LISTA 2 – KERNELIZATION

1. In the Point Line Cover problem, we are given a set of n points on the plane and an integer k, and the goal is to check if there exists a set of k lines on the plane that contain all the input points. Show a kernel for this problem with $O(k^2)$ points.

2. Show a kernel with $O(k^2)$ vertices for the following problem: given a graph G and an integer k, check if G contains a subgraph with exactly k edges, whose all vertices are of odd degree.

3. Show that Feedback Vertex Set admits a polynomial kernel on undirected regular graphs.

4. A split graph is a graph in which the vertices can be partitioned into a clique and an independent set. In the Vertex Disjoint Paths problem, we are given an undirected graph G and k pairs of vertices (s_i, t_i), $i \in \{1, \dots, k\}$, and the objective is to decide whether there exists paths P_i joining s_i to t_i such that these paths are pairwise vertex disjoint.
    Show that Vertex Disjoint Paths admits a polynomial kernel on split graphs (when parameterized by k).

5. A graph G is called cluster graph if every connected component of G is a clique. In the Cluster Editing problem, we are given as input a graph G and an integer k, and the objective is to check whether one can edit (add or delete) at most k edges in G to obtain a cluster graph.
    a) Show that a graph G is a cluster graph if and only if it does not have an induced path on 3 vertices.
    b) Show a kernel for Cluster Editing with $O(k^2)$ vertices.