), the Iterated Local Search (ILS) of Subramanian et al. (2013) and the Unified Hybrid Genetic Algorithm (UHGS) of Vidal et al. (2014). The success of UTS can be mainly explained by very simple neighborhoods and diversity mechanisms. To achieve high quality solutions for a variety of problems, ALNS uses a family of destruction and reconstruction procedures, along with an adaptive selection strategy. ILS exploits again simple neighborhoods along with mathematical programming over a set partitioning formulation. Finally, UHGS relies on generic route evaluation opera-

tors, a giant-tour solution representation (see [Prins et al. 2014](#) for a survey on VRP algorithms based on this representation), as well as advanced diversity management mechanisms that promote different and good solutions for survival.

Table 1 summarizes a large variety of works related to rich HFVRPs. The table is organized by method family (constructive heuristic or local search; neighborhood-centered; population-based; mathematical programming). The first column, **Authors**, shows the authors names. The second, named **Approach** list the method used by the authors to solve the problem. Next, columns **Fleet** and **Costs** present the main attributes of HFVRP, the fleet size and the associated vehicles costs, respectively. Finally, the last set of columns, **Additional Attributes**, shows the multi-attributes characteristic of variant studied by the authors, where **MD**, **TW**, **RD**, **SDep**, **SD**, **MT**, **Op** and **Others** are, respectively, Multi-Depot, Time Windows, Route Duration, Site Dependency, Split Delivery Multi-Trip, Open and others attributes.

In general, most research emphasis has been focused, in recent years, on metaheuristic principles and new problems rather than studies on neighborhoods and elementary building blocks of the methods. We aim to contribute to this latter point, in order to progress towards unified methods and simple concepts which can be efficiently applied to a good variety of HFRVRPs. Finally, even after nearly a hundred articles dealing with HFVRPs, some important questions remain open:

- What is a good neighborhood for HFFVRP?
- Is it beneficial or not to revise frequently vehicle-to-route assignments during computations?

This is a second gap in the literature on which we aim to contribute.

4 HILS-RVRP algorithm

The proposed unified hybrid heuristic, called HILS-RVRP, extends the algorithms of [Penna et al. \(2013\)](#) and [Subramanian et al. \(2012, 2013\)](#). It combines a multi-start ILS ([Lourenço et al. 2010](#)) with mathematical programming over a SP formulation.

Algorithm 1 presents the pseudocode of HILS-RVRP. This matheuristic performs I_{MS} restarts (Lines 6–12). At each restart, an ILS-RVND heuristic is responsible for improving an initial solution (Lines 7–8), as well as populating a pool of routes (r_{pool}) associated with locally optimal solutions. The pool is then used to build a restricted SP model, which is solved by a MIP solver. Two strategies are proposed to exploit the SP model. If the instance is considered to be of sufficient size ($n \geq n_{large}$), then the algorithm calls the SP approach after each restart (Line 10). Otherwise, if the problem is of smaller size, then the SP model is solved only once at the end of the last restart. As in [Subramanian et al. \(2013\)](#), we have assumed that $n_{large} = 150$.

The pseudocode of the ILS-RVND sub-procedure is depicted in Algorithm 2. This heuristic iteratively performs local search and perturbation operations until a maximum number of consecutive iterations without improvements of the best current solution (I_{ILS}) is achieved (lines 5–12). The pool of routes is updated after each call to the local search procedure (Lines 3 and 8). In order to avoid r_{pool} to grow arbitrarily large, the algorithm only adds to the pool those routes associated with solutions whose gap with respect to the best current solution is relatively small, as described in [Subramanian et al. \(2013\)](#).

Algorithm 3 describes the SolveSP procedure. The routes associated with the local optima of each iteration are permanently stored in the pool (Lines 2 and 9), while the remaining ones are treated as temporary routes that are removed at each iteration (Line 12). A time limit

Table 1 HFRVRP related works and attributes

Authors	Approach(es)	Fleet		Costs			Additional attributes									
		U	L	F	V	FV	MD	RD	TW	SDep	SD	MT	Op	Others		
Liu and Shen (1999)	CH	✓		✓					✓							
Dell'Amico et al. (2007)	CH + RR	✓		✓					✓							
Dondo and Cerdá (2007)	CH		✓			✓			✓						✓	
Cáceres-Cruz et al. (2013)	CH	✓	✓	✓				✓								A
Cáceres-Cruz et al. (2014b)	CH	✓	✓	✓				✓							✓	
Prins (2002)	CH + LS/TS	✓	✓	✓				✓							✓	
Salhi and Sari (1997)	LS	✓				✓										
Cordeau and Laporte (2001)	TS		✓					✓								
Cordeau et al. (2004)	TS		✓					✓								
Paraskevopoulos et al. (2008)	TS	✓		✓					✓							
Li et al. (2012)	TS + AMP		✓			✓										✓
Yousefikhoshbakt et al. (2014)	TS		✓			✓										✓
Cordeau and Mayschberger (2012)	ILS + TS		✓					✓								
Tavakkoli-Moghaddam et al. (2007)	SA		✓			✓								✓		
Bräysy et al. (2008)	SA	✓		✓					✓							
Bräysy et al. (2009)	TA + GLS	✓		✓					✓							
Pisinger and Røpke (2007)	ALNS		✓					✓								
Amorim et al. (2014)	ALNS	✓							✓							GI
Mancini (2016)	ALNS	✓				✓			✓							P, PI
Repoussis and Taramitis (2010)	AMP	✓		✓					✓							
Tütüncü (2010)	GRASP + AMP		✓			✓										B
Duhamel et al. (2011)	GRASP + ELS		✓					✓								
Duhamel et al. (2013)	GRASP + ELS		✓			✓										
Mar-Ortiz et al. (2013)	GRASP		✓					✓						✓		✓

Table 1 continued

Authors	Approach(es)	Fleet			Costs			Additional attributes							
		U	L	F	F	V	FV	MD	RD	TW	SDep	SD	MT	Op	Others
Goel and Gruhn (2008)	VNS/LNS		✓			✓				✓				✓	PD, MC
Salhi et al. (2014)	VNS	✓				✓		✓							
de Armas et al. (2015)	VNS		✓			✓			✓		✓				CPr
de Armas and Melián-Batista (2015)	VNS		✓			✓			✓		✓				CPr, DR
Dominguez et al. (2016)	MS	✓				✓									2L
Reyes et al. (2007)	ACO		✓				✓			✓	✓				VS
Pellegrini et al. (2007)	ACO		✓			✓			✓						P
Belmecheri et al. (2013)	ACO/PSO		✓			✓			✓						MB
Belfiore and Yoshizaki (2009)	SS		✓			✓				✓	✓				
Vidal et al. (2014)	GA		✓			✓			✓		✓				
Berghida and Boukra (2015)	EA		✓			✓				✓					MB
Koç et al. (2015)	EA		✓			✓				✓					
Yao et al. (2016)	PSO		✓			✓				✓					CD
Ceselli et al. (2009)	CG		✓			✓				✓	✓				WT, EC, PI
Goel (2010)	CG		✓			✓				✓				✓	PD, MC
Bettinelli et al. (2011)	BCP/CG		✓			✓				✓					
Ozfirat and Ozkarahan (2010)	CP		✓			✓					✓				
Rieck and Zimmermann (2010)	Math. Model		✓			✓									SPD, WT, D
Salhi et al. (2013)	Math. Model/LS		✓			✓									B
HILS-RVRP (this work)	ILS + SP		✓			✓				✓	✓		✓	✓	B, MB

2L two-dimensional loading, A asymmetric, B backhauls, CD collection depot, Cpr customer priority, D docking, DR dynamic request, EC external courier, MB mixed backhauls, MC multi-dimensional capacity, P periodic, PD pickup and delivery, PI products incompatibility, SPD simultaneous pickup and delivery, VS vehicle scheduling, WT working time Regulations

Algorithm 1: HILS-RVRP($I_{MS}, I_{ILS}, T_{max}, n_{large}, RGap_{max}$)

```

1 begin
2   Initialize fleet
3    $v \leftarrow$  total number of vehicles
4    $f(s^*) \leftarrow \infty$ 
5    $r_{pool} \leftarrow \emptyset$ 
6   for  $i \leftarrow 1$  to  $I_{MS}$  do
7      $s \leftarrow$  GenerateInitialSolution( $v$ )
8      $[s^{*'}, r_{pool}] \leftarrow$  ILS-RVND( $s, I_{ILS}, r_{pool}$ )
9     if ( $n \geq n_{large}$  or  $i = I_{MS}$ ) then
10       $[s^{*'}, r_{pool}] \leftarrow$  SolveSP( $r_{pool}, s^{*'}, I_{ILS}, T_{max}, RGap_{max}$ )
11      if ( $f(s^{*'}) < f(s^*)$ ) then
12         $s^* \leftarrow s^{*'}$ 
13   return  $s^*$ 
14 end

```

Algorithm 2: ILS-RVND(s, I_{ILS}, r_{pool})

```

1 begin
2    $s^{*'} \leftarrow$  LocalSearch( $s$ )
3   AddTemporaryRoutes( $r_{pool}, s^{*'}, f(s^*)$ )
4    $iter_{ILS} \leftarrow 0$ 
5   while ( $iter_{ILS} \leq I_{ILS}$ ) do
6      $s' \leftarrow$  Perturbation( $s^{*'}$ )
7      $s'' \leftarrow$  LocalSearch( $s'$ )
8     AddTemporaryRoutes( $r_{pool}, s'', f(s^*)$ )
9     if ( $f(s'') < f(s^{*'})$ ) then
10       $s^{*'} \leftarrow s''$ 
11       $iter_{ILS} \leftarrow 0$ 
12     $iter_{ILS} \leftarrow iter_{ILS} + 1$ 
13   return  $[s^{*'}, r_{pool}]$ 
14 end

```

T_{max} is imposed to the solver to avoid prohibitively large CPU time. During the resolution of the SP, the solver may find a new best integer solution. In that case, it calls the ILS-RVND to improve it (Line 6). If this improvement leads to a solution whose value is better than the current lower bound of the SP model, then the solver execution is naturally interrupted, otherwise the value of the solution is used as a cut-off bound. Finally, when solving FSM variants, this model can be further modified in case the gap between the linear relaxation of the the root node and the incumbent solution exceeds a given input value ($RGap_{max}$). As thoroughly detailed in [Subramanian et al. \(2012\)](#), the modification consists of adding constraints to forbid the vehicle fleet associated with the best current solution to be changed. This may remove potential improved solutions from the integer linear program, but decreases the root gap and lets the model be more computationally tractable.

Our HILS-RVND also significantly differs from the previously cited works, as it has been extended to a much wider family of VRP variants with heterogeneous fleet and integrates several key improvements: (i) the ability of handling infeasible solutions with respect to time constraints; (ii) the incorporation of new neighborhoods and perturbations to manage multiple-depots, backhauls, split and time-windows attributes; and (iii) the addition of pre-

Algorithm 3: SolveSP($r_{pool}, s^*, I_{ILS}, T_{max}, RGap_{max}$)

```

1 begin
2   AddPermanentRoutes( $r_{pool}, s^*$ )
3   improvement  $\leftarrow$  true
4   while (improvement) do
5      $SP_{model} \leftarrow$  CreateSPModel( $r_{pool}, v$ )
6      $s' \leftarrow$  MIPSolver( $SP_{model}, s^*, T_{max}, RGap_{max}, ILS\text{-}RVND(s^*, I_{ILS}, r_{pool})$ )
7     if ( $f(s') < f(s^*)$ ) then
8        $s^* \leftarrow s'$ 
9       AddPermanentRoutes( $r_{pool}, s^*$ )
10    else
11      improvement  $\leftarrow$  false
12  RemoveTemporaryRoutes( $r_{pool}$ )
13  return [ $s^*, r_{pool}$ ]
14 end

```

processing phases and auxiliary data structures for efficient move evaluations. We finally investigate, within this method, a class of large neighborhoods which aim to jointly change the sequence of visits and optimize the assignment of vehicles to routes. The next subsections now provide a detailed description of each component of the method.

4.1 Constructive procedure

Initial solutions are generated via a simple insertion heuristic. Firstly, each route is filled with a random customer, and the remaining ones are inserted either according to (i) a nearest insertion criterion, or (ii) a modified cheapest insertion criterion which promotes the insertion of customers located far from the depot. At each restart, one of these criteria is randomly selected. Infeasible solutions are incorporated and penalized in accordance with the characteristics of the problem as follows:

Fixed fleet When it is no longer possible to perform a feasible insertion of an unrouted customer due to fleet capacity, an extra vehicle with large fixed/dependent costs and capacity is added to the partial solution to accommodate the remaining customers. The routes associated with this extra vehicle tend to be emptied during the local search, leading to a feasible solution. Once this happens, this additional vehicle is eliminated from the solution.

Unlimited fleet In this case, it is always possible to generate feasible solutions with respect to the vehicle capacity because there is no limit on the number of vehicles of each type. Once a complete initial solution is generated, a vehicle associated with each type is added to the solution so as to allow for a possible fleet resizing during the local search.

Multiple depots The same rationale as in the previous cases is used when multiple-depots are considered, but an extra vehicle is added for each depot.

Backhauls As specified by the problem, infeasible solutions are avoided by not allowing backhaul customers to be inserted before linehaul customers in the route. The insertion of backhaul customers in empty routes is also forbidden. This is done via a simple modification

of the distance matrix, by setting a large cost to those arcs which connect the depot to one backhaul customer, or a backhaul customer to a linehaul customer.

Site-dependencies Customers are only allowed to be inserted in compatible routes. The cheapest insertion criterion also takes into account vehicle restrictions of the customers by including an insertion incentive that is inversely proportional to the number of vehicle types that a customer can be visited by. Moreover, a similar policy as the one used for the fixed fleet case is adopted when feasible insertions are no longer possible. In this case, the extra vehicle does not consider site-dependency constraints.

Split deliveries Splits are not allowed during the construction phase.

Time windows Time-window constraints are ignored during the construction phase.

4.2 Local search

The local search is performed by a procedure based on Randomized Variable Neighborhood Descent (RVND). Inter-route neighborhood classes are explored in random order as in a VND (Hansen et al. 2010). Intra-route neighborhoods are further applied to those routes that have been modified by one of the inter-route neighborhoods.

4.2.1 Inter-route neighborhood structures

The method relies on several inter-route neighborhoods, described in the following. As indicated below, a few neighborhood structures are only applied for some specific attributes.

General neighborhoods A set of seven inter-route neighborhood structures were adopted for all variants, namely: (i) SHIFT(1,0); (ii) SHIFT(2,0); (iii) SWAP(1,1); (iv) SWAP(2,1); (v) SWAP(2,2); (vi) 2-OPT*; and (vii) k -SHIFT. The first two consists of transferring one and two customers, respectively, from one route to another one. Neighborhoods (iii), (iv) and (v) consists of interchanging customers between two routes. For example, SWAP(2,1) interchanges two consecutive customers from one route with one customer from another one. Neighborhood (vi) is an inter-route version of the classical 2-OPT neighborhood. Finally, neighborhood (vii) consists of moving k consecutive customers from one route to the end of another one.

Multi-depot neighborhoods In the presence of multiple depots, two additional neighborhoods are considered: SHIFTDEPOT and SWAPDEPOT. The first one transfers an entire route from one depot to another one, whereas the second interchanges routes between two different depots.

Split-delivery neighborhoods When customer demands are allowed to be split, four additional neighborhoods are included in the RVND, namely: (i) SWAP(1,1)*; (ii) SWAP(2,1)*; (iii) ROUTEADDITION; and (iv) k -SPLIT. The first two were proposed by Boudia et al. (2007), and they generalize the neighborhoods SWAP(1,1) and SWAP(2,1) by changing the quantities to be delivered to the customers that are moved. The third one was proposed by Dror and Trudeau (1990). It consists in adding an extra route by combining the segments of two routes that contain a customer that is visited more than once. The last neighborhood removes a customer from the solution and inserts it back using a procedure called SPLITREINSERTION (Boudia et al. 2007). A detailed description of these four neighborhoods can be found in Silva et al. (2015).

All neighborhoods are examined in an exhaustive fashion, that is, all possible moves are evaluated and the best improving move is applied.

4.2.2 Intra-route neighborhood structures

Five well-known intra-route neighborhood structures were adopted, namely: REINSERTION, OR-OPT (with two and three adjacent customers), 2-OPT and EXCHANGE. Such neighborhoods were also implemented using a RVND procedure, and they are called every time a solution is modified during the intra-route search.

4.3 Perturbation mechanisms

Five perturbation mechanisms were adopted. They may generate infeasible solutions, but only with respect to time window constraints. Three of them are used in all variants, namely, MULTIPLE-SWAP(1,1), MULTIPLE-SHIFT(1,1) and SPLIT. The first two consists of applying random SWAP(1,1) and SHIFT(1,1) moves consecutively, respectively. Note that the SHIFT(1,1) operator moves one customer from route R_1 to route R_2 and vice-versa, but not necessarily interchanging their positions as in SWAP(1,1). The third perturbation divides a route into smaller ones as described in Penna et al. (2013).

The fourth mechanism is only applied when the deliveries are allowed to be split. Such mechanism, called MULTIPLE-K-SPLIT, was proposed by Silva et al. (2015) and consists in applying the K-SPLIT operator several times at random. The last MERGE operator, introduced in this work, selects a route whose capacity is smaller than the largest one and attempts to merge it with another route that is selected using a criterion similar to classical savings heuristics of Clarke and Wright (1964).

4.4 Efficient move evaluations

One important component of any local search based heuristic is the ability to efficiently perform move evaluations. This evaluation can be done in $\mathcal{O}(1)$ time for most VRPs, e.g., when the objective function is to minimize the total distance (or travel time) and also when penalties due to constraint violations are not incurred, that is, only feasible solutions are considered during the search. In our case, since we accept and penalize infeasible solutions with respect to time windows (Vidal et al. 2013b, 2015), a more sophisticated approach must be adopted to compute the cost of moves in constant time.

To evaluate moves in the presence of time-window infeasible solutions, we apply the methodology of Vidal et al. (2013b). This technique has been designed for a specific relaxation where one is penalized for a *return in time* (time warp) rather than for a late arrival (Nagata et al. 2010). Any classical move (e.g., SWAP or SHIFT) produces a pair of routes which result from the concatenation of a bounded number of sequences of consecutive visits from the incumbent solution. By preprocessing and updating some meaningful information on sequences of the incumbent solution, it is possible to speed-up the subsequent evaluation of the moves. In our context, any sequence σ will be characterized by six values: distance $DIST(\sigma)$, demand $Q(\sigma)$, duration $D(\sigma)$, earliest visit $E(\sigma)$, latest visit $L(\sigma)$, and return in time $TW(\sigma)$. For a sequence σ_0 containing only one customer i , $DIST(\sigma_0) = Q(\sigma_0) = TW(\sigma_0) = 0$, $D(\sigma_0) = s_i$, $E(\sigma_0) = a_i$, and $L(\sigma_0) = b_i$. Let $\sigma_1 = (\sigma_{1(i)}, \dots, \sigma_{1(j)})$ and $\sigma_2 = (\sigma_{2(i)}, \dots, \sigma_{2(j)})$ be two sequences, then the sequence corresponding to the concatenation of $\sigma_1 \oplus \sigma_2$ (where the first customer of σ_2 is visited immediately after the last customer of σ_1) can be evaluated as:

$$DIST(\sigma_1 \oplus \sigma_2) = DIST(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + DIST(\sigma_2) \tag{1}$$

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2) \tag{2}$$

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2) + \Delta_{WT} \tag{3}$$

$$E(\sigma_1 \oplus \sigma_2) = \max\{E(\sigma_2) - \Delta, E(\sigma_1)\} - \Delta_{TW} \tag{4}$$

$$L(\sigma_1 \oplus \sigma_2) = \min\{L(\sigma_2) - \Delta, L(\sigma_1)\} + \Delta_{TW} \tag{5}$$

$$TW(\sigma_1 \oplus \sigma_2) = TW(\sigma_1) + TW(\sigma_2) + \Delta_{TW}, \tag{6}$$

with $\Delta = D(\sigma_1) - TW(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)}$ (7)

$$\Delta_{WT} = \max\{E(\sigma_2) - \Delta - L(\sigma_1), 0\} \tag{8}$$

$$\Delta_{TW} = \max\{E(\sigma_1) + \Delta - L(\sigma_2), 0\}. \tag{9}$$

By using Eqs. (3)–(9) one can obtain in $\mathcal{O}(1)$ time the penalized cost of a new route σ issued from a move and assigned to a vehicle k , which is formally expressed in Eq. (10).

$$F(\sigma, k) = f_k + r_k \times DIST(\sigma) + \omega \times TW(\sigma) \tag{10}$$

The term $f_k + r_k \times DIST(\sigma)$ corresponds to the total distance of the route σ when using the vehicle of type k . The term $\omega \times TW(\sigma)$ computes the penalty due to time windows violations existing in route σ , where ω is parameter that controls the level of intensity of such penalty. Note that if all customers are served within their time windows, then the total $TW(\sigma)$ violation equals zero. It is finally worth mentioning that the partial loads on sequences are used during the search, as in Penna et al. (2013), to filter subsets of moves that are known in advance to be infeasible with respect to the vehicle capacity.

4.5 Compound neighborhood structures

In most methods from the literature, the number of feasible moves is limited by the fleet composition, i.e., the current vehicle type associated with each route. In an attempt to perform more systematic fleet optimization along with route improvements, we designed a generalized version of our local search, that we will reference as *combined neighborhood search* (CNS). In the proposed approach, the same inter-route neighborhoods (SWAP, SHIFT, 2- OPT* etc...) are generalized with a combined optimization of route-to-vehicle assignments during the move evaluations. This means that a generalized SWAP move, for example, is evaluated by jointly swapping the visits **and** determining the best assignment of vehicle types to each route in the newly created solution. Besides this change, the overall local search scheme and exploration strategy remains the same as described in Sect. 4.2.

To optimize fleet-assignment decisions we tested two approaches. The first uses an exact method based on the Primal–Dual Algorithm to solve the Assignment Problem (AP) and finds the optimal fleet composition according to the neighborhood tested. Although very slow, this procedure should be viewed as a benchmark, to evaluate what can be gained by means of optimal assignment decisions with each move. The second approach uses a simple reassignment heuristic, considering only the vehicles that are still unemployed.

Primal–dual algorithm For a given solution, the optimal fleet composition and assignment to routes can be found by solving an Assignment Problem (AP) expressed in Eqs. (11–14). Let \mathcal{R} be the set of routes and \mathcal{P} be the set of available vehicles. The model relies on binary decision variables x_{ij} , which take value 1 if and only if route i is associated to vehicle j . For each route i , let q_i be the load and d_i the distance associated to the route. For each vehicle k , let Q_k be the capacity, F_k the fixed cost and U_k the cost per distance unit. The cost of an

assignment of a vehicle $j \in \mathcal{P}$ to a route $i \in \mathcal{R}$ is given by c_{ij} , where $c_{ij} = F_j + U_j \times d_i$ if $q_i \leq Q_j$, otherwise $c_{ij} = \infty$.

$$\text{Min } \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{P}} c_{ij} x_{ij} \quad (11)$$

subject to

$$\sum_{j \in \mathcal{P}} x_{ij} = 1 \quad \forall i \in \mathcal{R} \quad (12)$$

$$\sum_{i \in \mathcal{R}} x_{ij} = 1 \quad \forall j \in \mathcal{P} \quad (13)$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in \mathcal{R}, \forall j \in \mathcal{P}. \quad (14)$$

The objective function (11) minimizes the sum of the costs by choosing the best assignment of routes to vehicles. Constraints (12) state that a single route from the set \mathcal{R} is associated to only one vehicle $j \in \mathcal{P}$. Constraints (13) requires that a single vehicle from the set \mathcal{P} is assigned to only one route $i \in \mathcal{R}$. Constraints (14) define the domain of the decision variables. Note that AP requires $|\mathcal{R}| = |\mathcal{P}|$, if $|\mathcal{R}| < |\mathcal{P}|$ some dummy routes are created and assigned to vehicles with null costs. This AP is solved in $\mathcal{O}(|\mathcal{R}|^3)$ operations using the primal–dual algorithm (PDA) of McGinnis (1983).

Simple fleet reassignment The previously-described PDA is exact but computationally expensive. Thus, we developed an alternative heuristic, called Simple Fleet Reassignment (SFR), which takes into account only the vehicles that are still available (i.e., not assigned to a route) and the routes involved in the move. First, a list with all available vehicles LV is created. Next, for routes r_1 and r_2 associated with the move, sequentially, the method finds in LV the vehicle type that best fits, i.e., the vehicle that meets the demands of the customers of the route with the least associated cost. If a better vehicle is found for these routes, the vehicle type assigned to routes r_1 and r_2 is updated and the solution cost is returned.

5 Computational experiments

HILS-RVRP was coded in C++ (g++ 4.8.2) and executed in an Intel Core i7 Processor 2.93 GHz with 8 GB of RAM running Ubuntu Linux 14.04 (Kernel 3.10–64 bits). A single thread was used for all tests. The SP models were solved using CPLEX 12.5.1. The proposed algorithm was tested on well-known benchmark instances, available in the literature for each variant considered.

The values of most of the parameters are the same as in Subramanian et al. (2012), that is, $I_{MS} = 30$, $T_{max} = 30$ seconds and $RGap_{max} = 0.02$. The maximum number of iterations per ILS was set to $I_{ILS} = n + 5 \times v$ (n and v being the number of customers and vehicles, respectively) as in Penna et al. (2013). Infeasibility can occur when an extra vehicle is used while building an initial solution for the fixed fleet variants. In this case, the extra vehicle has the following penalty values: $f_{|M|+1} = 10 \times f_{|M|}$, $r_{|M|+1} = 100 \times r_{|M|}$ and $Q_{|M|} = \sum_{i=1}^n q_i$ (w.l.o.g., the set of vehicle types M is in increasing order of costs and the vehicle type $|M|$ is the one with the largest cost). When the fleet is non-correlated (see Sect. 5.2), we assume that $|M|$ is the vehicle type associated with the largest fixed cost (or with the largest variable cost in case there are no fixed costs in the problem).

Table 2 Percentage of feasible runs for each value of ω on time windows variants

ω	FSMTW (duration)	FSMTW (distance)	HFFVRPMBTW	SDepVRPTW	Average
1	21.30	17.97	66.50	40.00	36.44
10	59.86	39.28	91.00	74.00	66.04
100	77.25	61.30	97.50	93.00	82.26
1000	100.00	100.00	100.00	100.00	100.00

Since [Subramanian et al. \(2012\)](#) and [Penna et al. \(2013\)](#) did not consider HFVRPs with time windows or infeasible solutions, the value of the time-window penalty ω , in Eq. (10), could not be inherited from these works. Therefore, we conducted a series of experiments with different values of ω on 168 challenging instances of the FSMTW (138), SDepVRPTW (10) and HFFVRPMBTW (20) to calibrate the value of ω . We ran our algorithm ten times for each instance and for $\omega \in \{1, 10, 100, 1000\}$. The idea behind this experiment is to determine a value of ω for which all runs yield feasible solutions. Table 2 reports the results of the experiments. The value ω is the only one to lead to feasible solutions on all test instances, this value has thus been used in our computational experiments.

The next set of computational experiments aims to compare the proposed algorithm with the current state-of-the-art methods on a wide range of VRP variants with heterogeneous fleet. The benchmark instances used in those tests are described in Table 3. In this table, **Authors** and **Acr.** indicate the name and the acronym of the authors who proposed the instances, respectively. **#Inst.** represents the number of instances of the dataset. **Variante** denotes the name of the VRP variant, while n is the number of customers and m the number of vehicles types. **Costs** corresponds to the types of costs considered, that is, fixed, variable, or both fixed and variable. **Fleet** represents the fleet size, i.e., limited (L) or unlimited (U), while **Depots** is the number of depots and **RD** indicates if the instances of the referred benchmark impose route duration constraints (Y = yes and N = no). Letters F, V and FV in problem names indicate whether fixed costs, variable costs or both are tackled.

The algorithm was executed 10 times on each instance with different random seeds, and a summary of the results is presented in Table 4. Detailed results, for each instance, are provided in Appendix. A comparison was performed with the best known algorithms reported in the literature.

5.1 Comparison with state-of-the-art methods for HFRVRPs

Table 4 describes the results obtained by HILS-RVRP. In this table, **Variante** denotes the HFVRP variant name, **Bench.** denotes the benchmark set name, n is the number of customers, **Authors** represents the authors of state-of-the-art methods reported in the literature, **Best Gap** indicates the average gap between the best solution found by HILS-RVRP and the best known solution, **Avg. Gap** corresponds to the gap between the average solution found by HILS-RVRP and the best known solution, **#Sol. found** represents the number of best known solutions found, **Time** indicates the running in seconds on each CPU model, scaled for our 2.93 GHz PC using the performance factors listed in [Dongarr \(2010\)](#). The best algorithms, according to the **Best Gap**, are highlighted in boldface.

The algorithm was tested on 13 HFRVRP variants using 12 well-known benchmark datasets with up to 1008 customers. The table is divided into 3 groups of results. The first

Table 3 List of benchmark instances

Authors	Acq.	#Inst.	Variant	n	m	Costs	Fleet	Depots	RD
Golden et al. (1984)	G84	12	FSMVRP-F HFFVRPSD	[20–100]	[3–6]	F	U	1	N
Taillard (1999)	T99	12	FSMVRP-V FSMVRP-FV HFFVRP-V HFFVRP-FV HFFOVRP	[20–100]	[3–6]	V/FV	L/U	1	N
Brandão (2011)	B11	5	FSMVRP-V HFFVRP-V	[100–199]	[4–6]	V	L/U	1	N
Li et al. (2007)	LGW07	5	HFFVRP-V	[200–360]	[5–6]	V	L	1	N
Duhamel et al. (2011)	DLP11	96	HFFVRP-FV	[20–256]	[2–8]	FV	L	1	N
Salhi and Sari (1997)	SS97	23	MDFSMVRP	[50–360]	5	FV	U	[2–9]	N/Y
Tütüncü (2010)	T10	18	HFFVRPB	[50–100]	[3–5]	V	L	1	N
Salhi et al. (2013)	SNM13	36	FSMVRPB	[20–100]	[3–6]	F	U	1	N
Cordeau and Laporte (2001)	CL01	35	SDepVRP SDepVRPTW	[27–1008] [48–1008]	[2–6]	–	L	1	N/Y
Liu and Shen (1999)	LS99a	168	FSMVRPTW	100	[3–6]	F	U	1	N
Liu and Shen (1999)	LS99b	168	FSMVRPTW	100	[3–6]	F	U	1	N
Belmecheri et al. (2013)	BPYA13	56	HFFVRPMBTW	100	5	V	L	1	N

group describes the results on a classical version of the HFFVRP. The second group presents the results for HFRVRP variants that involves heterogeneous vehicles and another attribute, while the third group displays the results for more complex HFRVRP variants, adding time windows attributes to previously described HFRVRP versions.

For classical HFVRP variants (group 1), a total of 106 benchmark instances were considered, and they were divided into three different datasets (LGW07, B11, DLP11) containing 5, 5 and 96 instances respectively. HILS-RVRP was capable of obtaining 68 new solutions and equaling 26 of them, meaning that the proposed algorithm found or improved the BKS in 88.7% of the instances. The gap between the best solution found by HILS-RVRP and the BKS varied from -0.14 to -0.02% . Note that LGW07 contains larger instances than B11, involving up to 360 customers, and HILS-RVRP was capable of obtaining three new best solutions for this dataset. The DLP11 instances, that are based on real distances from French cities, the HILS-RVND found 61 new solutions.

The second group includes six variants, each with a different set of instances, leading to a total 125 test-problems involving up to 1008 customers. From Table 4, we observe that the proposed algorithm always found an average gap lower or equal than 0.76%, except for the SDepVRP variant, for which HILS-RVRP obtained a value of 1.44%. One possible reason for not achieving improved solutions for this latter problem is that we did not implement any particular neighborhood structure or perturbation mechanism for this case. Furthermore, several improved solutions were found for problems HHHFOVRP, HFFVRPB and FSMVRPB. Regarding the CPU times, HILS-RVRP appears to be in many cases faster than other algorithms from the literature, such as those of [Salhi et al. \(2013\)](#) and [Yousefikhoshbakhth et al. \(2014\)](#).

Finally, the third group considers four variants with time-window constraints. HILS-RVRP has been tested on three benchmark datasets, also considering two cases for the FSMVRPTW: either minimizing the sum of the durations, or the total travel distance. Overall, we considered a total of 392 instances involving up to 288 customers. The proposed heuristic found average gaps always smaller than 1% and many improved solutions were obtained, even for problem FSMVRPTW, which is the most well-studied variant after the classical ones. Variant HFFVRPMBTW is the one that combines the most attributes, and an average improvement of 20% was achieved, thus suggesting that HILS-RVRP is robust enough to cope with problems that consider several attributes at once. Moreover, the proposed algorithm appears to be competitive in terms of CPU time, except for problem SDepVRPTW, where HILS-RVRP was slower than the state-of-the-art method of [Vidal et al. \(2014\)](#).

5.2 Sensitivity analysis: combined neighborhoods

The impact of the CNS was investigated on two instance sets of the classical HFVRP. The first set, including T99 and B11 benchmark problems, involves correlated vehicle costs and capacities, i.e., if vehicle types are considered in ascending order of capacities, the fixed costs and variable costs also increase (Fig. 1a). This situation is the rule when one considers vehicle with the same age and technology. The second set, named DLP11, is based on road distances between major cities in different districts of France. The fleet composition is uncorrelated in most problems, see Fig. 1b, for instance the larger fixed cost of a hybrid vehicle is compensated by a smaller operating cost. Figure 1 shows the route cost per distance for each vehicle type for one instance of each set. In Fig. 1b, a route with a customer demand of 100 can be associated to vehicle type B, C or D. If the distance is smaller than 20 it is better use vehicle C. Otherwise, if the route distance is greater than 50, then vehicle D leads to smaller costs. This threshold effect does not happen when fleet capacity and costs are correlated.

Table 4 Summary of the results

Variant	Bench.	n	State-of-the-art methods		Best gap	Avg. gap	Time (s)	#Sol. found	CPU (GHz)
			Authors	Authors					
HFFVRP-V	B11	[100–199]	Brandão (2011)	Brandão (2011)	0.35	–	–	5/5	P4 2.6
			Subramanian et al. (2012)	Subramanian et al. (2012)	0.00	0.39	53.59	5/5	I7 2.93
			HILS-RVRP	HILS-RVRP	-0.05	0.20	42.25	5/5	I7 2.93
HFFVRP-V	LGW07	[200–360]	Brandão (2011)	Brandão (2011)	0.09	–	1246.28	2/5	P4 2.6
			Subramanian et al. (2012)	Subramanian et al. (2012)	0.92	2.15	302.77	1/5	I7 2.93
			HILS-RVRP	HILS-RVRP	-0.02	1.42	549.41	4/5	I7 2.93
HFFVRP-FV	DLP11	[20–256]	Duhamel et al. (2011)	Duhamel et al. (2011)	0.86	–	468.57	7/96	P 2.2
			HILS-RVRP	HILS-RVRP	-0.14	0.26	203.84	85/96	I7 2.93
HFFOVRP	T99	[50–100]	Yousefikhoshbakht et al. (2014)	Yousefikhoshbakht et al. (2014)	0.00	–	116.72	1/8	P4 3.0
			HILS-RVRP	HILS-RVRP	-6.24	-6.15	6.31	7/8	I7 2.93
MDFSMVRP	SS97	[50–360]	Salhi et al. (2013)	Salhi et al. (2013)	1.52	–	247.30	12/23	P4-M
			Vidal et al. (2014)	Vidal et al. (2014)	0.05	–	715.07	21/23	Xe 2.93
			HILS-RVRP	HILS-RVRP	0.06	0.76	91.43	17/23	I7 2.93
HFFVRPB	T10	[50–100]	Tütüncü (2010)	Tütüncü (2010)	0.00	–	–	16/16	P4 2.66
			HILS-RVRP	HILS-RVRP	-10.41	-9.76	3.26	11/11	I7 2.93
FSMVRPB	SNM13	[50–100]	Salhi et al. (2013)	Salhi et al. (2013)	1.24	–	668.66	21/36	P4 3.0
			HILS-RVRP	HILS-RVRP	-1.42	-0.77	5.06	33/36	I7 2.93
SDepVRP	CL01	[27–1008]	Pisinger and Røpke (2007)	Pisinger and Røpke (2007)	0.24	–	186.82	19/35	P4 3
			Cordeau and Maischberger (2012)	Cordeau and Maischberger (2012)	0.04	–	–	33/35	X7 2.93
			HILS-RVRP	HILS-RVRP	0.31	1.44	451.51	22/35	I7 2.93
HFFVRPSD	G84	[20–100]	Ozfirat and Ozkarahan (2010)	Ozfirat and Ozkarahan (2010)	0.00	–	259.42	12/12	P4 3
			HILS-RVRP^a	HILS-RVRP^a	-1.80	-1.53	278.44	12/12	I7 2.93

Table 4 continued

Variant	Bench.	n	State-of-the-art methods					CPU (GHz)
			Authors	Best gap	Avg. gap	Time (s)	#Sol. found	
FSMVRPTW	LS99	[100]	Vidal et al. (2014)	0.24	–	304.90	120/168	Opt 2.2
			Koç et al. (2015)	0.36	–	326.51	29/168	Xe 2.6
FSMVRPTW	Min. duration LS99	[100]	HILS-RVRP	0.26	0.70	122.02	91/168	I7 2.93
			Vidal et al. (2014)	0.10	–	282.88	124/168	Opt 2.2
HFFVRPMBTW	Min. distance BPYA13	[100]	Koç et al. (2015)	0.07	–	266.42	118/168	Xe 2.6
			HILS-RVRP	0.10	0.39	142.19	110/168	I7 2.93
SDepVRPTW	CL01	[48–288]	Belmecheri et al. (2013)	6.96	–	–	15/56	–
			Berghida and Boukra (2015)	2.86	–	–	23/56	i7 2.20
			HILS-RVRP	–20.02	–19.37	103.46	55/56	I7 2.93
			Cordeau and Maischberger (2012)	0.56	–	–	6/20	X7 2.93
			Vidal et al. (2014)	0.10	–	328.95	11/20	Xe 2.93
			HILS-RVRP	0.34	0.97	1424.57	8/20	I7 2.93

^a HILS-RVRP without the SP module

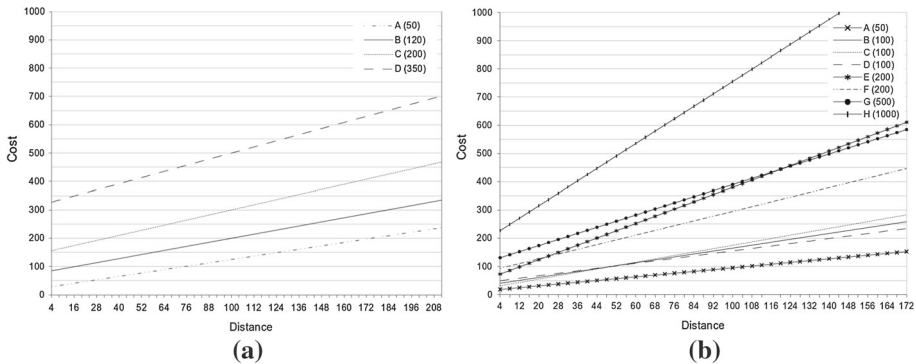


Fig. 1 Correlated and uncorrelated instances. **a** Instance #17 (T99). **b** Instance #06 (DLP11)

We implemented three versions of the algorithm to study the impact of the CNS:

- **HILS-RVRP**: The HILS-RVRP without the CNS;
- **HILS-RVRP-SFR**: The HILS-RVRP with CNS based on the Simple Fleet Reassignment procedure;
- **HILS-RVRP-PDA**: The HILS-RVRP with CNS based on the Primal–Dual procedure.

These three method variants have been tested with and without the MERGE ($P^{(4)}$) neighborhood, leading to overall six versions of the proposed algorithm. Each version was executed 10 times for each instance and the number of multi-start iterations of the HILS-RVRP parameter was set to 100. The results are presented in Tables 5 and 6. In these two tables, **Gap** denotes the gap between the average solution, on 10 runs, found by each version of the algorithm and the best known solution of the literature. **Time** corresponds to the average time, in seconds, of these runs. The best average gap for each version is highlighted in boldface.

Table 5 displays the results on the 8 HFFVRP-D and HFFVRP-FD benchmark instances T99, and the HFFVRP-D instances B11. All versions achieved a very similar performance in terms of average gap, but the version HILS-RVRP-PDA without the MERGE perturbation slightly outperformed the others. In this set of instances, the use of the MERGE procedure also generated slightly worse solutions. This can be related to the structure of the instances, since this perturbation drives the search towards fewer routes, associated to vehicles with larger capacities, and consequently higher costs. Still, these differences of performance remain very limited. In terms of the average CPU time, using the MERGE leads to a time reduction of 3.42%, while the CNS displays larger CPU times due to the resolution of vehicle-to-route assignment problems.

Table 6 describes the results on the more realistic HFFVRP-FD benchmark instances of Duhamel et al. (2011). This set contains 96 instances, ranging from 20 to 256 customers, and with 3 to 8 types of vehicles. This set of instances is divided into four subsets, a “small” subset containing 15 instances with less than 100 customers, 38 instances with 100 to 150 customers, 31 instances with 150 to 200 customers, and finally 12 instances and with more than 200 customers. Due the CPU time requirements of the CNS with PDA, it was only possible to test it on the “small” instance subset. On this benchmark, HILS-RVRP-SFR with the MERGE perturbation slightly outperforms the other versions in terms of average solution gap, but at the expense of a higher CPU time. Several new best known solutions have also been generated during these tests for DLP11 benchmark instances. These solutions are presented

Table 5 Comparative results for all versions of the algorithm on correlated instances

Problem type	n	$P^{(1)} + P^{(2)} + P^{(3)}$				$P^{(1)} + P^{(2)} + P^{(3)} + P^{(4)}$							
		MS – ILS		MS – ILS + SFR		MS – ILS + PDA		MS – ILS		MS – ILS + SFR		MS – ILS + PDA	
		Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)
HFFVRP-FD ^a	50–100	0.37	30.54	0.35	32.02	0.34	1639.91	0.34	27.36	0.36	39.51	0.39	1448.92
HFFVRP-D ^a	50–100	0.30	30.19	0.29	30.96	0.28	1900.57	0.30	26.37	0.29	38.06	0.31	1621.06
HFFVRP-D ^b	100–199	0.21	271.40	0.21	384.50	0.21	50,779.58	0.21	267.04	0.21	389.86	0.32	51,331.29
Average		0.29	110.71	0.28	149.16	0.27	18,106.69	0.28	106.92	0.29	155.81	0.34	18,133.76

^a T99

^b B11

Table 6 Comparative results for all versions of the algorithm on uncorrelated Instances (Duhamel et al. 2011)

Problem type	n	$P^{(1)} + P^{(2)} + P^{(3)}$		$P^{(1)} + P^{(2)} + P^{(3)} + P^{(4)}$									
		MS – ILS	MS – ILS + SFR	MS – ILS + PDA	MS – ILS	MS – ILS + SFR	MS – ILS + PDA						
		Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)		
HFFVRP-FD	20–95	0.18	30.52	0.58	40.95	0.57	23,579.44	0.19	29.22	0.17	42.37	0.18	23,064.81
HFFVRP-FD	102–147	0.58	164.04	0.98	255.80	–	–	0.55	169.19	0.53	252.49	–	–
HFFVRP-FD	152–196	0.65	542.13	0.84	820.84	–	–	0.67	538.01	0.64	772.70	–	–
HFFVRP-FD	203–256	0.25	1352.96	0.95	2197.56	–	–	0.21	1244.18	0.19	1935.74	–	–
Average		0.42	522.41	0.84	828.79	–	–	0.41	495.15	0.38	750.83	–	–

in the Appendix, and a comparison is established with the best-known solutions obtained by different versions of the GRASP \times ELS of [Duhamel et al. \(2011, 2013\)](#).

The conclusion of this experiment is quite counterintuitive. In many previous works on the HFVRP, we commonly assumed that the inherent difficulty of HFVRPs comes from more complex vehicle assignment decisions. However, our attempts to optimize more systematically this decision set, in combination with changes of sequences, did not lead to large solution improvements as they drove the search towards shorter routes. In this context, the CPU effort may be better distributed on a more intensive search on classical neighborhoods, rather than on an extensive search for alternative fleet assignment choices. For this reason, we kept the simpler version of the algorithm, without combined neighborhoods, for the complete tests on all HFVRP variants, and we hope that future research can help to point out efficient strategies for joint assignment and sequencing optimization.

6 Conclusions and perspectives

In this work we proposed an ILS + SP based heuristic algorithm capable of addressing a broad class of VRPs with heterogeneous fleet. We considered variants with asymmetric costs, simultaneous pickup and delivery operations, backhauls, multiple depots, site dependency, split deliveries, and time windows. Computational experiments were carried out in hundreds of instances involving up to 1008 customers from more than 10 benchmark datasets. We compared the results found by HILS-RVRP with those obtained by state-of-the-art heuristics in 12 different variants that include at least one of the aforementioned attributes. The proposed heuristic was capable of equaling or improving 71.70% of the best known solutions and to achieve, in almost all cases, an average gap between the average solutions and the best known solutions below 1%. The results suggest that HILS-RVRP is a robust and flexible heuristic algorithm that, despite its generality, is still quite competitive with problem-specific algorithm both in terms of solution quality and CPU time.

We finally investigated a larger neighborhood which considers joint changes in the sequences and a systematic optimization of route-to-vehicle assignment decisions. Even if this neighborhood opened the way to a larger class of possible solution refinements, we also observed that it tended to drive the search towards shorter routes and missed high-quality solutions built with longer routes. For future works, the interaction between assignment and sequencing decisions still deserve attention, and other neighborhood and guidance techniques, which aim to better integrate the two decision classes while circumventing the previously-mentioned issues, should be more thoroughly investigated.

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A Appendix: Detailed results

A.1 HFFVRP-V

Detailed results obtained for the HFFVRP-V instances of: (i) [\(Brandão 2011, B11\)](#), compared with the TSA of [\(Brandão 2011, B11\)](#) and the ILS-RVND-SP of [\(Subramanian et al. 2012,](#)

SPUO12) (Table 7); and (ii) (Li et al. 2007, LGW07), compared with (Brandão 2011, B11) and (Subramanian et al. 2012, SPUO12) (Table 8).

A.2 HFFVRP-FV

Detailed results obtained for the HFFVRP instances of (Duhamel et al. 2011, DLP11), compared with the sequential version of the GRASP \times ELS of (Duhamel et al. 2013, DLP13). Column BKS in Tables 9–12 indicates the best results considering all versions of the GRASP \times ELS (sequential or parallel). The instances were divided into four sets:

- DLP11–set 1: [20–95] customers (Table 9);
- DLP11–set 2: [102–147] customers (Table 10);
- DLP11–set 3: [152–196] customers (Table 11);
- DLP11–set 4: [203–256] customers (Table 12).

A.3 HFFOVRP

Detailed results obtained for the instances of (Taillard 1999, T99) as considered in (Yousefikhoshbakht et al. 2014, YDR14). The results obtained by HILS-RVRP were compared with those found by the BRMTS heuristic of the referred authors. Moreover, although Yousefikhoshbakht et al. (2014) mentioned that they used fixed and variant vehicle costs, it appears, according to our testing, that they only used variable costs. Table 13 presents the results involving only variable costs, while Table 14 reports the results involving both fixed and variable costs.

A.4 Results for the MDFSM

Detailed results obtained for the MDFSM instances of (Salhi and Sari 1997, SS97), compared with those found by the VNS2 of (Salhi et al. 2014, SIW14) and the HGSADC of (Vidal et al. 2014, VCGP14) (Table 15). Column *RD* in Table 15 indicates the instances with route duration.

A.5 HFFVRPB

Detailed results obtained for the HFFVRPB instances of (Tütüncü 2010, T10), compared with those found by the GRAMPS and ADVISER heuristics from the referred authors (Table 16 describes the results found). Note that we did not report the results for some instances because, according to the values suggested by the authors, they are infeasible.

A.6 FSMB

Detailed results obtained for the FSMB instances of (Salhi et al. 2013, SWH13), compared with those found by *Framework-2* from the same authors (Table 17).

A.7 SDepVRP

Detailed results obtained for the SDepVRP instances of (Cordeau and Laporte 2001, CL01), compared with those found by the ALNS 50k of (Pisinger and Røpke 2007, PR07) and the ITS of (Cordeau and Maischberger 2012, CM12) (Table 18, old instances without route duration and Table 19, new instances with route duration).

Table 7 Results for the HFFVRP-V (Brandão 2011, B11)

Inst.	n	BKS	TSA B11		ILS-RVND-SP SPUO12		HILS-RVRP		Sol. ^a	Time ^a (s)	Gap ^a (%)
			Best Sol.	Time (s)	Best Sol.	Time ^a (s)	Best Sol.	Gap (%)			
N1	150	2235.87	2243.76	-	2235.87	51.50	2234.13	2241.91	39.10	-0.43	-0.08
N2	199	2864.83	2874.13	-	2864.83	102.77	2859.82	2881.54	102.25	-0.50	0.26
N3	120	2378.99	2386.90	-	2378.99	51.71	2378.99	2381.97	21.89	-0.33	-0.21
N4	100	1839.22	1839.22	-	1839.22	9.64	1839.22	1839.22	10.24	0.00	0.00
N5	134	2047.81	2062.48	-	2047.81	52.33	2047.81	2047.81	37.78	-0.71	-0.71
Average			0.35	-	0.00	53.59			42.25	-0.05	0.20

^a Average of 10 runs

Table 8 Results for the HFFVRP-V (Li et al. 2007, LGW07)

Inst.	n	TSA B11		ILS-RVND-SP SPU012		HILS-RVVRP				
		BKS	Time (s)	Best Sol.	Time ^b (s)	Best Sol.	Gap (%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
H1	200	12,050.08	1395	12,050.08	72.10	12,050.08	0.00	12,061.45	91.38	0.09
H2	240	10,208.32 ^a	3650	10,226.17	176.43	10,329.15	-0.01	10,349.17	282.37	1.38
H3	280	16,223.39 ^a	2822	16,230.21	259.61	16,282.41	-0.18	16,317.62	364.01	0.58
H4	320	17,458.65	8734	17,458.65	384.52	17,743.68	0.18	17,842.46	741.72	2.20
H5	360	23,166.56 ^a	13,321	23,220.72	621.17	23,493.87	-0.06	23,822.40	1267.57	2.83
Average				0.09	1246.28	0.92	-0.02		549.41	1.42

^a Obtained by TSA with a different parameter tuning^b Average of 10 runs

Table 9 Results for HFFVRP-FV (Duhamel et al. 2011, DLP11-set 1)

Inst.	n	BKS	GRASP × ELS DLP13		HILLS-RVRP		Gap (%)	Sol. ^a	Time ^a (s)	Gap ^a (%)
			Best Sol.	Time (s)	Best Sol.	Sol. ^a				
HVRP_01_DLP	92	9210.14	9210.14	52.29	9210.14	9213.71	0.00	9213.71	44.49	0.04
HVRP_08_DLP	84	4591.75	4598.49	304.85	4591.75	4595.65	0.00	4595.65	12.00	0.08
HVRP_10_DLP	69	2107.55	2107.55	24.83	2107.55	2107.83	0.00	2107.83	12.06	0.01
HVRP_11_DLP	95	3367.41	3370.47	264.61	3367.41	3373.77	0.00	3373.77	22.10	0.19
HVRP_36_DLP	85	5684.61	5759.34	104.39	5684.61	5700.14	0.00	5700.14	29.75	0.27
HVRP_39_DLP	77	2923.72	2934.55	182.11	2921.40	2932.75	-0.08	2932.75	19.51	0.31
HVRP_43_DLP	86	8737.02	8764.75	219.91	8737.02	8746.38	0.00	8746.38	75.33	0.11
HVRP_52_DLP	59	4027.27	4029.42	39.97	4027.27	4030.44	0.00	4030.44	16.08	0.08
HVRP_55_DLP	56	10,244.34	10,247.86	190.76	10,244.34	10,250.98	0.00	10,250.98	13.23	0.06
HVRP_70_DLP	78	6685.24	6689.61	120.60	6684.56	6694.43	-0.01	6694.43	15.67	0.14
HVRP_75_DLP	20	452.85	452.85	0.02	452.85	452.85	0.00	452.85	1.15	0.00
HVRP_82_DLP	79	4766.74	4774.26	144.51	4766.74	4771.33	0.00	4771.33	36.22	0.10
HVRP_92_DLP	35	564.39	564.39	20.63	564.39	564.65	0.00	564.65	4.52	0.05
HVRP_93_DLP	39	1036.99	1036.99	27.39	1036.99	1038.34	0.00	1038.34	6.98	0.13
HVRP_94_DLP	46	1378.25	1378.66	15.68	1378.25	1378.25	0.00	1378.25	31.37	0.00
Average			0.17	114.17			-0.01		19.17	0.10

^a Average of 10 runs

Table 10 Results for HFFVRP-FV (Duhamel et al. 2011, DLP11-set 2)

Inst.	<i>n</i>	BKS		GRASP × ELS DLP13		HILS-RVRP				
		Best Sol.	Time (s)	Best Sol.	Time (s)	Best Sol.	Gap (%)	Sol. ^a	Time ^a (s)	Gap ^a (%)
HVRP_03_DLP	124	10,738.28	512.1	11,320.58	512.1	10,709.66	-0.27	10,727.52	91.32	-0.10
HVRP_05_DLP	116	10,903.63	488.63	10,963.62	488.63	10,876.48	-0.25	10,897.93	26.59	-0.05
HVRP_06_DLP	121	11,692.85	367.91	11,792.94	367.91	11,688.64	-0.04	11,734.52	43.64	0.36
HVRP_07_DLP	108	8095.88	306.09	8130.50	306.09	8089.46	-0.08	8144.80	27.47	0.60
HVRP_12_DLP	112	3543.99	71.46	3543.99	71.46	3543.99	0.00	3547.92	57.60	0.11
HVRP_13_DLP	119	6696.43	303.37	6713.14	303.37	6696.43	0.00	6703.23	50.97	0.10
HVRP_16_DLP	129	4156.97	180.91	4161.61	180.91	4156.97	0.00	4164.03	68.70	0.17
HVRP_17_DLP	105	5362.83	172.82	5370.05	172.82	5362.83	0.00	5367.76	42.09	0.09
HVRP_2A_DLP	113	7793.16	298.92	7885.93	298.92	7793.16	0.00	7796.54	43.38	0.04
HVRP_2B_DLP	107	8464.69	303.14	8537.31	303.14	8462.56	-0.03	8499.95	54.40	0.42
HVRP_21_DLP	126	5141.49	330.23	5154.38	330.23	5139.84	-0.03	5166.11	47.20	0.48
HVRP_25_DLP	143	7206.64	518.28	7228.54	518.28	7209.29	0.04	7230.50	123.79	0.33
HVRP_26_DLP	126	6446.31	350.71	6481.93	350.71	6433.21	-0.20	6461.05	149.06	0.23
HVRP_28_DLP	141	5531.06	343.06	5542.76	343.06	5530.55	-0.01	5542.80	101.82	0.21
HVRP_30_DLP	112	6313.39	201.39	6321.69	201.39	6315.70	0.04	6342.42	82.91	0.46
HVRP_31_DLP	131	4091.52	308.39	4103.88	308.39	4091.52	0.00	4112.64	102.25	0.52
HVRP_34_DLP	136	5758.089	405.62	5800.12	405.62	5747.25	-0.19	5785.59	65.57	0.48
HVRP_40_DLP	132	11,123.56	614.92	11,172.98	614.92	11,118.57	-0.04	11,171.17	90.62	0.43
HVRP_41_DLP	135	7616.17	325.8	7679.32	325.8	7597.27	-0.25	7672.27	68.18	0.74
HVRP_47_DLP	111	16,206.14	333.85	16,222.94	333.85	16,156.12	-0.31	16,247.77	41.18	0.26
HVRP_48_DLP	111	21,318.04	371.3	21,413.92	371.3	21,309.94	-0.04	21,391.58	45.75	0.34
HVRP_51_DLP	129	7721.47	315.6	7780.88	315.6	7721.47	0.00	7787.85	58.47	0.86

Table 10 continued

Inst.	n	BKS	GRASP × ELS DLPI3		HILS-RVRP		Sol. ^a	Time ^a (s)	Gap ^a (%)
			Best Sol.	Time (s)	Best Sol.	Gap (%)			
HVRP_53_DLP	115	6434.83	6470.49	418.17	6434.83	6454.77	36.09	0.00	0.31
HVRP_60_DLP	137	17,037.39	17,067.85	444.32	17,036.59	17,055.35	73.38	0.00	0.11
HVRP_61_DLP	111	7295.67	7300.10	108.21	7292.03	7302.40	37.38	-0.05	0.09
HVRP_66_DLP	150	12,830.82	13,319.73	442.89	12,783.94	12,922.52	113.74	-0.37	0.71
HVRP_68_DLP	125	8976.53	9135.23	269.63	8970.63	9123.03	67.86	-0.07	1.63
HVRP_73_DLP	137	10,195.33	10,243.66	598.34	10,195.33	10,195.36	73.57	0.00	0.00
HVRP_74_DLP	125	11,586.87	11,732.54	246.66	11,586.58	11,591.23	82.46	0.00	0.04
HVRP_79_DLP	147	7259.54	7314.89	473.69	7259.54	7289.26	122.29	0.00	0.41
HVRP_81_DLP	106	10,700.47	10,715.28	83.71	10,686.31	10,700.27	58.03	-0.13	0.00
HVRP_83_DLP	124	10,019.15	10,019.83	332.47	10,020.07	10,048.17	72.48	0.01	0.29
HVRP_84_DLP	105	7227.88	7269.55	206.41	7227.88	7237.93	54.37	0.00	0.14
HVRP_85_DLP	146	8779.76	8874.31	382.98	8773.08	8818.55	91.21	-0.08	0.44
HVRP_87_DLP	108	3753.87	3753.87	104.11	3753.87	3756.97	31.41	0.00	0.08
HVRP_88_DLP	127	12,402.85	12,443.41	632.22	12,388.23	12,405.80	46.52	-0.12	0.02
HVRP_89_DLP	134	7106.84	7135.36	245.63	7086.36	7102.98	76.51	-0.29	-0.05
HVRP_90_DLP	102	2346.13	2360.83	15.36	2346.43	2356.31	47.78	0.01	0.43
Average			0.71	327.09			67.59	-0.07	0.31

^a Average of 10 runs

Table 11 Results for HFFVRP-FV (Duhamel et al. 2011, DLP11-set 3)

Inst.	n	BKS		GRASP × ELS DLP13		HILS-RVRP				
		Best Sol.	Time (s)	Best Sol.	Time (s)	Best Sol.	Gap (%)	Sol. ^a	Time ^a (s)	Gap ^a (%)
HVRP_02_DLP	181	11,678.44	689.81	11,678.44	689.81	11,675.26	-0.03	11,695.78	187.92	0.15
HVRP_04_DLP	183	10,808.31	667.11	11,030.42	667.11	10,748.17	-0.56	10,775.93	171.66	-0.30
HVRP_09_DLP	167	7619.19	319.39	7654.45	319.39	7603.38	-0.21	7630.55	232.55	0.15
HVRP_14_DLP	176	5644.92	361.72	5676.98	361.72	5657.62	0.22	5697.17	357.30	0.93
HVRP_15_DLP	188	8236.4	905.21	8367.71	905.21	8220.64	-0.19	8285.79	158.34	0.60
HVRP_24_DLP	163	9101.47	443.1	9186.30	443.1	9119.92	0.20	9189.22	163.77	0.96
HVRP_29_DLP	164	9143.69	122.02	9176.51	122.02	9142.86	-0.01	9149.12	232.18	0.06
HVRP_33_DLP	189	9421.01	606.39	9563.18	606.39	9470.99	-0.11	9471.26	344.96	0.53
HVRP_35_DLP	168	9574.71	811.07	9817.94	811.07	9555.92	-0.20	9585.91	144.06	0.12
HVRP_37_DLP	161	6858.23	571.37	6963.61	571.37	6850.77	-0.11	6875.28	245.33	0.25
HVRP_42_DLP	178	10,902.84	966.84	11,118.66	966.84	10,817.90	-0.78	10,995.75	246.34	0.85
HVRP_44_DLP	172	12,197.46	744.39	12,351.49	744.39	12,191.48	-0.05	12,314.24	159.25	0.96
HVRP_45_DLP	170	10,484.23	415.02	10,546.69	415.02	10,476.25	-0.08	10,614.48	147.07	1.24
HVRP_50_DLP	187	12,374.04	365.46	12,538.63	365.46	12,370.94	-0.03	12,430.18	374.43	0.45
HVRP_54_DLP	172	10,393.23	565.12	10,426.98	565.12	10,351.97	-0.40	10,435.58	203.19	0.41
HVRP_56_DLP	153	31,090.71	339.08	31,292.64	339.08	31,030.19	-0.19	31,144.98	260.09	0.17
HVRP_57_DLP	163	44,818.18	471.94	45,112.39	471.94	44,781.64	-0.08	44,899.36	250.64	0.18
HVRP_59_DLP	193	14,282.59	476.61	14,367.47	476.61	14,304.46	0.15	14,357.81	312.23	0.53
HVRP_63_DLP	174	19,951.76	253.1	20,513.10	253.1	20,022.94	0.36	20,281.49	213.10	1.89
HVRP_64_DLP	161	17,157.37	70.38	17,157.37	70.38	17,135.16	-0.13	17,157.79	106.02	0.00
HVRP_67_DLP	172	10,937.67	506.65	11,090.66	506.65	10,884.91	-0.48	10,945.00	275.36	0.07

Table 11 continued

Inst.	n	BKS	GRASP × ELS DLPI3		HILS-RVRP		Sol. ^a	Time ^a (s)	Gap ^a (%)
			Best Sol.	Time (s)	Best Sol.	Gap (%)			
HVRP_69_DLP	152	9162.78	9241.75	205.32	9147.54	9190.46	117.84	0.30	
HVRP_71_DLP	186	9870.22	9936.35	389.13	9834.40	9915.73	108.20	0.46	
HVRP_72_DLP	186	5905.58	5948.99	458.28	5903.81	5949.29	225.19	0.74	
HVRP_76_DLP	152	12,018.26	12,086.57	426.51	11,994.40	12,040.78	138.37	0.19	
HVRP_77_DLP	190	6930.44	7004.97	278.69	6916.01	6974.86	271.78	0.64	
HVRP_78_DLP	190	7035.01	7066.17	439.7	7053.62	7122.27	524.00	1.24	
HVRP_80_DLP	171	6816.89	6864.75	410.38	6819.71	6843.88	211.74	0.40	
HVRP_86_DLP	153	9030.68	9085.66	440.02	9027.84	9048.94	252.05	0.20	
HVRP_91_DLP	196	6377.48	6419.23	672.65	6374.27	6403.29	423.12	0.40	
HVRP_95_DLP	183	6181.6	6237.61	206.09	6175.62	6232.75	554.497	0.83	
Average			0.98	470.92			245.57	0.50	

^a Average of 10 runs

Table 12 Results for HFFVRP-FV (Duhamel et al. 2011, DLPI1-set 4)

Inst.	<i>n</i>	BKS		GRASP × ELS DLPI3		HILS-RVRP				
		Best Sol.	Time (s)	Best Sol.	Time (s)	Best Sol.	Gap (%)	Sol. ^a	Time ^a (s)	Gap ^a (%)
HVRP_18_DLP	256	9702.75	1216.10	9797.61	1216.10	9652.74	-0.52	9688.96	885.35	-0.14
HVRP_19_DLP	224	11,702.77	1009.87	11,805.34	1009.87	11,686.39	-0.14	11,745.69	274.81	0.37
HVRP_22_DLP	239	13,068.03	835.87	13,162.90	835.87	13,091.16	0.18	13,134.19	765.90	0.51
HVRP_23_DLP	203	7750.27	802.30	7809.20	802.30	7741.01	-0.12	7782.68	383.48	0.44
HVRP_27_DLP	220	8469.19	995.85	8520.74	995.85	8422.92	-0.55	8442.97	372.15	-0.32
HVRP_32_DLP	244	9417.62	1131.44	9537.48	1131.44	9382.60	-0.37	9436.70	511.89	0.20
HVRP_38_DLP	205	11,242.95	421.50	11,439.58	421.50	11,194.68	-0.43	11,254.27	531.13	0.10
HVRP_46_DLP	250	24,674.26	1475.05	24,805.27	1475.05	24,566.23	-0.44	24,698.60	495.57	0.10
HVRP_49_DLP	246	16,377.69	990.34	16,417.30	990.34	16,181.17	-1.20	16,322.51	650.74	-0.34
HVRP_58_DLP	220	23,397.76	1028.25	23,530.10	1028.25	23,370.42	-0.12	23,641.18	294.93	1.04
HVRP_62_DLP	225	23,149.61	828.76	23,434.56	828.76	23,070.35	-0.60	23,097.54	342.94	-0.22
HVRP_65_DLP	223	13,053.8	635.64	13,077.63	635.64	13,043.54	-0.08	13,063.89	288.39	0.08
Average				0.81	947.58		-0.37		483.11	0.15

^a Average of 10 runs

Table 13 Results for the HFFOVRP-V

Inst.	n	BKS	BRMTS YDR14		HILS-RVRP				
			Best Sol.	Time (s)	Best Sol.	Gap (%)	Sol. ^a	Time ^a (s)	Gap ^a (%)
13	50	981.32	981.32	45.63	<i>914.12</i>	-6.85	<i>914.12</i>	2.56	-6.85
14	50	448.25	448.25	38.72	<i>436.32</i>	-2.66	<i>436.32</i>	1.82	-2.66
15	50	703.69	703.69	50.93	<i>681.46</i>	-3.16	<i>681.71</i>	2.01	-3.12
16	50	788.12	788.12	60.34	<i>769.46</i>	-2.37	<i>770.37</i>	2.26	-2.25
17	75	815.05	815.05	102.61	<i>762.64</i>	-6.43	<i>763.60</i>	5.63	-6.31
18	75	1596.45	1596.45	159.54	<i>1297.92</i>	-18.70	<i>1299.87</i>	6.73	-18.58
19	100	956.62	956.62	208.84	<i>851.94</i>	-10.94	<i>853.89</i>	14.48	-10.74
20	100	1031.94	1031.94	267.14	1044.55	1.22	1045.69	15.02	1.33
Average			0.00	116.72		-6.24		6.31	-6.15

^a Average of 10 runs

Table 14 Results for the HFFOVRP-FV

Inst.	n	BKS	HILS-RVRP				
			Best Sol.	Gap (%)	Sol. ^a	Time ^a (s)	Gap ^a (%)
13	50	2588.65	<i>2588.65</i>	0.00	2589.09	4.26	0.02
14	50	9961.81	<i>9961.81</i>	0.00	9968.21	3.74	0.06
15	50	2731.46	<i>2731.46</i>	0.00	2731.78	30.39	0.01
16	50	2929.78	<i>2929.78</i>	0.00	2962.68	29.06	1.12
17	75	1792.20	<i>1792.20</i>	0.00	1796.87	14.39	0.26
18	75	3228.14	<i>3228.14</i>	0.00	3236.72	31.99	0.27
19	100	10,179.70	<i>10,179.70</i>	0.00	10,188.56	44.94	0.09
20	100	4344.55	<i>4344.55</i>	0.00	4351.39	42.39	0.16
Average				0.00		25.15	0.25

^a Average of 10 runs

Table 15 Results for the MDFSM

Inst.	<i>n</i>	<i>t</i>	<i>m</i>	<i>RD</i>	BKS		VNS2 SIW14		HGSADC VCGP14		HILS-RVRP			
					Best Sol.	Time (s) ^b	Best Sol.	Time (s) ^b	Best Sol.	Time (s) ^b	Best Sol.	Gap (%)	Sol. ^a	Time ^a (s)
p01	50	5	4	-	1477.73	1499.30	12.0	1477.73	112.2	1477.73	0.00	1494.76	4.31	1.15
p02	50	5	4	-	957.73	984.50	12.0	957.73	146.4	957.73	0.00	966.81	4.12	0.95
p03	75	5	5	-	1569.67	1588.40	30.0	1569.67	202.2	1569.67	0.00	1585.29	9.82	0.99
p04	100	5	2	-	2292.64	2313.70	84.0	2292.64	278.4	2292.64	0.00	2313.42	25.41	0.91
p05	100	5	2	-	1453.64	1466.90	108.0	1453.64	445.2	1453.64	0.00	1471.05	16.65	1.20
p06	100	5	3	-	2208.66	2246.90	60.0	2208.66	320.4	2208.66	0.00	2229.31	25.29	0.93
p07	100	5	4	-	2198.91	2256.40	66.0	2198.91	311.4	2198.91	0.00	2227.96	26.61	1.32
p08	249	5	2	310	6441.36	6996.00	822.0	6441.36	1200.0	6448.93	0.12	6525.14	165.05	1.30
p09	249	5	3	311	5998.70	6068.80	438.0	5998.70	1200.0	6011.85	0.22	6074.65	171.76	1.27
p10	249	5	4	312	5807.53	6043.00	372.0	5807.53	1200.0	5826.61	0.33	5868.29	173.56	1.05
p11	249	5	5	313	5770.42	5882.80	384.0	5770.42	1184.4	5779.64	0.16	5843.27	181.71	1.26
p12	80	5	2	-	2072.18	2076.20	72.0	2072.18	216.0	2072.18	0.00	2075.22	6.96	0.15
p13	80	5	2	200	2096.39	2096.40	66.0	2096.39	213.6	2096.39	0.00	2096.63	3.48	0.01
p14	80	5	2	180	2139.30	2139.30	66.0	2160.12	250.8	2160.12	0.97	2169.66	2.89	1.42
p15	160	5	4	-	3973.47	4024.90	162.0	3973.47	576.6	3973.47	0.00	3991.43	51.51	0.45
p16	160	5	4	200	4119.76	4148.70	162.0	4119.76	595.8	4119.76	0.00	4128.48	23.31	0.21
p17	160	5	4	180	4309.09	4338.20	198.0	4309.09	837.0	4309.09	0.00	4327.05	20.12	0.42
p18	240	5	6	-	5887.43	5970.50	288.0	5887.43	1188.0	5887.43	0.00	5917.18	166.19	0.51
p19	240	5	6	200	6130.36	6196.30	306.0	6130.36	1168.2	6130.36	0.00	6164.98	66.08	0.56
p20	240	5	6	180	6469.21	6567.10	318.0	6469.21	1200.0	6458.07	-0.17	6486.35	56.60	0.26

Table 15 continued

Inst.	<i>n</i>	<i>t</i>	<i>m</i>	<i>RD</i>	BKS	VNS2 SIW14		HGSADC VCGP14		HILS-RVRP				
						Best Sol.	Time (s) ^b	Best Sol.	Time (s) ^b	Best Sol.	Gap (%)	Sol. ^a	Time ^a (s)	Gap ^a (%)
p21	360	5	9	–	8709.26	8883.10	654.0	8709.26	1200.0	8709.26	0.00	8738.45	513.06	0.34
p22	360	5	9	200	9151.64	9294.80	468.0	9151.64	1200.0	9151.91	0.00	9209.14	210.11	0.63
p23	360	5	9	180	9706.60	9887.70	540.0	9714.41	1200.0	<i>9678.75</i>	–0.29	9732.10	178.21	0.26
Average						1.52	247.30	0.05	715.07		0.06		91.43	0.76

^a Average of 10 runs

^b Original time in minutes, here converted to seconds

Table 16 Results for the HFFVRPB

Inst.	n	LH	BH	BKS	GRAMPS		ADVISER		HILS-RVRP				
					Best Sol.	Time (s)	Best Sol.	Time (s)	Best Sol.	Gap (%)	Sol. ^a	Time ^a (s)	Gap ^a (%)
1	50	25	25	1056.44	1111.67	-	1056.44	-	874.60	-17.21	874.76	0.89	-17.20
2	50	34	16	982.86	1067.28	-	982.86	-	911.20	-7.29	913.30	0.81	-7.08
3	50	40	10	998.22	1124.14	-	998.22	-	-	-	-	-	-
4	50	25	25	1070.06	1094.08	-	1070.06	-	1050.60	-1.82	1051.11	0.93	-1.77
5	50	34	16	1127.97	1135.21	-	1127.97	-	1051.30	-6.80	1052.00	0.83	-6.74
6	50	40	10	1183.36	1200.58	-	1183.36	-	-	-	-	-	-
7	75	37	38	1190.63	1190.63	-	1190.63	-	1073.90	-9.80	1096.80	3.13	-7.88
8	75	50	25	1182.66	1211.28	-	1182.66	-	-	-	-	-	-
9	75	60	15	1203.09	1222.66	-	1203.09	-	1003.20	-16.61	1013.97	2.35	-15.72
10	75	37	38	1781.50	1845.75	-	1781.50	-	1553.00	-12.83	1557.28	2.74	-12.59
11	75	50	25	1941.74	2035.39	-	1941.74	-	1659.80	-14.52	1667.85	3.19	-14.11
12	75	60	15	1917.54	1945.35	-	1917.54	-	-	-	-	-	-
13	100	50	50	1227.81	1228.24	-	1227.81	-	1181.70	-3.76	1195.00	7.98	-2.67
14	100	67	33	1109.02	1136.87	-	1109.02	-	-	-	-	-	-
15	100	80	20	1216.65	1228.56	-	1216.65	-	1114.90	-8.36	1135.00	6.27	-6.71
16	100	50	50	1555.35	1629.47	-	1555.35	-	1314.50	-15.49	1323.97	6.71	-14.88
Average					3.31		0.00					3.26	-9.76

^a Average of 10 runs

Table 17 Results for the FSMB

Inst.	<i>n</i>		BKS	Framework-2 SWH13		HILS-RVRP				
	<i>LH</i>	<i>BH</i>		Best Sol.	Time (s)	Best Sol.	Gap (%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
HWS1	20	10	720.57 ^a	726.48	0.61	720.57	0.00	720.57	0.12	0.00
HWS2	20	13	818.12 ^a	818.12	1.09	818.12	0.00	818.12	0.14	0.00
HWS3	20	16	848.23 ^a	848.59	1.64	848.32	0.01	848.32	0.10	0.01
HWS4	20	10	4342.48 ^a	4350.65	0.91	4342.48	0.00	4342.48	0.12	0.00
HWS5	20	13	5357.98 ^a	5366.39	2.75	5357.98	0.00	5361.47	0.12	0.07
HWS6	20	16	5421.65 ^a	5875.23	3.44	5421.65	0.00	5560.13	0.14	2.55
HWS7	20	10	729.50 ^a	767.93	0.58	729.50	0.00	729.50	0.14	0.00
HWS8	20	13	838.11 ^a	872.97	1.39	838.11	0.00	838.20	0.16	0.01
HWS9	20	16	890.76 ^a	903.18	2.09	890.76	0.00	890.76	0.13	0.00
HWS10	20	10	4349.12 ^a	4365.44	0.88	4349.12	0.00	4349.12	0.12	0.00
HWS11	20	13	5363.58 ^a	5414.50	2.72	5363.58	0.00	5380.38	0.15	0.31
HWS12	20	16	5497.98 ^a	5928.78	4.94	5497.98	0.00	5751.82	0.15	4.62
HWS13	50	25	1625.70	1625.70	17.88	1590.47	-2.17	1593.28	1.67	-1.99
HWS14	50	33	1811.63	1811.63	26.19	1771.53	-2.21	1778.19	1.37	-1.85
HWS15	50	40	2018.93	2018.93	38.42	1999.05	-0.98	2004.69	1.21	-0.71
HWS16	50	25	5561.67	5561.67	330.34	5551.19	-0.19	5551.34	1.53	-0.19
HWS17	50	33	6570.39	6570.39	996.55	6547.93	-0.34	6547.93	2.29	-0.34
HWS18	50	40	7599.08	7599.08	1120.50	7120.52	-6.30	7523.17	2.86	-1.00
HWS19	50	25	1704.41	1704.41	39.81	1616.21	-5.17	1627.49	1.46	-4.51
HWS20	50	33	2037.23	2037.23	84.95	2015.67	-1.06	2018.79	2.32	-0.91
HWS21	50	40	2340.09	2340.09	103.52	2295.57	-1.90	2304.64	2.67	-1.51
HWS22	50	25	1774.71	1774.71	18.41	1717.60	-3.22	1722.60	1.79	-2.94
HWS23	50	33	2166.52	2166.52	64.77	2096.10	-3.25	2127.37	1.93	-1.81
HWS24	50	40	2430.88	2430.88	49.72	2401.04	-1.23	2407.88	1.39	-0.95

Table 17 continued

Inst.	<i>n</i>	<i>LH</i>	<i>BH</i>	BKS	Framework-2 SWH13		HILS-RVRP				
					Best Sol.	Time (s)	Best Sol.	Gap (%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
HWS25	75	37	38	1332.02	1332.02	1006.28	1285.86	-3.47	1292.21	5.66	-2.99
HWS26	75	50	25	1421.04	1421.04	1779.88	1399.36	-1.53	1401.82	5.19	-1.35
HWS27	75	60	15	1534.65	1534.65	1996.59	1513.10	-1.40	1524.69	4.60	-0.65
HWS28	75	37	38	1617.85	1617.85	1351.92	1572.38	-2.81	1574.08	5.24	-2.71
HWS29	75	50	25	1799.76	1799.76	1513.30	1760.95	-2.16	1761.04	4.45	-2.15
HWS30	75	60	15	1990.46	1990.46	2662.15	1950.99	-1.98	1951.30	3.80	-1.97
HWS31	100	50	50	4943.29	5201.81	4257.41	4963.08	0.40	4966.57	12.66	0.47
HWS32	100	66	34	6035.96	-	-	5993.30	-0.71	5993.91	36.35	-0.70
HWS33	100	80	20	7601.09	-	-	7097.81	-6.62	7330.43	32.22	-3.56
HWS34	100	50	50	2465.41	2646.52	2871.74	2494.95	1.20	2522.53	17.35	2.32
HWS35	100	66	34	2971.98	2971.98	651.80	2927.20	-1.51	2931.90	15.01	-1.35
HWS36	100	80	20	3533.90	3533.90	1729.30	3450.73	-2.35	3458.24	15.40	-2.14
Average					1.24	668.66		-1.42		5.06	-0.77

^a Optimality proved^b Average of 10 runs

Table 18 Results for the SDepVRP (old set)

Inst.	n	t	BKS	ALNS 50k PR07		ITS CM12		HILS-RVRP			Time ^a (s)	Gap ^a (%)
				Best Sol.	Time (s)	Best Sol.	Time (s)	Best Sol.	Gap (%)	Sol. ^a		
p01	55	3	640.32	640.32	20	640.32	-	640.32	0.00	640.53	1.00	0.03
p02	52	2	598.10	598.10	19	598.10	-	598.10	0.00	598.10	0.93	0.00
p03	80	3	954.32	957.04	40	954.32	-	954.32	0.00	956.37	4.26	0.22
p04	76	2	854.43	854.43	36	854.43	-	854.43	0.00	855.30	3.03	0.10
p05	103	3	1003.57	1003.57	68	1003.57	-	1003.57	0.00	1008.67	11.02	0.51
p06	104	2	1028.52	1028.52	69	1028.52	-	1028.52	0.00	1036.96	7.85	0.82
p07	27	3	391.30	391.30	8	391.30	-	391.30	0.00	391.30	0.11	0.00
p08	54	3	664.46	664.46	24	664.46	-	664.46	0.00	664.46	0.69	0.00
p09	81	3	948.23	948.23	47	948.23	-	948.23	0.00	948.23	4.19	0.00
p10	108	3	1218.75	1218.75	76	1218.75	-	1218.75	0.00	1231.41	11.21	1.04
p11	135	3	1448.17	1463.33	116	1448.17	-	1448.17	0.00	1482.58	49.62	2.55
p12	162	3	1665.55	1678.40	157	1665.55	-	1682.94	1.04	1708.05	64.58	2.55
p13	54	3	1194.18	1194.18	24	1194.18	-	1194.18	0.00	1196.12	0.93	0.16
p14	108	3	1959.96	1960.62	72	1959.96	-	1959.96	0.00	1960.90	7.59	0.05
p15	162	3	2685.09	2685.09	152	2685.09	-	2685.09	0.00	2701.85	33.21	0.62
p16	216	3	3393.55	3396.36	213	3393.55	-	3393.86	0.01	3431.81	83.85	1.13
p17	270	3	4066.15	4085.61	291	4066.15	-	4078.19	0.30	4147.14	260.77	1.99
p18	324	3	4751.27	4755.50	346	4751.27	-	4768.23	0.36	4910.04	505.80	3.34
p19	104	3	843.15	846.07	85	843.15	-	843.15	0.00	848.59	8.32	0.64
p20	156	3	1030.78	1030.78	168	1030.78	-	1030.78	0.00	1044.11	24.30	1.29
p21	209	3	1263.71	1271.75	217	1263.71	-	1260.01	-0.29	1278.05	61.59	1.13
p22	122	3	1008.71	1008.71	130	1008.71	-	1008.71	0.00	1009.09	14.02	0.04
p23	102	3	803.29	803.29	73	803.29	-	803.29	0.00	805.30	5.62	0.25
Average				0.16	106.57		0.00		0.06		89.22	0.80

^a Average of 10 runs

Table 19 Results for the SDepVRP (new instances)

Inst.	n	t	BKS	ALNS 50k PR07		ITS CMI2		HILS-VRP				
				Best Sol.	Time (s)	Best Sol.	Time (s)	Best Sol.	Gap (%)	Sol. ^a	Time ^a (s)	Gap ^a (%)
pr01	48	4	1380.77	1380.77	19	1380.77	–	1380.77	0.00	1408.22	0.62	1.99
pr02	96	4	2303.89	2311.54	63	2303.89	–	2303.89	0.00	2335.95	5.32	1.39
pr03	144	4	2575.36	2602.13	140	2575.36	–	2578.93	0.14	2598.95	16.16	0.92
pr04	192	4	3449.84	3474.01	191	3449.84	–	3454.85	0.15	3524.97	44.90	2.18
pr05	240	4	4377.35	4416.38	251	4377.35	–	4379.41	0.05	4481.48	77.01	2.38
pr06	288	4	4422.02	4444.52	314	4422.02	–	4451.34	0.66	4531.44	141.38	2.47
pr07	72	6	1889.82	1889.82	39	1889.82	–	1889.82	0.00	1935.09	2.53	2.40
pr08	144	6	2971.01	2977.50	135	2971.01	–	2973.26	0.08	3057.03	17.43	2.90
pr09	216	6	3536.20	3536.20	226	3536.20	–	3559.69	0.66	3624.11	64.06	2.49
pr10	288	6	4639.62	4648.76	322	4639.62	–	4663.38	0.51	4743.69	160.19	2.24
pr11	1008	4	12,719.65	12,719.65	847	12,845.60	–	13,091.91	2.93	13,305.68	6775.12	4.61
pr12	720	6	9388.07	9388.07	658	9392.84	–	9612.09	2.39	9665.67	2872.05	2.96
Average				267.08	0.32	0.09			0.63		848.16	2.39

^a Average of 10 runs

Table 20 Results for the HFFVRPSD

Inst.	n	BKS	CP OO10		HILS-RVRP		HFFVRPSD		
			Best Sol.	Time ^a (s)	Best Sol.	Gap (%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
3	20	970.53	970.53	104	943.12	-2.82	943.12	11.03	-2.82
4	20	6421.88	6421.88	45	6399.20	-0.35	6400.01	5.20	-0.34
5	20	998.74	998.74	84	970.77	-2.80	970.77	10.11	-2.80
6	20	6514.09	6514.09	137	6497.44	-0.26	6582.88	4.54	1.06
13	50	2440.78	2440.78	146	2374.03	-2.73	2376.20	198.26	-2.65
14	50	9138.25	9138.25	412	9114.07	-0.26	9203.83	90.81	0.72
15	50	2616.11	2616.11	298	2568.96	-1.80	2569.86	106.94	-1.77
16	50	2719.89	2719.89	302	2690.18	-1.09	2691.51	98.02	-1.04
17	75	1783.33	1783.33	317	1725.40	-3.25	1726.62	385.42	-3.18
18	75	2394.16	2394.16	486	2333.26	-2.54	2336.85	781.51	-2.39
19	100	8722.49	8722.49	506	8654.31	-0.78	8655.85	845.61	-0.76
20	100	4130.49	4130.49	396	4011.30	-2.89	4033.08	803.84	-2.36
Average			0.00	269.42		-1.80		278.44	-1.53

^a Time for one run (Pentium 4 3.0 GHz)

^b Average of 30 runs

A.8 HFFVRPSD

Detailed results obtained for the FSM instances of (Golden et al. 1984, G84) and adapted for the HFFVRPSD by (Ozfirat and Ozkarahan 2010, OO10), compared with those found by the CP heuristic of the same authors (Table 20). As HFFVRPSD allows visiting the customers more than once, the SP procedure was not considered for this variant.

A.9 FSMVRPTW: minimize route duration

Detailed results obtained for the FSMVRPTW instances of (Liu and Shen 1999, LS99), considering the objective of minimizing the sum of the route durations, compared with those found by the UHGS of (Vidal et al. 2014, VCGP14) and the HEA of (Koç et al. 2015, KBJL15) (Tables 21, 22, 23).

Table 21 Results for the FSMTW (minimize duration, fleet A)

Inst.	n	VCGP14		KBIL15		HILS-RVRP				
		Best Sol.	Time(s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap (%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
R101a	100	4536.40	361.8	4541.70	315.6	4608.62	1.59	4616.92	62.56	1.77
R102a	100	4348.92	382.8	4355.10	352.2	4368.70	0.45	4380.77	76.74	0.73
R103a	100	4119.04	279.0	4131.23	251.4	4144.96	0.63	4152.64	77.66	0.82
R104a	100	3961.39	313.8	3992.10	301.2	3962.87	0.04	3971.32	71.20	0.25
R105a	100	4209.84	306.6	4232.54	283.8	4204.87	-0.12	4222.45	76.06	0.30
R106a	100	4109.08	379.2	4138.30	307.8	4103.65	-0.13	4125.17	84.09	0.39
R107a	100	4007.87	327.0	4034.32	324.0	4003.95	-0.10	4014.98	75.44	0.18
R108a	100	3934.48	307.2	3966.10	286.8	3934.48	0.00	3946.40	69.80	0.30
R109a	100	4020.75	303.6	4059.02	276.0	4022.90	0.05	4034.21	78.46	0.33
R110a	100	3965.88	318.0	3996.31	250.2	3963.80	-0.05	3973.92	75.62	0.20
R111a	100	3985.68	363.0	4020.10	298.8	3984.92	-0.02	3999.81	83.57	0.35
R112a	100	3918.88	406.2	3957.60	346.8	3918.88	0.00	3927.38	68.16	0.22
C101a	100	7226.51	191.4	7226.51	178.2	7226.51	0.00	7231.05	83.95	0.06
C102a	100	7119.35	169.2	7145.65	186.0	7119.35	0.00	7119.35	90.42	0.00
C103a	100	7102.86	147.6	7143.88	162.0	7102.86	0.00	7103.02	86.13	0.00
C104a	100	7081.50	133.2	7082.92	120.6	7081.51	0.00	7081.51	73.36	0.00
C105a	100	7175.00	204.0	7175.00	147.0	7196.06	0.29	7196.39	98.76	0.30
C106a	100	7163.32	220.2	7163.32	180.6	7176.68	0.19	7180.15	98.72	0.23
C107a	100	7140.20	191.4	7140.20	166.8	7144.49	0.06	7144.53	97.32	0.06
C108a	100	7111.23	166.8	7120.98	147.0	7111.23	0.00	7111.23	87.48	0.00
C109a	100	7091.66	143.4	7091.66	142.2	7091.66	0.00	7091.66	82.51	0.00
RC101a	100	5217.90	301.8	5235.42	298.2	5213.66	-0.08	5227.92	61.18	0.19

Table 21 continued

Inst.	n	BKS	VCGP14		KBIL15		HILS-RVRP				
			Best Sol.	Time(s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap (%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
RC102a	100	5018.47	5018.47	342.6	5029.69	338.4	5019.99	0.03	5042.64	66.00	0.48
RC103a	100	4822.21	4822.21	361.8	4870.00	308.4	4822.21	0.00	4859.48	66.50	0.77
RC104a	100	4737.00	4737.00	246.0	4769.30	298.2	<i>4736.13</i>	-0.02	4748.10	54.96	0.23
RC105a	100	5097.35	5097.35	336.6	5118.10	319.2	<i>5096.66</i>	-0.01	5111.72	70.68	0.28
RC106a	100	4935.91	4935.91	396.0	4958.62	360.6	<i>4928.60</i>	-0.15	4946.41	65.98	0.21
RC107a	100	4783.08	4783.08	319.2	4825.21	322.2	4783.08	0.00	4803.81	61.07	0.43
RC108a	100	4708.85	4708.85	310.2	4754.77	282.6	4708.98	0.00	4723.87	51.10	0.32
R201a	100	3753.42	3782.88	459.6	3760.43	538.2	3787.55	0.91	3838.10	265.18	2.26
R202a	100	3540.03	3540.03	802.2	3554.20	598.8	3540.59	0.02	3562.54	296.59	0.64
R203a	100	3311.35	3311.35	544.2	3315.50	525.6	3315.16	0.12	3318.61	288.80	0.22
R204a	100	3075.95	3075.95	532.2	3075.95	478.8	3075.95	0.00	3080.01	209.39	0.13
R205a	100	3334.27	3334.27	555.0	3334.27	507.0	3334.27	0.00	3361.92	217.21	0.83
R206a	100	3242.40	3242.40	540.6	3263.40	490.2	3242.40	0.00	3259.19	226.03	0.52
R207a	100	3145.08	3145.08	564.0	3152.29	557.4	3145.83	0.02	3157.37	210.22	0.39
R208a	100	3017.12	3017.12	484.2	3017.12	510.6	3017.12	0.00	3021.98	181.73	0.16
R209a	100	3183.36	3183.36	569.4	3194.28	562.2	3184.41	0.03	3190.30	174.19	0.22
R210a	100	3287.66	3287.66	612.6	3309.26	527.4	3288.46	0.02	3301.07	219.59	0.41
R211a	100	3019.93	3019.93	544.8	3020.56	479.4	3019.93	0.00	3022.14	155.30	0.07
C201a	100	5820.78	5878.54	310.2	5830.20	300.0	5853.90	0.57	5892.77	271.05	1.24
C202a	100	5776.88	5776.88	309.0	5776.88	310.2	5776.88	0.00	5776.88	242.04	0.00
C203a	100	5736.94	5741.12	343.2	5741.12	285.6	5741.12	0.07	5742.71	196.11	0.10
C204a	100	5680.46	5680.46	258.6	5680.46	252.6	5680.46	0.00	5680.46	180.73	0.00
C205a	100	5747.67	5781.15	393.6	5751.40	407.4	5781.15	0.58	5789.29	254.67	0.72

Table 21 continued

Inst.	<i>n</i>	<i>BKS</i>	VCGP14		KBIL15		HILS-RVRP		Time ^b (s)	Gap ^b (%)	
			Best Sol.	Time(s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap (%)			Sol. ^b
C206a	100	5738.09	5767.70	284.4	5741.30	258.0	5767.70	0.52	5771.59	213.11	0.58
C207a	100	5721.16	5731.44	308.4	5725.10	250.2	5731.44	0.18	5732.97	203.22	0.21
C208a	100	5725.03	5725.03	271.2	5725.03	312.6	5725.03	0.00	5727.22	171.01	0.04
RC201a	100	4701.88	4737.59	316.8	4707.80	270.0	4737.59	0.76	4747.24	107.41	0.96
RC202a	100	4487.48	4487.48	268.8	4519.40	280.2	4487.48	0.00	4487.57	125.19	0.00
RC203a	100	4305.49	4305.49	352.8	4319.10	316.2	4305.49	0.00	4314.27	137.30	0.20
RC204a	100	4137.93	4137.93	400.8	4155.77	311.4	<i>4137.84</i>	0.00	4142.65	124.05	0.11
RC205a	100	4585.20	4615.04	384.0	4595.67	413.4	4615.40	0.66	4634.25	125.16	1.07
RC206a	100	4405.16	4405.16	308.4	4434.30	301.8	4405.16	0.00	4420.53	116.34	0.35
RC207a	100	4290.14	4290.14	391.2	4315.90	376.2	4290.14	0.00	4301.27	123.83	0.26
RC208a	100	4075.04	4075.04	344.4	4081.37	310.2	4075.04	0.00	4075.96	112.59	0.02
Average			0.14	351.50	0.36	326.51		0.13		131.13	0.38

^a Original time in minutes^b Average of 10 runs

Table 22 Results for the FSMTW (minimize duration, fleet B)

Inst.	n	BKS		VCGP14		KBJL15		HILS-RVRP				
		Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
R101b	100	2421.19	233.40	2486.77	233.40	2425.10	226.80	2486.77	2.71	2494.10	54.59	3.01
R102b	100	2209.50	258.60	2222.15	258.60	2212.37	238.20	2222.15	0.57	2226.88	59.11	0.79
R103b	100	1930.21	250.80	1930.21	250.80	1951.99	256.80	1930.69	0.02	1941.17	65.88	0.57
R104b	100	1688.12	260.40	1688.12	260.40	1714.86	240.60	1688.12	0.00	1710.55	65.13	1.33
R105b	100	2017.56	229.80	2017.56	229.80	2024.91	220.80	2017.56	0.00	2024.55	62.85	0.35
R106b	100	1913.84	306.00	1913.84	306.00	1922.10	251.40	<i>1913.04</i>	-0.04	1918.48	70.66	0.24
R107b	100	1774.50	256.20	1774.50	256.20	1783.20	318.00	1774.50	0.00	1785.81	66.61	0.64
R108b	100	1649.24	349.80	1654.68	349.80	1661.58	286.80	1654.78	0.34	1663.92	61.99	0.89
R109b	100	1818.15	305.40	1818.15	305.40	1829.10	294.60	1818.15	0.00	1829.18	65.23	0.61
R110b	100	1761.53	346.20	1761.53	346.20	1778.80	312.60	1764.64	0.18	1777.41	76.36	0.90
R111b	100	1751.10	334.20	1751.10	334.20	1775.24	286.80	1754.89	0.22	1773.07	77.50	1.25
R112b	100	1663.09	379.80	1663.09	379.80	1677.00	372.60	<i>1661.97</i>	-0.07	1672.61	58.08	0.57
C101b	100	2417.52	123.60	2417.52	123.60	2417.52	119.40	2417.52	0.00	2417.52	50.16	0.00
C102b	100	2350.54	178.80	2350.55	178.80	2350.54	147.00	2350.54	0.00	2350.54	70.56	0.00
C103b	100	2345.31	241.20	2345.31	241.20	2345.31	208.20	2345.31	0.00	2345.31	72.54	0.00
C104b	100	2325.78	145.80	2327.84	145.80	2330.59	185.40	2327.84	0.09	2327.98	77.14	0.09
C105b	100	2373.53	201.00	2373.53	201.00	2376.45	183.60	2373.53	0.00	2377.74	67.68	0.18
C106b	100	2381.14	190.20	2386.03	190.20	2386.43	177.00	2386.03	0.21	2390.56	66.59	0.40
C107b	100	2357.52	186.00	2364.21	186.00	2359.00	147.00	2364.21	0.28	2366.44	72.71	0.38
C108b	100	2346.38	196.80	2346.38	196.80	2348.15	167.40	2346.38	0.00	2347.66	68.35	0.05
C109b	100	2336.29	156.00	2336.29	156.00	2337.60	153.60	2336.29	0.00	2336.29	90.44	0.00
RC101b	100	2456.10	281.40	2456.10	281.40	2464.19	268.20	2456.10	0.00	2463.85	61.82	0.32

Table 22 continued

Inst.	<i>n</i>	BKS		VCGP14		KBJL15		HILS-RVRP				
		Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
RC102b	100	2259.25	254.40	2259.25	247.20	2270.43	247.20	2259.25	0.00	2267.13	61.31	0.35
RC103b	100	2025.30	284.40	2025.30	238.80	2041.20	238.80	2025.30	0.00	2033.25	63.21	0.39
RC104b	100	1901.04	262.20	1901.04	252.60	1922.27	252.60	1901.04	0.00	1923.40	61.79	1.18
RC105b	100	2308.59	285.60	2329.14	273.60	2327.70	273.60	2329.14	0.89	2332.74	60.53	1.05
RC106b	100	2146.00	220.80	2146.00	252.60	2147.14	252.60	<i>2140.94</i>	-0.24	2151.26	64.45	0.25
RC107b	100	1989.34	245.40	1989.34	251.40	1996.09	251.40	1989.34	0.00	1998.37	60.63	0.45
RC108b	100	1898.96	195.60	1898.96	186.60	1908.89	186.60	1898.96	0.00	1908.89	54.64	0.52
R201b	100	1953.42	383.40	1973.43	372.60	1956.21	372.60	1980.60	1.39	1989.18	217.75	1.83
R202b	100	1740.03	483.00	1740.03	480.00	1752.40	480.00	1743.82	0.22	1775.69	258.74	2.05
R203b	100	1511.35	386.40	1511.35	346.80	1515.17	346.80	1515.08	0.25	1518.14	254.35	0.45
R204b	100	1275.95	454.80	1275.95	413.40	1279.57	413.40	1275.95	0.00	1279.16	190.43	0.25
R205b	100	1534.27	387.00	1534.27	389.40	1549.39	389.40	1542.47	0.53	1566.91	196.58	2.13
R206b	100	1441.35	355.20	1441.35	312.60	1450.37	312.60	1448.03	0.46	1464.74	193.32	1.62
R207b	100	1345.08	418.20	1345.08	378.60	1359.18	378.60	1346.18	0.08	1362.59	191.29	1.30
R208b	100	1217.12	360.00	1217.12	328.20	1220.36	328.20	1217.12	0.00	1220.92	168.08	0.31
R209b	100	1380.79	465.00	1380.79	428.40	1385.65	428.40	1384.42	0.26	1391.29	172.35	0.76
R210b	100	1485.65	463.20	1485.65	415.80	1495.75	415.80	1492.12	0.44	1507.17	205.78	1.45
R211b	100	1219.93	441.60	1219.93	447.00	1219.93	447.00	1219.93	0.00	1226.01	152.18	0.50
C201b	100	1816.14	174.00	1820.64	186.60	1820.64	186.60	1820.64	0.25	1820.64	194.23	0.25
C202b	100	1768.51	313.20	1768.51	274.80	1770.10	274.80	1768.51	0.00	1778.44	173.71	0.56
C203b	100	1733.63	197.40	1733.63	191.40	1733.63	191.40	1733.63	0.00	1733.63	159.40	0.00
C204b	100	1680.46	198.00	1680.46	190.20	1680.46	190.20	1680.46	0.00	1680.46	164.82	0.00
C205b	100	1747.68	328.80	1778.30	312.60	1756.54	312.60	1778.30	1.75	1784.42	193.49	2.10

Table 22 continued

Inst.	n	BKS	VCGP14		KBLL15		HILS-RVRP				
			Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
C206b	100	1756.01	1767.70	230.40	1773.17	207.60	1767.70	0.67	1776.92	178.08	1.19
C207b	100	1729.39	1729.49	208.80	1729.39	178.20	1729.49	0.01	1733.41	167.76	0.23
C208b	100	1723.20	1724.20	204.00	1724.20	187.80	1724.20	0.06	1724.20	167.91	0.06
RC201b	100	2230.54	2329.59	260.40	2235.90	250.20	2331.14	4.51	2345.62	124.50	5.16
RC202b	100	2002.62	2057.66	401.40	2022.00	328.20	2057.66	2.75	2078.10	144.73	3.77
RC203b	100	1824.54	1824.54	319.80	1840.40	307.20	1824.88	0.02	1847.01	165.37	1.23
RC204b	100	1555.74	1555.75	330.00	1555.74	298.80	1555.75	0.00	1560.82	146.18	0.33
RC205b	100	2166.62	2174.74	325.80	2169.00	388.20	2174.74	0.37	2196.93	144.16	1.40
RC206b	100	1883.08	1883.08	259.80	1898.70	248.40	1883.08	0.00	1892.52	137.24	0.50
RC207b	100	1714.14	1714.14	339.00	1730.00	308.40	1714.14	0.00	1733.17	145.07	1.11
RC208b	100	1483.20	1483.20	288.00	1490.64	265.80	1483.20	0.00	1487.17	141.23	0.27
Average			0.29	288.15	0.46	271.48		0.34		118.83	0.85

^a Original time in minutes

^b Average of 10 runs

Table 23 Results for the FSMTW (minimize duration, fleet C)

Inst.	n	BKS		VCGP14		KBJL15		HILS-RVRP				
		Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
R101c	100	2134.90	202.80	2199.79	202.80	2137.20	188.40	2199.79	3.04	2200.87	54.19	3.09
R102c	100	1913.37	303.60	1925.56	303.60	1914.87	372.60	1925.56	0.64	1927.82	62.85	0.76
R103c	100	1609.94	217.20	1615.38	217.20	1621.20	194.40	1615.38	0.34	1619.49	67.40	0.59
R104c	100	1363.26	274.80	1363.26	274.80	1375.60	268.20	1363.26	0.00	1373.84	66.13	0.78
R105c	100	1722.05	216.00	1722.05	216.00	1722.05	190.20	1722.05	0.00	1724.62	62.42	0.15
R106c	100	1599.04	286.20	1599.04	286.20	1610.40	244.80	1599.04	0.00	1605.26	69.50	0.39
R107c	100	1442.97	223.20	1442.97	223.20	1454.30	210.60	1442.97	0.00	1458.92	70.03	1.11
R108c	100	1321.68	326.40	1321.68	326.40	1329.92	319.80	<i>1317.43</i>	-0.32	1333.87	62.38	0.92
R109c	100	1506.59	301.20	1506.59	301.20	1507.10	283.80	1506.59	0.00	1511.26	68.29	0.31
R110c	100	1443.92	343.80	1443.92	343.80	1451.06	327.60	<i>1443.33</i>	-0.04	1457.89	76.92	0.97
R111c	100	1423.47	419.40	1423.47	419.40	1436.32	368.40	1425.18	0.12	1444.20	71.06	1.46
R112c	100	1329.07	286.20	1329.07	286.20	1341.10	250.20	1329.07	0.00	1346.41	65.15	1.30
C101c	100	1628.31	109.80	1628.94	109.80	1628.94	118.20	1628.94	0.04	1628.94	49.98	0.04
C102c	100	1610.96	145.80	1610.96	145.80	1610.96	151.80	1610.96	0.00	1610.96	63.04	0.00
C103c	100	1607.14	166.20	1607.14	166.20	1607.14	227.40	1607.14	0.00	1607.14	61.19	0.00
C104c	100	1598.50	162.00	1599.90	162.00	1599.21	173.40	1599.90	0.09	1599.90	62.97	0.09
C105c	100	1628.38	112.80	1628.94	112.80	1628.94	118.20	1628.94	0.03	1628.94	61.15	0.03
C106c	100	1628.94	112.80	1628.94	112.80	1628.94	120.60	1628.94	0.00	1628.94	56.94	0.00
C107c	100	1628.38	117.60	1628.94	117.60	1628.94	119.40	1628.94	0.03	1628.94	59.20	0.03
C108c	100	1622.89	180.00	1622.89	180.00	1625.00	147.00	1622.89	0.00	1627.13	58.86	0.26
C109c	100	1614.99	217.20	1615.93	217.20	1618.61	212.40	1614.99	0.00	1615.95	64.59	0.06
RC101c	100	2082.95	294.60	2082.95	294.60	2092.10	272.40	2082.95	0.00	2086.57	60.68	0.17
RC102c	100	1895.05	256.80	1895.05	256.80	1901.89	251.40	1895.56	0.03	1901.70	63.18	0.35
RC103c	100	1650.30	238.80	1650.30	238.80	1660.70	213.60	1650.30	0.00	1666.02	62.18	0.95

Table 23 continued

Inst.	n	BKS	VCGP14		KBIL15		HILS-RVRP				
			Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
RC104c	100	1526.04	1526.04	214.20	1540.60	208.20	1526.04	0.00	1551.14	62.96	1.64
RC105c	100	1953.99	1957.14	282.60	1956.09	249.60	1957.14	0.16	1958.54	59.19	0.23
RC106c	100	1774.94	1774.94	228.00	1780.45	209.40	1774.94	0.00	1778.88	64.92	0.22
RC107c	100	1607.11	1607.11	229.80	1620.30	184.20	1607.11	0.00	1620.06	59.59	0.81
RC108c	100	1523.96	1523.96	202.80	1532.60	213.60	1523.96	0.00	1526.00	57.50	0.13
R201c	100	1716.02	1716.02	272.40	1731.20	406.80	1716.02	0.00	1720.65	200.64	0.27
R202c	100	1515.03	1515.03	530.40	1529.70	488.40	1519.41	0.29	1536.94	234.74	1.45
R203c	100	1286.35	1286.35	374.40	1296.72	390.00	1290.16	0.30	1297.61	266.88	0.87
R204c	100	1050.95	1050.95	457.20	1052.90	473.40	1050.95	0.00	1057.00	192.24	0.58
R205c	100	1309.27	1309.27	386.40	1315.20	402.60	1309.27	0.00	1317.49	183.91	0.63
R206c	100	1216.35	1216.35	320.40	1226.93	395.40	1223.07	0.55	1234.31	180.06	1.48
R207c	100	1120.08	1120.08	433.80	1125.50	418.80	1121.80	0.15	1137.13	201.30	1.52
R208c	100	992.12	992.12	360.60	997.97	352.20	992.12	0.00	1000.17	182.55	0.81
R209c	100	1155.79	1155.79	450.00	1164.31	428.40	1159.67	0.34	1168.88	170.29	1.13
R210c	100	1257.89	1257.89	392.40	1269.70	368.40	1264.38	0.52	1272.87	201.36	1.19
R211c	100	994.93	994.93	395.40	995.58	370.20	994.93	0.00	1000.99	156.78	0.61
C201c	100	1250.97	1269.41	171.60	1250.97	178.20	1269.41	1.47	1277.27	176.97	2.10
C202c	100	1239.54	1239.54	231.00	1240.86	212.40	1239.54	0.00	1242.29	165.61	0.22
C203c	100	1193.63	1193.63	181.80	1193.63	188.40	1193.63	0.00	1193.63	148.25	0.00
C204c	100	1176.52	1176.52	234.00	1176.52	220.20	1176.52	0.00	1176.52	161.86	0.00
C205c	100	1238.30	1238.30	261.60	1240.10	257.40	1238.30	0.00	1245.38	180.27	0.57
C206c	100	1229.23	1238.30	292.20	1229.23	262.80	1238.30	0.74	1241.36	163.15	0.99
C207c	100	1209.48	1209.49	180.00	1209.48	213.60	1209.49	0.00	1214.57	158.96	0.42
C208c	100	1204.20	1204.20	181.80	1204.20	180.60	1204.20	0.00	1204.20	158.52	0.00

Table 23 continued

Inst.	n	BKS	VCGP14		KBIL15		HILS-RVRP				
			Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
RC201c	100	1915.42	1996.79	220.20	1917.90	279.00	1996.79	4.25	2010.99	127.73	4.99
RC202c	100	1677.62	1732.66	391.80	1680.00	366.00	1732.66	3.28	1753.28	150.22	4.51
RC203c	100	1496.11	1496.11	369.00	1500.20	376.20	1499.54	0.23	1515.94	168.38	1.33
RC204c	100	1220.75	1220.75	327.00	1222.16	328.20	1220.75	0.00	1229.53	146.27	0.72
RC205c	100	1822.07	1844.74	304.20	1823.00	317.40	1844.74	1.24	1866.61	161.43	2.44
RC206c	100	1553.65	1553.65	267.00	1564.30	282.00	1558.08	0.29	1572.54	138.66	1.22
RC207c	100	1377.52	1377.52	363.60	1381.71	340.20	1379.14	0.12	1393.87	153.51	1.19
RC208c	100	1140.10	1140.10	379.20	1151.40	310.20	1140.10	0.00	1148.27	143.06	0.72
Average			0.28	275.04	0.37	271.74		0.32		116.04	0.87

^a Original time in minutes^b Average of 10 runs

A.10 FSMTW: minimize total distance

Detailed results obtained for the FSMVRPTW instances of ([Liu and Shen 1999](#), LS99), considering the objective of minimizing the sum of the total distance, compared with those found by the UHGS of ([Vidal et al. 2014](#), VCGP14) and the HEA of ([Koç et al. 2015](#), KBJL15) (Tables [24](#), [25](#), [26](#)).

Table 24 Results for the FSMTW (minimize distance, fleet A)

Inst.	n	BKS		VCGP14		KBIL15		HILS-RVRP				
		Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
R101	100	4314, 36	276.60	4314.36	4317.52	4317.52	248.40	4314.36	0.00	4325.76	120.56	0.26
R102	100	4166.28	361.80	4166.28	4173.84	4173.84	358.80	4166.28	0.00	4181.25	124.45	0.36
R103	100	4027.36	321.00	4027.36	4031.40	4031.40	312.60	4024.14	-0.08	4038.85	134.79	0.29
R104	100	3936.40	288.60	3936.40	3946.44	3946.44	247.20	3936.55	0.00	3948.00	113.83	0.29
R105	100	4122.50	389.40	4122.50	4134.06	4134.06	360.60	4122.50	0.00	4133.06	131.61	0.26
R106	100	4048.59	334.20	4048.59	4060.05	4060.05	307.20	4050.17	0.04	4059.07	127.47	0.26
R107	100	3970.51	333.60	3970.51	3985.12	3985.12	286.80	3976.40	0.15	3988.73	126.16	0.46
R108	100	3928.12	280.80	3928.12	3932.60	3932.60	392.40	3928.12	0.00	3935.45	108.28	0.19
R109	100	4015.71	288.00	4015.71	4024.83	4024.83	367.20	4015.71	0.00	4023.34	128.21	0.19
R110	100	3961.68	389.40	3961.68	3973.51	3973.51	312.60	3961.68	0.00	3965.89	114.65	0.11
R111	100	3964.99	316.80	3964.99	3988.00	3988.00	307.20	3971.90	0.17	3989.84	127.58	0.63
R112	100	3918.88	295.20	3918.88	3930.19	3930.19	282.60	3917.88	-0.03	3927.20	105.96	0.21
C101	100	7093.45	177.60	7093.45	7093.45	7093.45	148.20	7093.45	0.00	7093.59	167.56	0.00
C102	100	7080.17	128.40	7080.17	7080.17	7080.17	159.00	7080.17	0.00	7080.17	134.68	0.00
C103	100	7079.21	125.40	7079.21	7079.21	7079.21	120.60	7079.21	0.00	7079.21	119.00	0.00
C104	100	7075.06	131.40	7075.06	7075.06	7075.06	118.20	7075.06	0.00	7075.06	101.36	0.00
C105	100	7093.45	199.80	7093.45	7093.45	7093.45	159.00	7093.45	0.00	7093.60	154.47	0.00
C106	100	7083.87	136.80	7083.87	7083.87	7083.87	130.20	7083.87	0.00	7083.87	140.25	0.00
C107	100	7084.61	133.80	7084.61	7084.61	7084.61	143.40	7084.61	0.00	7084.61	142.94	0.00
C108	100	7079.66	131.40	7079.66	7079.66	7079.66	118.20	7079.66	0.00	7079.66	124.35	0.00
C109	100	7077.3	122.40	7077.30	7077.30	7077.30	131.40	7077.30	0.00	7077.30	107.98	0.00
RC101	100	5150.86	312.60	5150.86	5173.47	5173.47	308.40	5150.86	0.00	5160.03	121.21	0.18
RC102	100	4987.24	288.60	4987.24	5018.83	5018.83	255.60	4974.82	-0.25	4999.64	122.10	0.25
RC103	100	4804.61	424.80	4804.61	4850.20	4850.20	388.20	4804.61	0.00	4837.40	122.75	0.68

Table 24 continued

Inst.	n	BKS		VCGP14		KBIL15		HILS-RVRP				
		Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
RC104	100	4717.63	318.00	4717.63	318.00	4725.40	317.40	4721.44	0.08	4734.77	94.74	0.36
RC105	100	5035.35	334.20	5035.35	334.20	5048.86	286.80	5036.50	0.02	5047.72	119.73	0.25
RC106	100	4936.74	337.80	4936.74	337.80	4964.13	317.40	4921.13	-0.32	4941.27	118.48	0.09
RC107	100	4788.69	304.80	4788.69	304.80	4825.60	250.20	4787.59	-0.02	4807.65	107.49	0.40
RC108	100	4708.85	286.80	4708.85	286.80	4724.79	277.80	4711.31	0.05	4726.02	86.50	0.36
R201	100	3446.78	390.60	3446.78	390.60	3446.78	367.80	3446.78	0.00	3452.08	320.01	0.15
R202	100	3297.42	460.80	3308.16	460.80	3297.42	447.60	3308.16	0.33	3313.28	342.72	0.48
R203	100	3141.09	339.00	3141.09	339.00	3141.09	368.40	3141.09	0.00	3143.21	321.32	0.07
R204	100	3018.14	417.60	3018.14	417.60	3018.14	376.80	3018.14	0.00	3019.95	296.69	0.06
R205	100	3218.97	384.00	3218.97	384.00	3218.97	382.80	3218.97	0.00	3228.75	305.08	0.30
R206	100	3146.34	618.00	3146.34	618.00	3146.34	488.40	3147.41	0.03	3155.39	325.90	0.29
R207	100	3077.36	522.00	3077.58	522.00	3077.36	388.20	3077.58	0.01	3082.82	313.42	0.18
R208	100	2997.24	322.20	2997.24	322.20	2997.25	380.40	2997.24	0.00	2999.97	282.11	0.09
R209	100	3119.56	382.20	3122.42	382.20	3119.56	299.40	3122.42	0.09	3129.93	301.95	0.33
R210	100	3170.41	415.80	3174.85	415.80	3170.41	328.20	3174.31	0.12	3181.35	331.93	0.35
R211	100	3019.93	546.00	3019.93	546.00	3019.93	475.80	3019.93	0.00	3022.78	285.35	0.09
C201	100	5695.02	222.60	5695.02	222.60	5695.02	207.60	5695.02	0.00	5695.02	345.40	0.00
C202	100	5685.24	226.80	5685.24	226.80	5685.24	190.20	5685.24	0.00	5685.24	287.90	0.00
C203	100	5681.55	252.60	5681.55	252.60	5681.55	257.40	5681.55	0.00	5681.79	271.20	0.00
C204	100	5677.66	256.20	5677.66	256.20	5677.66	238.20	5677.66	0.00	5677.88	281.35	0.00
C205	100	5691.36	238.80	5691.36	238.80	5691.36	207.60	5691.36	0.00	5691.36	300.58	0.00
C206	100	5689.32	229.20	5689.32	229.20	5689.32	178.20	5689.32	0.00	5689.32	270.71	0.00
C207	100	5687.35	254.40	5687.35	254.40	5687.35	246.00	5687.35	0.00	5687.35	294.10	0.00
C208	100	5686.5	231.60	5686.50	231.60	5686.50	213.60	5686.50	0.00	5686.50	265.14	0.00

Table 24 continued

Inst.	n	BKS	VCGP14		KBIL15		HILS-RVRP				
			Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
RC201	100	4374.09	4374.09	355.20	4376.82	308.40	4374.09	0.00	4380.07	193.35	0.14
RC202	100	4244.63	4244.63	277.80	4244.63	255.60	4244.63	0.00	4246.90	203.69	0.05
RC203	100	4170.17	4170.17	463.80	4170.17	368.40	4170.17	0.00	4177.62	196.96	0.18
RC204	100	4087.11	4087.11	347.40	4087.11	328.20	4087.11	0.00	4094.04	171.97	0.17
RC205	100	4291.93	4291.93	327.60	4293.73	251.40	4291.93	0.00	4294.56	197.21	0.06
RC206	100	4251.88	4251.88	307.20	4251.88	256.20	4251.88	0.00	4257.78	207.92	0.14
RC207	100	4182.44	4185.98	289.20	4182.44	338.40	4185.98	0.08	4188.14	192.60	0.14
RC208	100	4075.04	4075.04	244.80	4075.04	318.60	4075.04	0.00	4077.57	166.98	0.06
Average			0.01	305.24	0.13	283.60		0.01		193.26	0.17

^a Original time in minutes^b Average of 10 runs

Table 25 Results for the FSMTW (minimize distance, fleet B)

Inst.	n	BKS		VCGP14		KBIL15		HILS-RVRP				
		Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
R101	100	2222.56	303.00	2228.67	2222.56	2222.56	256.20	2228.67	0.27	2229.99	67.06	0.33
R102	100	2048.12	215.40	2073.63	2048.12	2048.12	196.80	2071.90	1.16	2073.92	71.71	1.26
R103	100	1853.66	274.20	1853.66	1855.74	1853.66	316.20	1853.66	0.00	1857.83	73.56	0.23
R104	100	1683.33	322.20	1683.33	1686.42	1683.33	305.40	1685.49	0.13	1691.53	65.13	0.49
R105	100	1980.96	198.00	1988.86	1980.96	1980.96	202.20	1988.86	0.40	1992.22	70.02	0.57
R106	100	1888.31	278.40	1888.31	1890.28	1888.31	251.40	1888.31	0.00	1896.83	74.06	0.45
R107	100	1752.02	249.00	1753.35	1752.02	1752.02	315.60	1753.35	0.08	1765.13	73.84	0.75
R108	100	1647.88	272.40	1647.88	1649.37	1647.88	238.20	1647.88	0.00	1664.56	67.97	1.01
R109	100	1818.15	219.60	1818.15	1819.10	1818.15	239.40	1818.15	0.00	1827.03	73.66	0.49
R110	100	1758.64	306.60	1758.64	1761.96	1758.64	328.20	1762.39	0.21	1773.40	78.90	0.84
R111	100	1740.86	319.20	1740.86	1743.16	1740.86	341.40	1743.16	0.13	1761.93	81.04	1.21
R112	100	1661.85	321.60	1661.85	1663.09	1661.85	300.60	1663.09	0.07	1672.36	68.90	0.63
C101	100	2340.15	187.20	2340.15	2340.15	2340.15	178.80	2340.15	0.00	2342.43	70.09	0.10
C102	100	2325.70	156.60	2325.70	2325.70	2325.70	163.80	2325.70	0.00	2325.70	73.58	0.00
C103	100	2324.60	181.80	2324.60	2324.60	2324.60	218.40	2324.60	0.00	2324.60	80.46	0.00
C104	100	2318.04	146.40	2318.04	2318.04	2318.04	178.80	2318.04	0.00	2318.04	70.17	0.00
C105	100	2340.15	180.00	2340.15	2340.15	2340.15	162.60	2340.15	0.00	2340.28	72.24	0.01
C106	100	2340.15	207.60	2340.15	2340.15	2340.15	191.40	2340.15	0.00	2340.84	68.22	0.03
C107	100	2340.15	192.00	2340.15	2340.15	2340.15	176.40	2340.15	0.00	2340.28	70.80	0.01
C108	100	2338.58	190.80	2338.58	2338.58	2338.58	232.80	2338.58	0.00	2338.58	84.86	0.00
C109	100	2328.55	163.20	2328.55	2328.55	2328.55	187.20	2328.55	0.00	2328.55	80.69	0.00
RC101	100	2407.43	249.60	2412.71	2407.43	2407.43	207.60	2412.71	0.22	2416.42	67.53	0.37
RC102	100	2213.92	351.60	2213.92	2219.23	2213.92	308.40	2213.92	0.00	2221.66	70.19	0.35
RC103	100	2015.55	210.00	2016.28	2015.55	2015.55	221.40	2016.28	0.04	2022.06	75.49	0.32

Table 25 continued

Inst.	n	BKS		VCGP14		KBIL15		HILS-RVRP				
		Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
RC104	100	1896.40	244.20	1897.04	244.20	1896.40	274.20	1907.56	0.59	1920.20	68.47	1.26
RC105	100	2274.28	272.40	2287.51	272.40	2274.28	341.40	2287.51	0.58	2299.23	65.91	1.10
RC106	100	2132.13	237.00	2140.86	237.00	2132.13	187.20	2139.36	0.34	2143.93	68.42	0.55
RC107	100	1984.67	179.40	1989.34	179.40	1984.67	147.00	1989.34	0.24	1997.33	63.66	0.64
RC108	100	1895.97	234.60	1898.96	234.60	1895.97	160.20	1898.96	0.16	1905.85	61.43	0.52
R201	100	1646.78	357.60	1646.78	357.60	1646.78	407.40	1646.78	0.00	1658.07	178.63	0.69
R202	100	1501.81	549.00	1508.16	549.00	1501.81	433.80	1510.25	0.56	1516.94	196.30	1.01
R203	100	1341.09	225.00	1341.09	225.00	1341.09	273.60	1341.09	0.00	1346.55	183.62	0.41
R204	100	1218.14	306.60	1218.14	306.60	1218.14	246.60	1218.14	0.00	1222.38	169.45	0.35
R205	100	1418.97	436.80	1418.97	436.80	1420.81	388.20	1418.97	0.00	1429.55	176.47	0.75
R206	100	1346.34	433.80	1346.34	433.80	1347.41	419.40	1349.12	0.21	1360.02	190.02	1.02
R207	100	1277.58	376.80	1277.58	376.80	1278.57	406.80	1277.58	0.00	1282.99	179.80	0.42
R208	100	1197.24	259.80	1197.24	259.80	1198.70	328.20	1197.24	0.00	1200.17	167.17	0.24
R209	100	1322.42	369.00	1322.42	369.00	1322.42	328.20	1322.42	0.00	1330.11	182.65	0.58
R210	100	1370.41	418.20	1374.31	418.20	1370.41	355.80	1376.40	0.44	1382.57	186.79	0.89
R211	100	1219.93	407.40	1219.93	407.40	1220.57	468.60	1219.93	0.00	1222.20	169.81	0.19
C201	100	1695.02	178.20	1695.02	178.20	1695.02	126.60	1695.02	0.00	1695.02	201.24	0.00
C202	100	1685.24	169.80	1685.24	169.80	1685.24	139.80	1685.24	0.00	1685.24	165.84	0.00
C203	100	1681.55	201.60	1681.55	201.60	1681.55	154.20	1681.55	0.00	1682.06	160.47	0.03
C204	100	1677.66	208.20	1677.66	208.20	1677.66	221.40	1677.66	0.00	1678.07	177.00	0.02
C205	100	1691.36	192.60	1691.36	192.60	1691.36	184.20	1691.36	0.00	1691.36	170.75	0.00
C206	100	1689.32	179.40	1689.32	179.40	1689.32	191.40	1689.32	0.00	1689.32	165.75	0.00
C207	100	1687.35	206.40	1687.35	206.40	1687.35	225.60	1687.35	0.00	1687.35	165.04	0.00
C208	100	1686.50	194.40	1686.50	194.40	1686.50	144.60	1686.50	0.00	1686.50	150.51	0.00

Table 25 continued

Inst.	n	BKS		VCGP14		KBIL15		HILS-RVRP				
		Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap(%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
RC201	100	1938.36	356.40	1938.36	356.40	1941.16	418.80	1938.36	0.00	1943.46	131.74	0.26
RC202	100	1768.04	355.20	1772.81	355.20	1768.04	388.20	1772.49	0.25	1773.94	138.32	0.33
RC203	100	1603.55	359.40	1604.04	359.40	1603.55	369.00	1604.03	0.03	1622.55	153.21	1.18
RC204	100	1489.27	256.80	1490.25	256.80	1489.27	208.20	1490.25	0.07	1492.28	142.89	0.20
RC205	100	1832.53	306.60	1832.53	306.60	1833.34	238.80	1832.53	0.00	1836.37	139.21	0.21
RC206	100	1724.41	347.40	1725.44	347.40	1724.41	272.40	1725.44	0.06	1736.04	159.45	0.67
RC207	100	1646.37	339.00	1646.37	339.00	1650.23	300.60	1644.44	-0.12	1653.60	157.99	0.44
RC208	100	1481.74	263.40	1483.20	263.40	1481.74	244.80	1483.20	0.10	1486.01	156.95	0.29
Average			269.98	0.09	269.98	0.04	262.76		0.11		117.77	0.42

^a Original time in minutes

^b Average of 10 runs

Table 26 Results for the FSMTW (minimize distance, fleet C)

Inst.	n	BKS		VCGP14		KBJL15		HILS-RVRP				
		Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Gap (%)	Sol. ^b	Time ^b (s)	Gap ^b (%)
R101	100	1937.38	276.00	1951.20	276.00	1937.38	250.20	1951.20	0.71	1951.64	65.13	0.74
R102	100	1762.22	175.20	1785.35	175.20	1762.22	193.80	1778.29	0.91	1782.92	70.67	1.17
R103	100	1546.98	213.00	1552.34	213.00	1546.98	221.40	1550.73	0.24	1552.81	82.77	0.38
R104	100	1352.37	322.20	1355.15	322.20	1352.37	310.20	1355.15	0.21	1366.54	72.12	1.05
R105	100	1681.44	195.00	1694.56	195.00	1681.44	247.80	1694.56	0.78	1696.65	69.18	0.90
R106	100	1583.17	247.20	1583.17	247.20	1585.65	220.20	1583.17	0.00	1590.23	78.15	0.45
R107	100	1424.37	321.60	1428.08	321.60	1424.37	358.80	1428.08	0.26	1444.99	77.80	1.45
R108	100	1314.88	319.20	1314.88	319.20	1318.44	286.80	1314.88	0.00	1330.05	71.87	1.15
R109	100	1506.59	280.80	1506.59	280.80	1507.10	246.60	1506.59	0.00	1511.16	71.68	0.30
R110	100	1443.37	303.60	1443.92	303.60	1443.37	286.80	1439.42	-0.27	1452.53	86.86	0.63
R111	100	1419.43	334.80	1420.15	334.80	1419.43	308.40	1423.41	0.28	1439.63	79.32	1.42
R112	100	1327.58	298.20	1327.58	298.20	1328.01	280.20	1328.47	0.07	1346.27	71.60	1.41
C101	100	1628.94	127.20	1628.94	127.20	1628.94	119.40	1628.94	0.00	1628.94	56.82	0.00
C102	100	1597.66	141.00	1597.66	141.00	1597.66	128.40	1597.66	0.00	1597.84	60.67	0.01
C103	100	1596.56	172.80	1596.56	172.80	1596.56	159.00	1596.56	0.00	1596.56	65.06	0.00
C104	100	1590.76	139.20	1590.76	139.20	1590.76	126.60	1590.76	0.00	1590.76	62.50	0.00
C105	100	1628.94	117.60	1628.94	117.60	1628.94	144.60	1628.94	0.00	1628.94	57.33	0.00
C106	100	1628.94	123.00	1628.94	123.00	1628.94	104.40	1628.94	0.00	1628.94	55.11	0.00
C107	100	1628.94	130.20	1628.94	130.20	1628.94	121.80	1628.94	0.00	1628.94	58.33	0.00
C108	100	1622.75	192.00	1622.75	192.00	1622.75	153.60	1622.75	0.00	1628.32	62.69	0.34
C109	100	1614.99	232.80	1615.93	232.80	1614.99	178.20	1614.99	0.00	1616.05	70.56	0.07

Table 26 continued

Inst.	n	BKS		VCGP14		KBJL15		HILS-RVRP			
		Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a		
RC101	100	2033.89	263.40	2043.48	263.40	2033.89	249.60	2040.61	2046.28	63.45	0.61
RC102	100	1847.92	246.00	1847.92	246.00	1847.92	241.80	1847.92	1859.92	70.91	0.65
RC103	100	1646.35	251.40	1646.35	251.40	1646.35	250.20	1646.35	1656.22	77.59	0.60
RC104	100	1518.96	338.40	1522.04	338.40	1518.96	308.40	1522.04	1544.84	67.29	1.70
RC105	100	1884.92	240.60	1913.06	240.60	1884.92	274.20	1913.06	1926.98	64.25	2.23
RC106	100	1753.99	226.20	1770.95	226.20	1753.99	206.40	1770.95	1772.43	65.58	1.05
RC107	100	1601.12	244.80	1607.11	244.80	1601.12	208.20	1607.11	1614.00	63.87	0.80
RC108	100	1516.36	201.00	1523.96	201.00	1516.36	218.40	1523.96	1525.47	60.51	0.60
R201	100	1429.50	294.00	1443.41	294.00	1429.50	272.40	<i>1421.78</i>	1431.20	176.19	0.12
R202	100	1273.11	474.60	1283.16	474.60	1273.11	427.20	1283.86	1291.28	189.45	1.43
R203	100	1116.09	237.00	1116.09	237.00	1116.09	274.80	1116.09	1119.58	178.95	0.31
R204	100	993.14	397.80	993.14	397.80	993.14	408.60	993.14	995.54	173.31	0.24
R205	100	1193.97	439.80	1193.97	439.80	1193.97	372.60	1193.97	1207.74	174.27	1.15
R206	100	1121.34	360.00	1121.34	360.00	1121.34	308.40	1123.43	1129.96	180.35	0.77
R207	100	1052.58	418.20	1052.58	418.20	1052.58	313.80	1055.45	1061.29	171.75	0.83
R208	100	969.90	346.80	969.90	346.80	969.90	328.20	969.90	974.31	170.43	0.45
R209	100	1094.97	356.40	1097.42	356.40	1094.97	338.40	1097.42	1105.09	175.98	0.92
R210	100	1145.48	408.00	1149.85	408.00	1145.48	370.20	1153.08	1157.78	187.50	1.07
R211	100	994.93	390.60	994.93	390.60	994.93	370.20	994.93	997.74	167.85	0.28

Table 26 continued

Inst.	n	BKS		VCGP14		KBJL15		HILS-RVRP		Sol. ^b	Time ^b (s)	Gap ^b (%)
		Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a	Best Sol.	Time (s) ^a			
C201	100	1194.33	289.20	1194.33	270.00	1194.33	270.00	1194.33	201.61	1194.33	201.61	0.00
C202	100	1185.24	154.20	1185.24	141.60	1185.24	141.60	1185.24	168.50	1185.24	168.50	0.00
C203	100	1176.25	216.00	1176.25	184.20	1176.25	184.20	1176.25	161.78	1176.41	161.78	0.01
C204	100	1175.37	259.20	1175.37	185.40	1175.37	185.40	1175.37	182.77	1175.37	182.77	0.00
C205	100	1190.36	267.60	1190.36	270.00	1190.36	270.00	1190.36	176.91	1190.36	176.91	0.00
C206	100	1188.62	240.60	1188.62	239.40	1188.62	239.40	1188.62	162.28	1188.62	162.28	0.00
C207	100	1184.88	219.60	1184.88	190.20	1184.88	190.20	1184.88	159.37	1185.37	159.37	0.04
C208	100	1186.50	179.40	1186.50	172.20	1186.50	172.20	1186.50	156.34	1186.50	156.34	0.00
RC201	100	1623.36	408.60	1623.36	360.60	1625.71	360.60	1623.36	125.38	1627.62	125.38	0.26
RC202	100	1445.12	317.40	1447.27	247.20	1445.12	247.20	1447.27	140.32	1452.60	140.32	0.52
RC203	100	1273.55	269.40	1274.04	220.20	1273.55	220.20	1274.03	152.44	1291.12	152.44	1.38
RC204	100	1157.94	360.60	1159.00	308.40	1157.94	308.40	1159.00	152.38	1161.80	152.38	0.33
RC205	100	1512.53	314.40	1512.53	300.60	1515.34	300.60	1512.53	136.56	1517.15	136.56	0.31
RC206	100	1395.18	235.20	1395.18	196.20	1399.41	196.20	1400.44	161.53	1412.04	161.53	1.21
RC207	100	1314.44	374.40	1314.44	328.20	1317.50	328.20	1314.44	154.91	1324.55	154.91	0.77
RC208	100	1140.10	408.60	1140.10	359.40	1140.10	359.40	1140.10	151.26	1148.45	151.26	0.73
Average			0.19	0.19	0.03	0.03	0.03	0.03	115.54		115.54	0.59

^a Original time in minutes^b Average of 10 runs

A.11 HFFVRPMBTW

Detailed results obtained for the HFFVRPMBTW instances of ([Belmecheri et al. 2013](#), BPYA13), compared with the results obtained by the PSO of the same authors and the EBBO of ([Berghida and Boukra 2015](#), BB15) (Tables [27](#), [28](#), [29](#)).

Table 27 Results for the HFFVRPMBTW (Type C)

Inst.	n	BKS		PSO BPYA13		EBBO BB15		HILS-RVRP		Time ^a (s)	Gap ^a (%)
		Best Sol.	Time (s)	Best Sol.	Time (s)	Best Sol.	Time (s)	Best Sol.	Time (s)		
C101	100	2331.54	2560.02	2331.54	-	2331.54	-	1348.34	1348.34	79.32	-42.17
C102	100	2410.18	2615.32	2410.18	-	2410.18	-	1344.72	1344.72	77.69	-44.21
C103	100	2102.41	2405.30	2102.41	-	2102.41	-	1343.33	1343.33	72.41	-36.11
C104	100	2021.55	2333.95	2021.55	-	2021.55	-	1332.78	1333.68	78.20	-34.03
C105	100	1998.83	2055.90	1998.83	-	1998.83	-	1348.10	1348.10	80.27	-32.56
C106	100	2193.54	2366.05	2193.54	-	2193.54	-	1348.34	1348.34	91.37	-38.53
C107	100	1992.83	1992.83	1992.83	-	2188.36	-	1348.10	1348.10	82.29	-32.35
C108	100	1938.54	1938.54	1938.54	-	1950.36	-	1348.10	1348.10	76.58	-30.46
C109	100	1786.66	2234.79	1786.66	-	1786.66	-	1342.30	1342.30	60.48	-24.87
C201	100	1298.17	1420.62	1298.17	-	1326.76	-	1172.71	1174.37	99.42	-9.54
C202	100	1327.33	1590.20	1327.33	-	1327.33	-	1172.71	1174.06	99.10	-11.55
C203	100	1432.36	1823.84	1432.36	-	1432.36	-	1160.29	1167.44	105.78	-18.50
C204	100	1727.46	1856.26	1727.46	-	1884.86	-	1162.24	1162.24	119.88	-32.72
C205	100	1296.01	1504.98	1296.01	-	1296.01	-	1167.88	1168.89	103.38	-9.81
C206	100	1444.86	1528.31	1444.86	-	1687.04	-	1163.11	1168.35	97.39	-19.14
C207	100	1376.64	1391.91	1376.64	-	1416.27	-	1166.22	1166.32	91.84	-15.28
C208	100	1453.05	1626.96	1453.05	-	1453.05	-	1166.78	1169.48	96.51	-19.52
Average			10.76		2.43					88.94	-26.63

^a Average of 10 runs

Table 28 Results for the HFFVRPMBTW (Type R)

Inst.	n	BKS	PSO BPYA13		EBBO BB15		HILS-RVRP		Time ^a (s)	Gap ^a (%)	
			Best Sol.	Time (s)	Best Sol.	Time (s)	Best Sol.	Gap (%)			
R101	100	2567.14	2632.13	-	2567.14	2239.53	-	2247.28	-12.76	99.69	-12.46
R102	100	2369.30	2375.41	-	2369.30	1988.11	-	2000.44	-16.09	107.87	-15.57
R103	100	2006.80	2006.80	-	2080.34	1658.71	-	1680.60	-17.35	86.95	-16.25
R104	100	1853.21	1853.21	-	1971.83	1444.76	-	1461.90	-22.04	82.12	-21.12
R105	100	2253.42	2253.42	-	2334.56	1873.51	-	1881.50	-16.86	98.75	-16.50
R106	100	2031.19	2031.19	-	2121.66	1699.09	-	1714.60	-16.35	110.12	-15.59
R107	100	1905.43	1928.90	-	1943.65	1546.98	-	1567.01	-18.81	102.49	-17.76
R108	100	1877.52	1877.52	-	2002.36	1392.10	-	1416.40	-25.85	80.96	-24.56
R109	100	2001.56	2001.56	-	2069.38	1621.53	-	1641.62	-18.99	101.90	-17.98
R110	100	1979.53	1983.98	-	2065.76	1551.83	-	1576.86	-21.61	99.14	-20.34
R111	100	1881.21	1896.70	-	1881.21	1524.55	-	1551.63	-18.96	93.69	-17.52
R112	100	1689.12	1895.77	-	1689.12	1404.26	-	1423.15	-16.86	86.35	-15.75
R201	100	1344.47	1990.47	-	1344.47	1637.34	-	1661.42	21.78	132.41	23.57
R202	100	1922.72	1932.74	-	1941.04	1573.33	-	1596.65	-18.17	157.32	-16.96
R203	100	1736.20	1745.37	-	1910.74	1419.67	-	1430.72	-18.23	141.93	-17.59
R204	100	1522.50	1522.50	-	1876.82	1260.50	-	1275.06	-17.21	154.38	-16.25
R205	100	1753.49	1885.75	-	1753.49	1474.12	-	1505.58	-15.93	138.98	-14.14
R206	100	1758.78	1813.48	-	1792.46	1425.15	-	1444.85	-18.97	153.72	-17.85
R207	100	1650.12	1654.84	-	1806.25	1333.60	-	1353.71	-19.18	141.27	-17.96
R208	100	1536.68	1589.42	-	1737.67	1244.82	-	1256.92	-18.99	155.73	-18.21
R209	100	1729.58	1729.58	-	1798.76	1375.12	-	1415.13	-20.49	136.16	-18.18
R210	100	1754.44	1754.44	-	1868.55	1396.01	-	1430.41	-20.43	125.22	-18.47
R211	100	1615.85	1699.39	-	1641.38	1267.12	-	1282.46	-21.58	117.58	-20.63
Average			3.74		4.58				-16.95	117.60	-15.83

^a Average of 10 runs

Table 29 Results for the HFFVRPMBTW (Type RC)

Inst.	n	BKS		PSO BPYA13		EBBO BB15		HILS-RVRP				
		Best Sol.	Time (s)	Best Sol.	Time (s)	Best Sol.	Time (s)	Best Sol.	Gap (%)	Sol. ^a	Time ^a (s)	Gap ^a (%)
RC101	100	2387.96	-	2957.49	-	2387.96	-	2314.12	-3.09	2329.56	89.90	-2.45
RC102	100	2464.51	-	2464.51	-	2664.55	-	2069.34	-16.03	2076.25	84.58	-15.75
RC103	100	2426.88	-	2426.88	-	2553.62	-	1904.09	-21.54	1927.90	91.91	-20.56
RC104	100	2244.58	-	2244.58	-	2253.76	-	1691.27	-24.65	1708.22	81.14	-23.90
RC105	100	2385.27	-	2711.05	-	2385.27	-	2155.60	-9.63	2171.73	86.08	-8.95
RC106	100	2254.16	-	2495.57	-	2254.16	-	1968.21	-12.69	1987.42	90.58	-11.83
RC107	100	2414.86	-	2420.42	-	2420.42	-	1809.99	-25.05	1825.15	89.60	-24.42
RC108	100	2166.96	-	2381.45	-	2166.96	-	1670.53	-22.91	1694.15	70.62	-21.82
RC201	100	2401.11	-	2401.11	-	2571.17	-	1965.54	-18.14	1998.80	111.73	-16.76
RC202	100	2100.22	-	2251.39	-	2100.22	-	1782.27	-15.14	1800.03	122.01	-14.29
RC203	100	1931.69	-	2022.90	-	1941.74	-	1600.20	-17.16	1620.29	121.84	-16.12
RC204	100	1673.10	-	1827.48	-	1673.10	-	1422.68	-14.97	1428.31	136.25	-14.63
RC205	100	2226.16	-	2274.91	-	2304.21	-	1846.52	-17.05	1869.31	118.44	-16.03
RC206	100	1953.99	-	2123.08	-	1953.99	-	1779.05	-8.95	1793.24	131.24	-8.23
RC207	100	1867.97	-	2084.50	-	1867.97	-	1631.16	-12.68	1636.28	114.72	-12.40
RC208	100	1836.63	-	1836.63	-	1836.63	-	1396.52	-23.96	1406.85	120.99	-23.40
Average				6.37		1.57			-16.48		103.85	-15.72

^a Average of 10 runs

A.12 SDepVRPTW

Detailed results obtained for the SDepVRPTW instances of (Cordeau and Laporte 2001, CL01), compared with those found by the ITS1 of (Cordeau and Maischberger 2012, CM12) and by the HGSADC of (Vidal et al. 2013b, VCGP13) (Table 30).

Table 30 Results for the SDepVRPTW

Inst.	n	BKS		ITSI CM12		HGSADC VCGP13		HILS-RVRP	
		Best Sol.	Time (s)	Best Sol.	Time (s) ^a	Best Sol.	Time (s)	Best Sol.	Time ^b (s)
p01a	48	1655.42	–	1655.42	13.80	1655.42	18.34	1655.42	18.34
p02a	96	2904.13	–	2904.13	42.00	2904.13	134.18	2906.01	134.18
p03a	144	3304.13	–	3317.33	96.00	3304.13	522.57	3317.04	522.57
p04a	192	4427.25	–	4461.13	351.00	4427.25	1516.05	4488.39	1516.05
p05a	240	5626.42	–	5663.32	698.40	5647.76	2505.33	5729.89	2505.33
p06a	288	5627.82	–	5698.93	760.80	5637.48	4601.31	5710.12	4601.31
p07a	72	2166.88	–	2166.88	25.20	2166.88	64.52	2166.88	64.52
p08a	144	3873.40	–	3880.58	141.00	3873.40	502.51	3887.65	502.51
p09a	216	4772.55	–	4818.32	336.00	4777.61	1196.14	4834.70	1196.14
p10a	288	5817.28	–	5908.53	694.80	5858.82	3211.65	5926.17	3211.65
p01b	48	1429.35	–	1429.35	13.20	1429.35	16.58	1429.35	16.58
p02b	96	2479.56	–	2479.56	59.40	2479.56	151.82	2481.20	151.82
p03b	144	2774.30	–	2781.22	136.80	2775.61	499.96	2786.42	499.96
p04b	192	3649.72	–	3674.53	394.20	3649.72	1553.47	3716.93	1553.47
p05b	240	4609.20	–	4613.58	483.60	4611.16	3276.20	4671.35	3276.20
p06b	288	4716.36	–	4788.39	917.40	4729.96	4148.33	4828.04	4148.33
p07b	72	1837.94	–	1837.94	30.60	1837.94	69.52	1837.94	69.52
p08b	144	3144.91	–	3149.77	129.00	3149.77	446.55	3153.64	446.55
p09b	216	3883.94	–	3937.53	534.00	3883.94	1191.17	3944.60	1191.17
p10b	288	4927.95	–	4996.72	721.80	4932.40	2865.22	5070.12	2865.22
Average				0.56	328.95	0.1	1424.57	0.34	1424.57

^a Original time in minutes, here converted to seconds

^b Average of 10 runs

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